PG No. 6 D_2 222 [orthorhombic] (polar, internal polar dipole)

* Harmonics for rank 0

$\vec{\mathbb{Q}}_0^{(1,1)}[q](A)$

** symmetry

1

** expression

$$\frac{\sqrt{3}Q_x x}{3}+\frac{\sqrt{3}Q_y y}{3}+\frac{\sqrt{3}Q_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](B_1)$

** symmetry

z

** expression

Q_z

$\vec{\mathbb{Q}}_1^{(1,1)}[q](B_1)$

** symmetry

z

** expression

$$\frac{3\sqrt{10}Q_x xz}{10}+\frac{3\sqrt{10}Q_y yz}{10}-\frac{\sqrt{10}Q_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](B_2)$

** symmetry

y

** expression

Q_y

$\vec{\mathbb{Q}}_1^{(1,1)}[q](B_2)$

** symmetry

y

** expression

$$\frac{3\sqrt{10}Q_x xy}{10}-\frac{\sqrt{10}Q_y\left(x^2-2y^2+z^2\right)}{10}+\frac{3\sqrt{10}Q_z yz}{10}$$

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](B_3)$

** symmetry

x

** expression

Q_x

$\vec{\mathbb{Q}}_1^{(1,1)}[q](B_3)$

** symmetry

x

** expression

$$\frac{\sqrt{10}Q_x\left(2x^2-y^2-z^2\right)}{10}+\frac{3\sqrt{10}Q_y xy}{10}+\frac{3\sqrt{10}Q_z xz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{6}Q_x x}{6}-\frac{\sqrt{6}Q_y y}{6}+\frac{\sqrt{6}Q_z z}{3}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x x}{2} - \frac{\sqrt{2}Q_y y}{2}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A,1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}Q_x x\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}Q_y y\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}Q_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x x\left(3x^2-7y^2-2z^2\right)}{14} + \frac{\sqrt{7}Q_y y\left(7x^2-3y^2+2z^2\right)}{14} + \frac{5\sqrt{7}Q_z z\,(x-y)\,(x+y)}{14}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](B_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} + \frac{\sqrt{2}Q_y x}{2}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](B_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}Q_x y\left(4x^2-y^2-z^2\right)}{7} - \frac{\sqrt{7}Q_y x\left(x^2-4y^2+z^2\right)}{7} + \frac{5\sqrt{7}Q_z xyz}{7}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](B_2)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](B_2)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{7}Q_x z\left(4x^2-y^2-z^2\right)}{7} + \frac{5\sqrt{7}Q_y xyz}{7} - \frac{\sqrt{7}Q_z x\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](B_3)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_y z}{2} + \frac{\sqrt{2}Q_z y}{2}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](B_3)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{7}Q_x xyz}{7} - \frac{\sqrt{7}Q_y z\left(x^2-4y^2+z^2\right)}{7} - \frac{\sqrt{7}Q_z y\left(x^2+y^2-4z^2\right)}{7}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_3^{(1,-1)}[q](A)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$Q_x yz + Q_y xz + Q_z xy$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](A)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}Q_x yz\left(6x^2-y^2-z^2\right)}{6} - \frac{\sqrt{15}Q_y xz\left(x^2-6y^2+z^2\right)}{6} - \frac{\sqrt{15}Q_z xy\left(x^2+y^2-6z^2\right)}{6}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_1,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xz}{5} - \frac{\sqrt{15}Q_y yz}{5} - \frac{\sqrt{15}Q_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_1,2)$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$Q_x xz - Q_y yz + \frac{Q_z\,(x-y)\,(x+y)}{2}$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_1,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{5Q_x xz\left(3x^2+3y^2-4z^2\right)}{12} - \frac{5Q_y yz\left(3x^2+3y^2-4z^2\right)}{12} + \frac{Q_z\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_1,2)$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x xz\left(5x^2-9y^2-2z^2\right)}{12} + \frac{\sqrt{15}Q_y yz\left(9x^2-5y^2+2z^2\right)}{12} - \frac{\sqrt{15}Q_z\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_2,1)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xy}{5}-\frac{\sqrt{15}Q_y\left(x^2-2y^2+z^2\right)}{10}-\frac{\sqrt{15}Q_z yz}{5}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_2,2)$
** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$-Q_x xy-\frac{Q_y\,(x-z)\,(x+z)}{2}+Q_z yz$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_2,1)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{5Q_x xy\left(3x^2-4y^2+3z^2\right)}{12}+\frac{Q_y\left(3x^4-24x^2y^2+6x^2z^2+8y^4-24y^2z^2+3z^4\right)}{12}-\frac{5Q_z yz\left(3x^2-4y^2+3z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_2,2)$
** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xy\left(5x^2-2y^2-9z^2\right)}{12}+\frac{\sqrt{15}Q_y\,(x-z)\,(x+z)\left(x^2-6y^2+z^2\right)}{12}-\frac{\sqrt{15}Q_z yz\left(9x^2+2y^2-5z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_3,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x\left(2x^2-y^2-z^2\right)}{10}-\frac{\sqrt{15}Q_y xy}{5}-\frac{\sqrt{15}Q_z xz}{5}$$

$\vec{\mathbb{Q}}_3^{(1,-1)}[q](B_3,2)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{Q_x\,(y-z)\,(y+z)}{2}+Q_y xy-Q_z xz$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_3,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{Q_x\left(8x^4-24x^2y^2-24x^2z^2+3y^4+6y^2z^2+3z^4\right)}{12}+\frac{5Q_y xy\left(4x^2-3y^2-3z^2\right)}{12}+\frac{5Q_z xz\left(4x^2-3y^2-3z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(1,1)}[q](B_3,2)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x\,(y-z)\,(y+z)\left(6x^2-y^2-z^2\right)}{12}-\frac{\sqrt{15}Q_y xy\left(2x^2-5y^2+9z^2\right)}{12}+\frac{\sqrt{15}Q_z xz\left(2x^2+9y^2-5z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}Q_x x\left(2x^2-3y^2-3z^2\right)}{6}-\frac{\sqrt{3}Q_y y\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}Q_z z\left(3x^2+3y^2-2z^2\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}Q_x x\left(x^2-6y^2+3z^2\right)}{42}+\frac{\sqrt{105}Q_y y\left(6x^2-y^2-3z^2\right)}{42}-\frac{\sqrt{105}Q_z z\left(3x^2+3y^2-2z^2\right)}{42}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A,3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}Q_x x\left(x^2-3z^2\right)}{14}-\frac{\sqrt{35}Q_y y\left(y^2-3z^2\right)}{14}-\frac{3\sqrt{35}Q_z z\,(x-y)\,(x+y)}{14}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}Q_x x\left(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}Q_y y\left(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4\right)}{66}$$
$$+\frac{\sqrt{1155}Q_z z\left(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4\right)}{66}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}Q_x x\left(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4\right)}{132}-\frac{\sqrt{33}Q_y y\left(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4\right)}{132}$$
$$+\frac{\sqrt{33}Q_z z\left(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4\right)}{132}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A,3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{11}Q_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$+\frac{\sqrt{11}Q_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}+\frac{21\sqrt{11}Q_z z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_1,1)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x y\left(3x^2-y^2\right)}{4}+\frac{\sqrt{5}Q_y x\left(x^2-3y^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_1, 2)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}Q_xy\left(3x^2+y^2-6z^2\right)}{28}-\frac{\sqrt{35}Q_yx\left(x^2+3y^2-6z^2\right)}{28}+\frac{3\sqrt{35}Q_zxyz}{7}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_1, 1)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77}Q_xy\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}-\frac{\sqrt{77}Q_yx\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}+\frac{9\sqrt{77}Q_zxyz\left(x-y\right)\left(x+y\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_1, 2)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}Q_xy\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}Q_yx\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}Q_zxyz\left(x^2+y^2-2z^2\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_2, 1)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_xz\left(3x^2-z^2\right)}{4}-\frac{\sqrt{5}Q_zx\left(x^2-3z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_2, 2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}Q_xz\left(3x^2-6y^2+z^2\right)}{28}+\frac{3\sqrt{35}Q_yxyz}{7}-\frac{\sqrt{35}Q_zx\left(x^2-6y^2+3z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_2, 1)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_xz\left(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4\right)}{22}-\frac{9\sqrt{77}Q_yxyz\left(x-z\right)\left(x+z\right)}{22}+\frac{\sqrt{77}Q_zx\left(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_2, 2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}Q_xz\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{21\sqrt{11}Q_yxyz\left(x^2-2y^2+z^2\right)}{22}+\frac{\sqrt{11}Q_zx\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_3, 1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_y z\left(3y^2-z^2\right)}{4}+\frac{\sqrt{5}Q_z y\left(y^2-3z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](B_3,2)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{35}Q_x xyz}{7}+\frac{\sqrt{35}Q_y z\left(6x^2-3y^2-z^2\right)}{28}+\frac{\sqrt{35}Q_z y\left(6x^2-y^2-3z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_3,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{9\sqrt{77}Q_x xyz\left(y-z\right)\left(y+z\right)}{22}-\frac{\sqrt{77}Q_y z\left(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4\right)}{22}-\frac{\sqrt{77}Q_z y\left(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](B_3,2)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}Q_x xyz\left(2x^2-y^2-z^2\right)}{22}-\frac{\sqrt{11}Q_y z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{\sqrt{11}Q_z y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}$$