

\* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 1   | $\{1 0\}$   | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 2   | $\{4_{100}^+ \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 3   | $\{4_{100}^- \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 4   | $\{4_{010}^+ \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 5   | $\{4_{010}^- \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 6   | $\{4_{001}^+ \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 7   | $\{4_{001}^- \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 8   | $\{2_{100} \frac{1}{2}\frac{1}{2}0\}$             | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | 1  |
| 9   | $\{2_{010} 0\frac{1}{2}\frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 10  | $\{2_{001} \frac{1}{2}0\frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 11  | $\{2_{110} \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |
| 12  | $\{2_{1-10} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 13  | $\{2_{011} \frac{3}{4}\frac{1}{4}\frac{3}{4}\}$   | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | 1  |

continued ...

Table 1

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 14  | $\{2_{01-1} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 15  | $\{2_{101} \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$   | $\begin{bmatrix} 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | 1  |
| 16  | $\{2_{-101} \frac{1}{4}\frac{1}{4}\frac{1}{4}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | 1  |
| 17  | $\{3_{111}^+ 0\}$                                 | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 18  | $\{3_{111}^- 0\}$                                 | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$                                  | 1   | 1  |
| 19  | $\{3_{1-1-1}^- 0\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 20  | $\{3_{1-1-1}^+ \frac{1}{2}0\frac{1}{2}\}$         | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 21  | $\{3_{-11-1}^- \frac{1}{2}0\frac{1}{2}\}$         | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 22  | $\{3_{-11-1}^+ \frac{1}{2}\frac{1}{2}0\}$         | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \end{bmatrix}$            | 1   | 1  |
| 23  | $\{3_{-1-11}^- \frac{1}{2}\frac{1}{2}0\}$         | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \end{bmatrix}$            | 1   | 1  |
| 24  | $\{3_{-1-11}^+ 0\frac{1}{2}\frac{1}{2}\}$         | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$            | 1   | 1  |
| 25  | $\{1' 0\}$  | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$                                  | 1   | -1 |
| 26  | $\{4_{100}^+ \frac{3}{4}\frac{3}{4}\frac{1}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 27  | $\{4_{100}^- \frac{1}{4}\frac{3}{4}\frac{3}{4}\}$ | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |

continued ...

Table 1

| No. | tag   | matrix (polar)   | det | TR |
|-----|---|--|-----|----|
| 28  | $\{4_{010}^+   \frac{1}{4} \frac{3}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |
| 29  | $\{4_{010}^-   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{3}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |
| 30  | $\{4_{001}^+   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$ | $\begin{bmatrix} 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |
| 31  | $\{4_{001}^-   \frac{3}{4} \frac{3}{4} \frac{1}{4}\}$ | $\begin{bmatrix} 0 & 1 & 0 & \frac{3}{4} \\ -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 32  | $\{2_{100}   \frac{1}{2} \frac{1}{2} 0\}$             | $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \end{bmatrix}$            | 1   | -1 |
| 33  | $\{2_{010}   0 \frac{1}{2} \frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \end{bmatrix}$            | 1   | -1 |
| 34  | $\{2_{001}   \frac{1}{2} 0 \frac{1}{2}\}$             | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix}$            | 1   | -1 |
| 35  | $\{2_{110}   \frac{1}{4} \frac{3}{4} \frac{3}{4}\}$   | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{4} \\ 1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & -1 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |
| 36  | $\{2_{1-10}   \frac{1}{4} \frac{1}{4} \frac{1}{4}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 37  | $\{2_{011}   \frac{3}{4} \frac{1}{4} \frac{3}{4}\}$   | $\begin{bmatrix} -1 & 0 & 0 & \frac{3}{4} \\ 0 & 0 & 1 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{3}{4} \end{bmatrix}$   | 1   | -1 |
| 38  | $\{2_{01-1}   \frac{1}{4} \frac{1}{4} \frac{1}{4}\}$  | $\begin{bmatrix} -1 & 0 & 0 & \frac{1}{4} \\ 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 39  | $\{2_{101}   \frac{3}{4} \frac{3}{4} \frac{1}{4}\}$   | $\begin{bmatrix} 0 & 0 & 1 & \frac{3}{4} \\ 0 & -1 & 0 & \frac{3}{4} \\ 1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$   | 1   | -1 |
| 40  | $\{2_{-101}   \frac{1}{4} \frac{1}{4} \frac{1}{4}\}$  | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{4} \\ 0 & -1 & 0 & \frac{1}{4} \\ -1 & 0 & 0 & \frac{1}{4} \end{bmatrix}$ | 1   | -1 |
| 41  | $\{3_{111}^+   0\}$                                   | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$                                  | 1   | -1 |

continued ...

Table 1

| No. | tag  | matrix (polar)  | det | TR |
|-----|--|---|-----|----|
| 42  | $\{3_{-111}^- 0\}$                         | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$                       | 1   | -1 |
| 43  | $\{3_{-1-1-1}^- 0\frac{1}{2}\frac{1}{2}\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 44  | $\{3_{-1-1-1}^+ \frac{1}{2}0\frac{1}{2}\}$ | $\begin{bmatrix} 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 45  | $\{3_{-11-1}^- \frac{1}{2}0\frac{1}{2}\}$  | $\begin{bmatrix} 0 & -1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |
| 46  | $\{3_{-11-1}^+ \frac{1}{2}\frac{1}{2}0\}$  | $\begin{bmatrix} 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1   | -1 |
| 47  | $\{3_{-1-11}^- \frac{1}{2}\frac{1}{2}0\}$  | $\begin{bmatrix} 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & -1 & \frac{1}{2} \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1   | -1 |
| 48  | $\{3_{-1-11}^+ 0\frac{1}{2}\frac{1}{2}\}$  | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & \frac{1}{2} \end{bmatrix}$ | 1   | -1 |