PG No. 2 C_{i} $\bar{1}$ [triclinic] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(1,1)}[g](A_u)$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3}+\frac{\sqrt{3}G_y y}{3}+\frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_g,1)$

** symmetry

x

** expression

G_x

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_g,2)$

** symmetry

y

** expression

G_y

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_g,3)$

** symmetry

z

** expression

G_z

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_g,1)$

** symmetry

x

** expression

$$\frac{\sqrt{10}G_x\left(2x^2-y^2-z^2\right)}{10}+\frac{3\sqrt{10}G_y xy}{10}+\frac{3\sqrt{10}G_z xz}{10}$$

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_g,2)$

** symmetry

y

** expression

$$\frac{3\sqrt{10}G_x xy}{10}-\frac{\sqrt{10}G_y\left(x^2-2y^2+z^2\right)}{10}+\frac{3\sqrt{10}G_z yz}{10}$$

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_g,3)$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x xz}{10}+\frac{3\sqrt{10}G_y yz}{10}-\frac{\sqrt{10}G_z\left(x^2+y^2-2z^2\right)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6}-\frac{\sqrt{6}G_y y}{6}+\frac{\sqrt{6}G_z z}{3}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x x}{2} - \frac{\sqrt{2}G_y y}{2}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 3)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}G_y z}{2} + \frac{\sqrt{2}G_z y}{2}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 4)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 5)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_x y}{2} + \frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 1)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_x x\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}G_y y\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}G_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}G_x x\left(3x^2-7y^2-2z^2\right)}{14} + \frac{\sqrt{7}G_y y\left(7x^2-3y^2+2z^2\right)}{14} + \frac{5\sqrt{7}G_z z\,(x-y)\,(x+y)}{14}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 3)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7} - \frac{\sqrt{7}G_y z\left(x^2-4y^2+z^2\right)}{7} - \frac{\sqrt{7}G_z y\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 4)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{7}G_x z\left(4x^2-y^2-z^2\right)}{7} + \frac{5\sqrt{7}G_y xyz}{7} - \frac{\sqrt{7}G_z x\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 5)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_x y\left(4x^2 - y^2 - z^2\right)}{7} - \frac{\sqrt{7}G_y x\left(x^2 - 4y^2 + z^2\right)}{7} + \frac{5\sqrt{7}G_z xyz}{7}$$

* Harmonics for rank 3
$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_x yz + G_y xz + G_z xy$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 2)$
** symmetry

$$\frac{x\left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(2x^2 - y^2 - z^2\right)}{10} - \frac{\sqrt{15}G_y xy}{5} - \frac{\sqrt{15}G_z xz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 3)$
** symmetry

$$-\frac{y\left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xy}{5} - \frac{\sqrt{15}G_y\left(x^2 - 2y^2 + z^2\right)}{10} - \frac{\sqrt{15}G_z yz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 4)$
** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xz}{5} - \frac{\sqrt{15}G_y yz}{5} - \frac{\sqrt{15}G_z\left(x^2 + y^2 - 2z^2\right)}{10}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 5)$
** symmetry

$$\frac{\sqrt{15}x\left(y - z\right)\left(y + z\right)}{2}$$

** expression

$$\frac{G_x\left(y - z\right)\left(y + z\right)}{2} + G_y xy - G_z xz$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 6)$
** symmetry

$$-\frac{\sqrt{15}y\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$-G_x xy - \frac{G_y\left(x - z\right)\left(x + z\right)}{2} + G_z yz$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 7)$
** symmetry

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$G_x xz - G_y yz + \frac{G_z \left(x - y\right) \left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 1)$
** symmetry

$$\sqrt{15} xyz$$

** expression

$$\frac{\sqrt{15} G_x yz \left(6x^2 - y^2 - z^2\right)}{6} - \frac{\sqrt{15} G_y xz \left(x^2 - 6y^2 + z^2\right)}{6} - \frac{\sqrt{15} G_z xy \left(x^2 + y^2 - 6z^2\right)}{6}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 2)$
** symmetry

$$\frac{x \left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{G_x \left(8x^4 - 24x^2y^2 - 24x^2z^2 + 3y^4 + 6y^2z^2 + 3z^4\right)}{12} + \frac{5 G_y xy \left(4x^2 - 3y^2 - 3z^2\right)}{12} + \frac{5 G_z xz \left(4x^2 - 3y^2 - 3z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 3)$
** symmetry

$$-\frac{y \left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$-\frac{5 G_x xy \left(3x^2 - 4y^2 + 3z^2\right)}{12} + \frac{G_y \left(3x^4 - 24x^2y^2 + 6x^2z^2 + 8y^4 - 24y^2z^2 + 3z^4\right)}{12} - \frac{5 G_z yz \left(3x^2 - 4y^2 + 3z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 4)$
** symmetry

$$-\frac{z \left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{5 G_x xz \left(3x^2 + 3y^2 - 4z^2\right)}{12} - \frac{5 G_y yz \left(3x^2 + 3y^2 - 4z^2\right)}{12} + \frac{G_z \left(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 5)$
** symmetry

$$\frac{\sqrt{15} x \left(y - z\right) \left(y + z\right)}{2}$$

** expression

$$\frac{\sqrt{15} G_x \left(y - z\right) \left(y + z\right) \left(6x^2 - y^2 - z^2\right)}{12} - \frac{\sqrt{15} G_y xy \left(2x^2 - 5y^2 + 9z^2\right)}{12} + \frac{\sqrt{15} G_z xz \left(2x^2 + 9y^2 - 5z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 6)$
** symmetry

$$-\frac{\sqrt{15} y \left(x - z\right) \left(x + z\right)}{2}$$

** expression

$$-\frac{\sqrt{15} G_x xy \left(5x^2 - 2y^2 - 9z^2\right)}{12} + \frac{\sqrt{15} G_y \left(x - z\right) \left(x + z\right) \left(x^2 - 6y^2 + z^2\right)}{12} - \frac{\sqrt{15} G_z yz \left(9x^2 + 2y^2 - 5z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 7)$
** symmetry

$$\frac{\sqrt{15} z \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{15} G_x xz \left(5x^2 - 9y^2 - 2z^2\right)}{12} + \frac{\sqrt{15} G_y yz \left(9x^2 - 5y^2 + 2z^2\right)}{12} - \frac{\sqrt{15} G_z \left(x - y\right) \left(x + y\right) \left(x^2 + y^2 - 6z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}G_x x\left(2x^2 - 3y^2 - 3z^2\right)}{6} - \frac{\sqrt{3}G_y y\left(3x^2 - 2y^2 + 3z^2\right)}{6} - \frac{\sqrt{3}G_z z\left(3x^2 + 3y^2 - 2z^2\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}G_x x\left(x^2 - 6y^2 + 3z^2\right)}{42} + \frac{\sqrt{105}G_y y\left(6x^2 - y^2 - 3z^2\right)}{42} - \frac{\sqrt{105}G_z z\left(3x^2 + 3y^2 - 2z^2\right)}{42}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 3)$

** symmetry

$$\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}G_x x\left(x^2 - 3z^2\right)}{14} - \frac{\sqrt{35}G_y y\left(y^2 - 3z^2\right)}{14} - \frac{3\sqrt{35}G_z z\left(x - y\right)\left(x + y\right)}{14}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 4)$

** symmetry

$$\frac{\sqrt{35}yz\left(y - z\right)\left(y + z\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_y z\left(3y^2 - z^2\right)}{4} + \frac{\sqrt{5}G_z y\left(y^2 - 3z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 5)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$-\frac{\sqrt{5}G_x z\left(3x^2 - z^2\right)}{4} - \frac{\sqrt{5}G_z x\left(x^2 - 3z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 6)$

** symmetry

$$\frac{\sqrt{35}xy\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y\left(3x^2 - y^2\right)}{4} + \frac{\sqrt{5}G_y x\left(x^2 - 3y^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 7)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2 - y^2 - z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{35}G_x xyz}{7} + \frac{\sqrt{35}G_y z\left(6x^2 - 3y^2 - z^2\right)}{28} + \frac{\sqrt{35}G_z y\left(6x^2 - y^2 - 3z^2\right)}{28}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u, 8)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x z\left(3x^2-6y^2+z^2\right)}{28}+\frac{3\sqrt{35}G_y xyz}{7}-\frac{\sqrt{35}G_z x\left(x^2-6y^2+3z^2\right)}{28}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,9)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x y\left(3x^2+y^2-6z^2\right)}{28}-\frac{\sqrt{35}G_y x\left(x^2+3y^2-6z^2\right)}{28}+\frac{3\sqrt{35}G_z xyz}{7}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}G_x x\left(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}G_y y\left(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4\right)}{66}$$
$$+\frac{\sqrt{1155}G_z z\left(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4\right)}{66}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}G_x x\left(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4\right)}{132}-\frac{\sqrt{33}G_y y\left(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4\right)}{132}$$
$$+\frac{\sqrt{33}G_z z\left(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4\right)}{132}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,3)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{11}G_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$+\frac{\sqrt{11}G_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}+\frac{21\sqrt{11}G_z z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,4)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{9\sqrt{77}G_x xyz\left(y-z\right)\left(y+z\right)}{22}-\frac{\sqrt{77}G_y z\left(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4\right)}{22}-\frac{\sqrt{77}G_z y\left(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,5)$
** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{77}G_x z\left(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4\right)}{22}-\frac{9\sqrt{77}G_y xyz\left(x-z\right)\left(x+z\right)}{22}+\frac{\sqrt{77}G_z x\left(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,6)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77}G_x y\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}-\frac{\sqrt{77}G_y x\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}+\frac{9\sqrt{77}G_z xyz\,(x-y)\,(x+y)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,7)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}G_x xyz\left(2x^2-y^2-z^2\right)}{22}-\frac{\sqrt{11}G_y z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{\sqrt{11}G_z y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,8)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{21\sqrt{11}G_y xyz\left(x^2-2y^2+z^2\right)}{22}+\frac{\sqrt{11}G_z x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,9)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}G_y x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}G_z xyz\left(x^2+y^2-2z^2\right)}{22}$$