# PG No. 31 $T_{\rm d}$ $\bar{4}3m$ [ cubic ] (axial, internal polar quadrupole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}^{(2,1)}_{1,1}[q](T_1)$, $\vec{\mathbb{G}}^{(2,1)}_{1,2}[q](T_1)$, $\vec{\mathbb{G}}^{(2,1)}_{1,3}[q](T_1)$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$-\frac{3\sqrt{10}Q_{u}yz}{10} - \frac{\sqrt{30}Q_{v}yz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}\left(y - z\right)\left(y + z\right)}{10}$$

$$\frac{3\sqrt{10}Q_{u}xz}{10} - \frac{\sqrt{30}Q_{v}xz}{10} - \frac{\sqrt{30}Q_{xy}yz}{10} + \frac{\sqrt{30}Q_{xz}\left(x - z\right)\left(x + z\right)}{10} + \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$\frac{\sqrt{30}Q_{v}xy}{5} - \frac{\sqrt{30}Q_{xy}\left(x - y\right)\left(x + y\right)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}^{(2,-1)}_{2,1}[q](E)$, $\vec{\mathbb{G}}^{(2,-1)}_{2,2}[q](E)$

** symmetry

$$-\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$-\frac{\sqrt{6}Q_{xy}z}{3} + \frac{\sqrt{6}Q_{xz}y}{6} + \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$\vec{\mathbb{G}}^{(2,1)}_{2,1}[q](E)$, $\vec{\mathbb{G}}^{(2,1)}_{2,2}[q](E)$

** symmetry

$$-\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{5\sqrt{42}Q_{u}xyz}{14} - \frac{\sqrt{14}Q_{xy}z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{28} + \frac{\sqrt{14}Q_{xz}y\left(9x^{2} - y^{2} - 6z^{2}\right)}{28} - \frac{\sqrt{14}Q_{yz}x\left(x^{2} - 9y^{2} + 6z^{2}\right)}{28}$$

$$\frac{5\sqrt{42}Q_{v}xyz}{14} - \frac{5\sqrt{42}Q_{xy}z\left(x - y\right)\left(x + y\right)}{28} - \frac{\sqrt{42}Q_{xz}y\left(x^{2} + y^{2} - 4z^{2}\right)}{28} + \frac{\sqrt{42}Q_{yz}x\left(x^{2} + y^{2} - 4z^{2}\right)}{28}$$

$\vec{\mathbb{G}}^{(2,-1)}_{2,1}[q](T_1)$, $\vec{\mathbb{G}}^{(2,-1)}_{2,2}[q](T_1)$, $\vec{\mathbb{G}}^{(2,-1)}_{2,3}[q](T_1)$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{u}x}{2} + \frac{\sqrt{6}Q_{v}x}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$$-\frac{\sqrt{2}Q_{u}y}{2} + \frac{\sqrt{6}Q_{v}y}{6} - \frac{\sqrt{6}Q_{xy}x}{6} + \frac{\sqrt{6}Q_{yz}z}{6}$$

$$-\frac{\sqrt{6}Q_{v}z}{3} + \frac{\sqrt{6}Q_{xz}x}{6} - \frac{\sqrt{6}Q_{yz}y}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{2,2}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{2,3}^{(2,1)}[q](T_1)$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}Q_{u}x\left(x^{2}+y^{2}-4z^{2}\right)}{28} - \frac{\sqrt{14}Q_{v}x\left(x^{2}-9y^{2}+6z^{2}\right)}{28} - \frac{\sqrt{14}Q_{xy}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14} + \frac{\sqrt{14}Q_{xz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$$\frac{\sqrt{42}Q_{u}y\left(x^{2}+y^{2}-4z^{2}\right)}{28} + \frac{\sqrt{14}Q_{v}y\left(9x^{2}-y^{2}-6z^{2}\right)}{28} - \frac{\sqrt{14}Q_{xy}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14} - \frac{\sqrt{14}Q_{yz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$$\frac{5\sqrt{42}Q_{u}z\left(x-y\right)\left(x+y\right)}{28} - \frac{\sqrt{14}Q_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{28} + \frac{\sqrt{14}Q_{xz}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14} + \frac{\sqrt{14}Q_{yz}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](A_2)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}Q_{u}\left(x-y\right)\left(x+y\right)}{4} + \frac{\sqrt{2}Q_{v}\left(x^{2}+y^{2}-2z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(2,1)}[q](A_2)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}Q_{u}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{6} - \frac{Q_{v}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{6} - \frac{7Q_{xy}xy\left(x-y\right)\left(x+y\right)}{6} + \frac{7Q_{xz}xz\left(x-z\right)\left(x+z\right)}{6} - \frac{7Q_{yz}yz\left(y-z\right)\left(y+z\right)}{6}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](T_1)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](T_1)$, $\vec{\mathbb{G}}_{3,3}^{(2,-1)}[q](T_1)$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$\frac{3\sqrt{10}Q_{u}yz}{20} + \frac{\sqrt{30}Q_{v}yz}{20} + \frac{\sqrt{30}Q_{xy}xz}{5} - \frac{\sqrt{30}Q_{xz}xy}{5} + \frac{\sqrt{30}Q_{yz}\left(y-z\right)\left(y+z\right)}{20}$$

$$-\frac{3\sqrt{10}Q_{u}xz}{20} + \frac{\sqrt{30}Q_{v}xz}{20} - \frac{\sqrt{30}Q_{xy}yz}{5} - \frac{\sqrt{30}Q_{xz}\left(x-z\right)\left(x+z\right)}{20} + \frac{\sqrt{30}Q_{yz}xy}{5}$$

$$-\frac{\sqrt{30}Q_{v}xy}{10} + \frac{\sqrt{30}Q_{xy}\left(x-y\right)\left(x+y\right)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{3,3}^{(2,1)}[q](T_1)$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_{u}yz\left(6x^{2}-y^{2}-z^{2}\right)}{4}-\frac{\sqrt{15}Q_{v}yz\left(6x^{2}-y^{2}-z^{2}\right)}{12}+\frac{\sqrt{15}Q_{xy}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{12}-\frac{\sqrt{15}Q_{xz}xy\left(4x^{2}-3y^{2}-3z^{2}\right)}{12}-\frac{\sqrt{15}Q_{yz}\left(y-z\right)\left(y+z\right)\left(6x^{2}-y^{2}-z^{2}\right)}{12}$$

$$-\frac{\sqrt{5}Q_{u}xz\left(x^{2}-6y^{2}+z^{2}\right)}{4}+\frac{\sqrt{15}Q_{v}xz\left(x^{2}-6y^{2}+z^{2}\right)}{12}+\frac{\sqrt{15}Q_{xy}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{12}-\frac{\sqrt{15}Q_{xz}\left(x-z\right)\left(x+z\right)\left(x^{2}-6y^{2}+z^{2}\right)}{12}-\frac{\sqrt{15}Q_{yz}xy\left(3x^{2}-4y^{2}+3z^{2}\right)}{12}$$

$$-\frac{\sqrt{15}Q_{v}xy\left(x^{2}+y^{2}-6z^{2}\right)}{6}+\frac{\sqrt{15}Q_{xy}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{12}-\frac{\sqrt{15}Q_{xz}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}+\frac{\sqrt{15}Q_{yz}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](T_2)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](T_2)$, $\vec{\mathbb{G}}_{3,3}^{(2,-1)}[q](T_2)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{u}yz}{4}-\frac{3\sqrt{2}Q_{v}yz}{4}+\frac{\sqrt{2}Q_{yz}\left(2x^{2}-y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{6}Q_{u}xz}{4}+\frac{3\sqrt{2}Q_{v}xz}{4}-\frac{\sqrt{2}Q_{xz}\left(x^{2}-2y^{2}+z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}Q_{u}xy}{2}-\frac{\sqrt{2}Q_{xy}\left(x^{2}+y^{2}-2z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](T_2)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](T_2)$, $\vec{\mathbb{G}}_{3,3}^{(2,1)}[q](T_2)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{u}yz\left(12x^{2}-9y^{2}+5z^{2}\right)}{12}-\frac{Q_{v}yz\left(36x^{2}+y^{2}-13z^{2}\right)}{12}+\frac{7Q_{xy}xz\left(2x^{2}-3y^{2}-z^{2}\right)}{12}+\frac{7Q_{xz}xy\left(2x^{2}-y^{2}-3z^{2}\right)}{12}-\frac{Q_{yz}\left(4x^{4}-12x^{2}y^{2}-12x^{2}z^{2}+5y^{4}-18y^{2}z^{2}+5z^{4}\right)}{12}$$

$$-\frac{\sqrt{3}Q_{u}xz\left(9x^{2}-12y^{2}-5z^{2}\right)}{12}+\frac{Q_{v}xz\left(x^{2}+36y^{2}-13z^{2}\right)}{12}-\frac{7Q_{xy}yz\left(3x^{2}-2y^{2}+z^{2}\right)}{12}-\frac{Q_{xz}\left(5x^{4}-12x^{2}y^{2}-18x^{2}z^{2}+4y^{4}-12y^{2}z^{2}+5z^{4}\right)}{12}-\frac{7Q_{yz}xy\left(x^{2}-2y^{2}+3z^{2}\right)}{12}$$

$$\frac{\sqrt{3}Q_{u}xy\left(x^{2}+y^{2}-6z^{2}\right)}{3}+\frac{7Q_{v}xy\left(x-y\right)\left(x+y\right)}{6}-\frac{Q_{xy}\left(5x^{4}-18x^{2}y^{2}-12x^{2}z^{2}+5y^{4}-12y^{2}z^{2}+4z^{4}\right)}{12}-\frac{7Q_{xz}yz\left(3x^{2}+y^{2}-2z^{2}\right)}{12}-\frac{7Q_{yz}xz\left(x^{2}+3y^{2}-2z^{2}\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-1)}[q](A_2)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$\frac{\sqrt{5}Q_{xy}z\left(x-y\right)\left(x+y\right)}{2} - \frac{\sqrt{5}Q_{xz}y\left(x-z\right)\left(x+z\right)}{2} + \frac{\sqrt{5}Q_{yz}x\left(y-z\right)\left(y+z\right)}{2}$$

$\vec{\mathbb{G}}_4^{(2,1)}[q](A_2)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$-\frac{3\sqrt{2310}Q_uxyz\left(x-y\right)\left(x+y\right)}{44} - \frac{3\sqrt{770}Q_vxyz\left(x^2+y^2-2z^2\right)}{44} + \frac{\sqrt{770}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{22} - \frac{\sqrt{770}Q_{xz}y\left(x-z\right)\left(x+z\right)\left(x^2-2y^2+z^2\right)}{22} - \frac{\sqrt{770}Q_{yz}x\left(y-z\right)\left(y+z\right)\left(2x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{6\sqrt{7}Q_uxyz}{7} - \frac{\sqrt{21}Q_{xy}z\left(3x^2+3y^2-2z^2\right)}{14} + \frac{\sqrt{21}Q_{xz}y\left(2x^2-y^2+z^2\right)}{14} - \frac{\sqrt{21}Q_{yz}x\left(x^2-2y^2-z^2\right)}{14}$$

$$-\frac{6\sqrt{7}Q_vxyz}{7} - \frac{\sqrt{7}Q_{xy}z\left(x-y\right)\left(x+y\right)}{14} + \frac{\sqrt{7}Q_{xz}y\left(4x^2-3y^2+5z^2\right)}{14} + \frac{\sqrt{7}Q_{yz}x\left(3x^2-4y^2-5z^2\right)}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$\frac{21\sqrt{22}Q_uxyz\left(x^2+y^2-2z^2\right)}{44} + \frac{21\sqrt{66}Q_vxyz\left(x-y\right)\left(x+y\right)}{44} - \frac{\sqrt{66}Q_{xy}z\left(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4\right)}{44} + \frac{\sqrt{66}Q_{xz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44} + \frac{\sqrt{66}Q_{yz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$$\frac{21\sqrt{66}Q_uxyz\left(x-y\right)\left(x+y\right)}{44} - \frac{21\sqrt{22}Q_vxyz\left(x^2+y^2-2z^2\right)}{44} + \frac{7\sqrt{22}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44} + \frac{\sqrt{22}Q_{xz}y\left(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4\right)}{44} - \frac{\sqrt{22}Q_{yz}x\left(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](T_1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](T_1)$, $\vec{\mathbb{G}}_{4,3}^{(2,-1)}[q](T_1)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{7}Q_u x\left(2x^2-5y^2-z^2\right)}{28}+\frac{\sqrt{21}Q_v x\left(2x^2+3y^2-9z^2\right)}{28}-\frac{\sqrt{21}Q_{xy} y\left(2x^2+y^2-5z^2\right)}{28}+\frac{\sqrt{21}Q_{xz} z\left(2x^2-5y^2+z^2\right)}{28}$$

$$\frac{3\sqrt{7}Q_u y\left(5x^2-2y^2+z^2\right)}{28}+\frac{\sqrt{21}Q_v y\left(3x^2+2y^2-9z^2\right)}{28}+\frac{\sqrt{21}Q_{xy} x\left(x^2+2y^2-5z^2\right)}{28}+\frac{\sqrt{21}Q_{yz} z\left(5x^2-2y^2-z^2\right)}{28}$$

$$\frac{3\sqrt{7}Q_u z\left(x-y\right)\left(x+y\right)}{7}+\frac{\sqrt{21}Q_v z\left(3x^2+3y^2-2z^2\right)}{14}-\frac{\sqrt{21}Q_{xz} x\left(x^2-5y^2+2z^2\right)}{28}-\frac{\sqrt{21}Q_{yz} y\left(5x^2-y^2-2z^2\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](T_1)$, $\vec{\mathbb{G}}_{4,3}^{(2,1)}[q](T_1)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{22}Q_u x\left(2x^4-3x^2y^2-17x^2z^2-5y^4+39y^2z^2+2z^4\right)}{88}-\frac{\sqrt{66}Q_v x\left(2x^4-31x^2y^2+11x^2z^2+9y^4+39y^2z^2-12z^4\right)}{88}$$
$$-\frac{\sqrt{66}Q_{xy} y\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}+\frac{\sqrt{66}Q_{xz} z\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}-\frac{21\sqrt{66}Q_{yz} xyz\left(y-z\right)\left(y+z\right)}{44}$$

$$-\frac{3\sqrt{22}Q_u y\left(5x^4+3x^2y^2-39x^2z^2-2y^4+17y^2z^2-2z^4\right)}{88}-\frac{\sqrt{66}Q_v y\left(9x^4-31x^2y^2+39x^2z^2+2y^4+11y^2z^2-12z^4\right)}{88}$$
$$+\frac{\sqrt{66}Q_{xy} x\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}+\frac{21\sqrt{66}Q_{xz} xyz\left(x-z\right)\left(x+z\right)}{44}-\frac{\sqrt{66}Q_{yz} z\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}$$

$$-\frac{21\sqrt{22}Q_u z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}-\frac{\sqrt{66}Q_v z\left(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4\right)}{88}-\frac{21\sqrt{66}Q_{xy} xyz\left(x-y\right)\left(x+y\right)}{44}$$
$$-\frac{\sqrt{66}Q_{xz} x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{66}Q_{yz} y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](T_2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](T_2)$, $\vec{\mathbb{G}}_{4,3}^{(2,-1)}[q](T_2)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3Q_u x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}Q_v x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}Q_{xy} y\left(y^2-3z^2\right)}{4}-\frac{\sqrt{3}Q_{xz} z\left(3y^2-z^2\right)}{4}+\sqrt{3}Q_{yz}xyz$$

$$\frac{3Q_u y\left(x-z\right)\left(x+z\right)}{4}-\frac{\sqrt{3}Q_v y\left(x-z\right)\left(x+z\right)}{4}+\frac{\sqrt{3}Q_{xy} x\left(x^2-3z^2\right)}{4}+\sqrt{3}Q_{xz}xyz-\frac{\sqrt{3}Q_{yz} z\left(3x^2-z^2\right)}{4}$$

$$-\frac{\sqrt{3}Q_v z\left(x-y\right)\left(x+y\right)}{2}+\sqrt{3}Q_{xy}xyz+\frac{\sqrt{3}Q_{xz} x\left(x^2-3y^2\right)}{4}-\frac{\sqrt{3}Q_{yz} y\left(3x^2-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](T_2)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](T_2)$, $\vec{\mathbb{G}}_{4,3}^{(2,1)}[q](T_2)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{154}Q_{u}x\left(x^{2}y^{2}-x^{2}z^{2}+y^{4}-9y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{462}Q_{v}x\left(x^{2}y^{2}-x^{2}z^{2}-5y^{4}+27y^{2}z^{2}-4z^{4}\right)}{88}$$
$$-\frac{\sqrt{462}Q_{xy}y\left(2x^{2}y^{2}-6x^{2}z^{2}-y^{4}+8y^{2}z^{2}-3z^{4}\right)}{44}+\frac{\sqrt{462}Q_{xz}z\left(6x^{2}y^{2}-2x^{2}z^{2}+3y^{4}-8y^{2}z^{2}+z^{4}\right)}{44}-\frac{\sqrt{462}Q_{yz}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{44}$$

$$-\frac{3\sqrt{154}Q_{u}y\left(x^{4}+x^{2}y^{2}-9x^{2}z^{2}-y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{462}Q_{v}y\left(5x^{4}-x^{2}y^{2}-27x^{2}z^{2}+y^{2}z^{2}+4z^{4}\right)}{88}$$
$$+\frac{\sqrt{462}Q_{xy}x\left(x^{4}-2x^{2}y^{2}-8x^{2}z^{2}+6y^{2}z^{2}+3z^{4}\right)}{44}+\frac{\sqrt{462}Q_{xz}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{44}+\frac{\sqrt{462}Q_{yz}z\left(3x^{4}+6x^{2}y^{2}-8x^{2}z^{2}-2y^{2}z^{2}+z^{4}\right)}{44}$$

$$\frac{9\sqrt{154}Q_{u}z\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{88}-\frac{\sqrt{462}Q_{v}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{462}Q_{xy}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}$$
$$+\frac{\sqrt{462}Q_{xz}x\left(x^{4}-8x^{2}y^{2}-2x^{2}z^{2}+3y^{4}+6y^{2}z^{2}\right)}{44}+\frac{\sqrt{462}Q_{yz}y\left(3x^{4}-8x^{2}y^{2}+6x^{2}z^{2}+y^{4}-2y^{2}z^{2}\right)}{44}$$