PG No. 15 D_{4h} $4/mmm$ [tetragonal] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(1,1)}[g](A_{1u})$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_{2g})$

** symmetry

z

** expression

G_z

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_{2g})$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x xz}{10} + \frac{3\sqrt{10}G_y yz}{10} - \frac{\sqrt{10}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](E_g)$

** symmetry

x

$-y$

** expression

G_x

$-G_y$

$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(1,1)}[g](E_g)$

** symmetry

x

$-y$

** expression

$$\frac{\sqrt{10}G_x\left(2x^2-y^2-z^2\right)}{10} + \frac{3\sqrt{10}G_y xy}{10} + \frac{3\sqrt{10}G_z xz}{10}$$

$$-\frac{3\sqrt{10}G_x xy}{10} + \frac{\sqrt{10}G_y\left(x^2-2y^2+z^2\right)}{10} - \frac{3\sqrt{10}G_z yz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_{1u})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_{1u})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_x x\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_y y\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_{1u})$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x x}{2}-\frac{\sqrt{2}G_y y}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_{1u})$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}G_x x\left(3x^2-7y^2-2z^2\right)}{14}+\frac{\sqrt{7}G_y y\left(7x^2-3y^2+2z^2\right)}{14}+\frac{5\sqrt{7}G_z z\,(x-y)\,(x+y)}{14}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_{2u})$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_x y}{2}+\frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_{2u})$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_x y\left(4x^2-y^2-z^2\right)}{7}-\frac{\sqrt{7}G_y x\left(x^2-4y^2+z^2\right)}{7}+\frac{5\sqrt{7}G_z xyz}{7}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_u)$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_u)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_y z}{2}+\frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2}-\frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_u)$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_u)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7}-\frac{\sqrt{7}G_y z\left(x^2-4y^2+z^2\right)}{7}-\frac{\sqrt{7}G_z y\left(x^2+y^2-4z^2\right)}{7}$$

$$-\frac{\sqrt{7}G_x z\left(4x^2-y^2-z^2\right)}{7}-\frac{5\sqrt{7}G_y xyz}{7}+\frac{\sqrt{7}G_z x\left(x^2+y^2-4z^2\right)}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_{2g})$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xz}{5}-\frac{\sqrt{15}G_y yz}{5}-\frac{\sqrt{15}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_{2g})$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{5G_x xz\left(3x^2+3y^2-4z^2\right)}{12}-\frac{5G_y yz\left(3x^2+3y^2-4z^2\right)}{12}+\frac{G_z\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_{1g})$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_x yz+G_y xz+G_z xy$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_{1g})$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}G_x yz\left(6x^2-y^2-z^2\right)}{6}-\frac{\sqrt{15}G_y xz\left(x^2-6y^2+z^2\right)}{6}-\frac{\sqrt{15}G_z xy\left(x^2+y^2-6z^2\right)}{6}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_{2g})$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$G_x xz-G_y yz+\frac{G_z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_{2g})$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x xz\left(5x^2-9y^2-2z^2\right)}{12}+\frac{\sqrt{15}G_y yz\left(9x^2-5y^2+2z^2\right)}{12}-\frac{\sqrt{15}G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_g,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(2x^2-y^2-z^2\right)}{10}-\frac{\sqrt{15}G_y xy}{5}-\frac{\sqrt{15}G_z xz}{5}$$

$$\frac{\sqrt{15}G_x xy}{5}+\frac{\sqrt{15}G_y\left(x^2-2y^2+z^2\right)}{10}+\frac{\sqrt{15}G_z yz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_g, 2)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_g, 2)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{G_x\,(y-z)\,(y+z)}{2} + G_y xy - G_z xz$$

$$-G_x xy - \frac{G_y\,(x-z)\,(x+z)}{2} + G_z yz$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_g, 1)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_g, 1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{G_x\left(8x^4-24x^2y^2-24x^2z^2+3y^4+6y^2z^2+3z^4\right)}{12} + \frac{5G_y xy\left(4x^2-3y^2-3z^2\right)}{12} + \frac{5G_z xz\left(4x^2-3y^2-3z^2\right)}{12}$$

$$\frac{5G_x xy\left(3x^2-4y^2+3z^2\right)}{12} - \frac{G_y\left(3x^4-24x^2y^2+6x^2z^2+8y^4-24y^2z^2+3z^4\right)}{12} + \frac{5G_z yz\left(3x^2-4y^2+3z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_g, 2)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_g, 2)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\,(y-z)\,(y+z)\left(6x^2-y^2-z^2\right)}{12} - \frac{\sqrt{15}G_y xy\left(2x^2-5y^2+9z^2\right)}{12} + \frac{\sqrt{15}G_z xz\left(2x^2+9y^2-5z^2\right)}{12}$$

$$-\frac{\sqrt{15}G_x xy\left(5x^2-2y^2-9z^2\right)}{12} + \frac{\sqrt{15}G_y\,(x-z)\,(x+z)\left(x^2-6y^2+z^2\right)}{12} - \frac{\sqrt{15}G_z yz\left(9x^2+2y^2-5z^2\right)}{12}$$

* Harmonics for rank 4
$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_{1u}, 1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}G_x x\left(2x^2-3y^2-3z^2\right)}{6} - \frac{\sqrt{3}G_y y\left(3x^2-2y^2+3z^2\right)}{6} - \frac{\sqrt{3}G_z z\left(3x^2+3y^2-2z^2\right)}{6}$$

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_{1u}, 2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}G_x x\left(x^2-6y^2+3z^2\right)}{42} + \frac{\sqrt{105}G_y y\left(6x^2-y^2-3z^2\right)}{42} - \frac{\sqrt{105}G_z z\left(3x^2+3y^2-2z^2\right)}{42}$$

$\vec{\mathbb{G}}_{4}^{(1,1)}[g](A_{1u}, 1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}G_x x\left(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}G_y y\left(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4\right)}{66}$$
$$+\frac{\sqrt{1155}G_z z\left(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4\right)}{66}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_{1u},2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}G_x x\left(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4\right)}{132}-\frac{\sqrt{33}G_y y\left(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4\right)}{132}$$
$$+\frac{\sqrt{33}G_z z\left(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4\right)}{132}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_{2u})$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y\left(3x^2-y^2\right)}{4}+\frac{\sqrt{5}G_y x\left(x^2-3y^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_{2u})$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77}G_x y\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}-\frac{\sqrt{77}G_y x\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}+\frac{9\sqrt{77}G_z xyz\left(x-y\right)\left(x+y\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_{1u})$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}G_x x\left(x^2-3z^2\right)}{14}-\frac{\sqrt{35}G_y y\left(y^2-3z^2\right)}{14}-\frac{3\sqrt{35}G_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_{1u})$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{11}G_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$+\frac{\sqrt{11}G_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}+\frac{21\sqrt{11}G_z z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_{2u})$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x y\left(3x^2+y^2-6z^2\right)}{28}-\frac{\sqrt{35}G_y x\left(x^2+3y^2-6z^2\right)}{28}+\frac{3\sqrt{35}G_z xyz}{7}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_{2u})$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}G_y x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}G_z xyz\left(x^2+y^2-2z^2\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u,1)$

** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{\sqrt{5}G_y z\left(3y^2-z^2\right)}{4}+\frac{\sqrt{5}G_z y\left(y^2-3z^2\right)}{4}$$

$$-\frac{\sqrt{5}G_x z\left(3x^2-z^2\right)}{4}-\frac{\sqrt{5}G_z x\left(x^2-3z^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{35}G_x xyz}{7}+\frac{\sqrt{35}G_y z\left(6x^2-3y^2-z^2\right)}{28}+\frac{\sqrt{35}G_z y\left(6x^2-y^2-3z^2\right)}{28}$$

$$\frac{\sqrt{35}G_x z\left(3x^2-6y^2+z^2\right)}{28}-\frac{3\sqrt{35}G_y xyz}{7}+\frac{\sqrt{35}G_z x\left(x^2-6y^2+3z^2\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u,1)$

** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{9\sqrt{77}G_x xyz\,(y-z)\,(y+z)}{22}-\frac{\sqrt{77}G_y z\left(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4\right)}{22}-\frac{\sqrt{77}G_z y\left(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4\right)}{22}$$

$$-\frac{\sqrt{77}G_x z\left(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4\right)}{22}-\frac{9\sqrt{77}G_y xyz\,(x-z)\,(x+z)}{22}+\frac{\sqrt{77}G_z x\left(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}G_x xyz\left(2x^2-y^2-z^2\right)}{22}-\frac{\sqrt{11}G_y z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{\sqrt{11}G_z y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}$$

$$\frac{\sqrt{11}G_x z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}+\frac{21\sqrt{11}G_y xyz\left(x^2-2y^2+z^2\right)}{22}-\frac{\sqrt{11}G_z x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}$$