PG No. 28 T 23 [cubic] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

  $\vec{\mathbb{G}}^{(1,0)}_{1,1}[q](T)$, $\vec{\mathbb{G}}^{(1,0)}_{1,2}[q](T)$, $\vec{\mathbb{G}}^{(1,0)}_{1,3}[q](T)$

  ** symmetry

  $$x$$

  $$y$$

  $$z$$

  ** expression

  $$\frac{\sqrt{2}Q_{y}z}{2} - \frac{\sqrt{2}Q_{z}y}{2}$$

  $$- \frac{\sqrt{2}Q_{x}z}{2} + \frac{\sqrt{2}Q_{z}x}{2}$$

  $$\frac{\sqrt{2}Q_{x}y}{2} - \frac{\sqrt{2}Q_{y}x}{2}$$

* Harmonics for rank 2

  $\vec{\mathbb{G}}^{(1,0)}_{2,1}[q](E)$, $\vec{\mathbb{G}}^{(1,0)}_{2,2}[q](E)$

  ** symmetry

  $$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

  $$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

  ** expression

  $$\frac{\sqrt{6}Q_{x}yz}{2} - \frac{\sqrt{6}Q_{y}xz}{2}$$

  $$\frac{\sqrt{2}Q_{x}yz}{2} + \frac{\sqrt{2}Q_{y}xz}{2} - \sqrt{2}Q_{z}xy$$

  $\vec{\mathbb{G}}^{(1,0)}_{2,1}[q](T)$, $\vec{\mathbb{G}}^{(1,0)}_{2,2}[q](T)$, $\vec{\mathbb{G}}^{(1,0)}_{2,3}[q](T)$

  ** symmetry

  $$\sqrt{3}yz$$

  $$\sqrt{3}xz$$

  $$\sqrt{3}xy$$

  ** expression

  $$\frac{\sqrt{2}Q_{x} \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{2}Q_{y}xy}{2} + \frac{\sqrt{2}Q_{z}xz}{2}$$

  $$\frac{\sqrt{2}Q_{x}xy}{2} - \frac{\sqrt{2}Q_{y} \left(x - z\right) \left(x + z\right)}{2} - \frac{\sqrt{2}Q_{z}yz}{2}$$

  $$- \frac{\sqrt{2}Q_{x}xz}{2} + \frac{\sqrt{2}Q_{y}yz}{2} + \frac{\sqrt{2}Q_{z} \left(x - y\right) \left(x + y\right)}{2}$$

* Harmonics for rank 3

  $\vec{\mathbb{G}}^{(1,0)}_{3}[q](A)$

  ** symmetry

  $$\sqrt{15}xyz$$

  ** expression

  $$\frac{\sqrt{5}Q_{x}x \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{5}Q_{y}y \left(x - z\right) \left(x + z\right)}{2} + \frac{\sqrt{5}Q_{z}z \left(x - y\right) \left(x + y\right)}{2}$$

  $\vec{\mathbb{G}}^{(1,0)}_{3,1}[q](T,1)$, $\vec{\mathbb{G}}^{(1,0)}_{3,2}[q](T,1)$, $\vec{\mathbb{G}}^{(1,0)}_{3,3}[q](T,1)$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{y}z\left(4x^{2}-y^{2}-z^{2}\right)}{4}-\frac{\sqrt{3}Q_{z}y\left(4x^{2}-y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{3}Q_{x}z\left(x^{2}-4y^{2}+z^{2}\right)}{4}-\frac{\sqrt{3}Q_{z}x\left(x^{2}-4y^{2}+z^{2}\right)}{4}$$

$$-\frac{\sqrt{3}Q_{x}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{3}Q_{y}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](T,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](T,2)$, $\vec{\mathbb{G}}_{3,3}^{(1,0)}[q](T,2)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\sqrt{5}Q_{x}xyz+\frac{\sqrt{5}Q_{y}z\left(2x^{2}+y^{2}-z^{2}\right)}{4}+\frac{\sqrt{5}Q_{z}y\left(2x^{2}-y^{2}+z^{2}\right)}{4}$$

$$\frac{\sqrt{5}Q_{x}z\left(x^{2}+2y^{2}-z^{2}\right)}{4}-\sqrt{5}Q_{y}xyz-\frac{\sqrt{5}Q_{z}x\left(x^{2}-2y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{5}Q_{x}y\left(x^{2}-y^{2}+2z^{2}\right)}{4}-\frac{\sqrt{5}Q_{y}x\left(x^{2}-y^{2}-2z^{2}\right)}{4}-\sqrt{5}Q_{z}xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A)$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_{x}yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}Q_{y}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}Q_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{3}Q_{x}yz\left(9x^{2}+2y^{2}-5z^{2}\right)}{6}+\frac{\sqrt{3}Q_{y}xz\left(2x^{2}+9y^{2}-5z^{2}\right)}{6}+\frac{7\sqrt{3}Q_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$$-\frac{Q_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}+\frac{Q_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}-\frac{Q_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](T,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](T,1)$, $\vec{\mathbb{G}}_{4,3}^{(1,0)}[q](T,1)$

** symmetry

$$\frac{\sqrt{35} y z \left(y - z\right) \left(y + z\right)}{2}$$

$$- \frac{\sqrt{35} x z \left(x - z\right) \left(x + z\right)}{2}$$

$$\frac{\sqrt{35} x y \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{7} Q_{x} \left(y^{2} - 2 y z - z^{2}\right) \left(y^{2} + 2 y z - z^{2}\right)}{4} - \frac{\sqrt{7} Q_{y} x y \left(y^{2} - 3 z^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} x z \left(3 y^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7} Q_{x} x y \left(x^{2} - 3 z^{2}\right)}{4} + \frac{\sqrt{7} Q_{y} \left(x^{2} - 2 x z - z^{2}\right) \left(x^{2} + 2 x z - z^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} y z \left(3 x^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7} Q_{x} x z \left(x^{2} - 3 y^{2}\right)}{4} + \frac{\sqrt{7} Q_{y} y z \left(3 x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} \left(x^{2} - 2 x y - y^{2}\right) \left(x^{2} + 2 x y - y^{2}\right)}{4}$$

$\vec{\mathbb{G}}^{(1,0)}_{4,1}[q](T,2)$, $\vec{\mathbb{G}}^{(1,0)}_{4,2}[q](T,2)$, $\vec{\mathbb{G}}^{(1,0)}_{4,3}[q](T,2)$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x z \left(x^{2} - 6 y^{2} + z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x} \left(y - z\right) \left(y + z\right) \left(6 x^{2} - y^{2} - z^{2}\right)}{4} - \frac{Q_{y} x y \left(6 x^{2} - y^{2} - 15 z^{2}\right)}{4} + \frac{Q_{z} x z \left(6 x^{2} - 15 y^{2} - z^{2}\right)}{4}$$

$$- \frac{Q_{x} x y \left(x^{2} - 6 y^{2} + 15 z^{2}\right)}{4} + \frac{Q_{y} \left(x - z\right) \left(x + z\right) \left(x^{2} - 6 y^{2} + z^{2}\right)}{4} + \frac{Q_{z} y z \left(15 x^{2} - 6 y^{2} + z^{2}\right)}{4}$$

$$\frac{Q_{x} x z \left(x^{2} + 15 y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{y} y z \left(15 x^{2} + y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{z} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{4}$$