# PG No. 8 $\quad D_{2\mathrm{h}} \quad mmm \quad$ [ orthorhombic ] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(1,0)}[g](B_{1u})$

** symmetry

$z$

** expression

$$\frac{\sqrt{2}G_x y}{2} - \frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{Q}}_1^{(1,0)}[g](B_{2u})$

** symmetry

$y$

** expression

$$-\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{Q}}_1^{(1,0)}[g](B_{3u})$

** symmetry

$x$

** expression

$$\frac{\sqrt{2}G_y z}{2} - \frac{\sqrt{2}G_z y}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(1,0)}[g](A_g, 1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}G_x yz}{2} - \frac{\sqrt{6}G_y xz}{2}$$

$\vec{\mathbb{Q}}_2^{(1,0)}[g](A_g, 2)$

** symmetry

$$\frac{\sqrt{3}\,(x - y)\,(x + y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x yz}{2} + \frac{\sqrt{2}G_y xz}{2} - \sqrt{2}G_z xy$$

$\vec{\mathbb{Q}}_2^{(1,0)}[g](B_{1g})$

** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{\sqrt{2}G_x xz}{2} + \frac{\sqrt{2}G_y yz}{2} + \frac{\sqrt{2}G_z\,(x - y)\,(x + y)}{2}$$

$\vec{\mathbb{Q}}_2^{(1,0)}[g](B_{2g})$

** symmetry

$\sqrt{3}xz$

** expression

$$\frac{\sqrt{2}G_x xy}{2} - \frac{\sqrt{2}G_y\,(x - z)\,(x + z)}{2} - \frac{\sqrt{2}G_z yz}{2}$$

$\vec{\mathbb{Q}}_2^{(1,0)}[g](B_{3g})$

** symmetry

$\sqrt{3}yz$

** expression

$$\frac{\sqrt{2}G_x(y-z)(y+z)}{2} - \frac{\sqrt{2}G_yxy}{2} + \frac{\sqrt{2}G_zxz}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}_3^{(1,0)}[g](A_u)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}G_xx(y-z)(y+z)}{2} - \frac{\sqrt{5}G_yy(x-z)(x+z)}{2} + \frac{\sqrt{5}G_zz(x-y)(x+y)}{2}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{1u},1)$

** symmetry

$$-\frac{z(3x^2+3y^2-2z^2)}{2}$$

** expression

$$-\frac{\sqrt{3}G_xy(x^2+y^2-4z^2)}{4} + \frac{\sqrt{3}G_yx(x^2+y^2-4z^2)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{1u},2)$

** symmetry

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_xy(x^2-y^2+2z^2)}{4} - \frac{\sqrt{5}G_yx(x^2-y^2-2z^2)}{4} - \sqrt{5}G_zxyz$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{2u},1)$

** symmetry

$$-\frac{y(3x^2-2y^2+3z^2)}{2}$$

** expression

$$\frac{\sqrt{3}G_xz(x^2-4y^2+z^2)}{4} - \frac{\sqrt{3}G_zx(x^2-4y^2+z^2)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{2u},2)$

** symmetry

$$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$$

** expression

$$\frac{\sqrt{5}G_xz(x^2+2y^2-z^2)}{4} - \sqrt{5}G_yxyz - \frac{\sqrt{5}G_zx(x^2-2y^2-z^2)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{3u},1)$

** symmetry

$$\frac{x(2x^2-3y^2-3z^2)}{2}$$

** expression

$$\frac{\sqrt{3}G_yz(4x^2-y^2-z^2)}{4} - \frac{\sqrt{3}G_zy(4x^2-y^2-z^2)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_{3u},2)$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

** expression

$$-\sqrt{5}G_xxyz + \frac{\sqrt{5}G_yz(2x^2+y^2-z^2)}{4} + \frac{\sqrt{5}G_zy(2x^2-y^2+z^2)}{4}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_g,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}G_x yz\left(y-z\right)\left(y+z\right)}{6} + \frac{\sqrt{105}G_y xz\left(x-z\right)\left(x+z\right)}{6} - \frac{\sqrt{105}G_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_g,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{3}G_x yz\left(9x^2 + 2y^2 - 5z^2\right)}{6} + \frac{\sqrt{3}G_y xz\left(2x^2 + 9y^2 - 5z^2\right)}{6} + \frac{7\sqrt{3}G_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_g,3)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{G_x yz\left(3x^2 - 4y^2 + 3z^2\right)}{2} + \frac{G_y xz\left(4x^2 - 3y^2 - 3z^2\right)}{2} - \frac{G_z xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_{1g},1)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{7}G_x xz\left(x^2 - 3y^2\right)}{4} + \frac{\sqrt{7}G_y yz\left(3x^2 - y^2\right)}{4} + \frac{\sqrt{7}G_z\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_{1g},2)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$\frac{G_x xz\left(x^2 + 15y^2 - 6z^2\right)}{4} - \frac{G_y yz\left(15x^2 + y^2 - 6z^2\right)}{4} - \frac{G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_{2g},1)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{7}G_x xy\left(x^2 - 3z^2\right)}{4} + \frac{\sqrt{7}G_y\left(x^2 - 2xz - z^2\right)\left(x^2 + 2xz - z^2\right)}{4} + \frac{\sqrt{7}G_z yz\left(3x^2 - z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_{2g},2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$-\frac{G_x xy\left(x^2 - 6y^2 + 15z^2\right)}{4} + \frac{G_y\left(x-z\right)\left(x+z\right)\left(x^2 - 6y^2 + z^2\right)}{4} + \frac{G_z yz\left(15x^2 - 6y^2 + z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_{3g},1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_x \left(y^2 - 2yz - z^2\right)\left(y^2 + 2yz - z^2\right)}{4} - \frac{\sqrt{7}G_y xy \left(y^2 - 3z^2\right)}{4} + \frac{\sqrt{7}G_z xz \left(3y^2 - z^2\right)}{4}$$

$\bar{\mathbb{Q}}_4^{(1,0)}[g](B_{3g}, 2)$

** symmetry

$$\frac{\sqrt{5}yz \left(6x^2 - y^2 - z^2\right)}{2}$$

** expression

$$\frac{G_x \left(y - z\right)\left(y + z\right)\left(6x^2 - y^2 - z^2\right)}{4} - \frac{G_y xy \left(6x^2 - y^2 - 15z^2\right)}{4} + \frac{G_z xz \left(6x^2 - 15y^2 - z^2\right)}{4}$$