PG No. 5 $C_{2\mathrm{h}}$ $2/m$ (b-axis setting) [monoclinic] (axial, internal axial octupole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(3,3)}[g](A_u)$

** symmetry

1

** expression

$$\frac{\sqrt{70}G_1y\left(3x^2-y^2\right)}{28}+\frac{\sqrt{70}G_2x\left(x^2-3y^2\right)}{28}+\frac{\sqrt{105}G_3xyz}{7}-\frac{\sqrt{42}G_{3x}x\left(x^2+y^2-4z^2\right)}{28}$$
$$-\frac{\sqrt{42}G_{3y}y\left(x^2+y^2-4z^2\right)}{28}-\frac{\sqrt{7}G_{az}z\left(3x^2+3y^2-2z^2\right)}{14}+\frac{\sqrt{105}G_{bz}z\left(x-y\right)\left(x+y\right)}{14}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(3,1)}[g](A_g)$

** symmetry

y

** expression

$$\frac{3\sqrt{14}G_1\left(x-y\right)\left(x+y\right)}{28}-\frac{3\sqrt{14}G_2xy}{14}+\frac{\sqrt{21}G_3xz}{7}-\frac{\sqrt{210}G_{3x}xy}{70}-\frac{\sqrt{210}G_{3y}\left(x^2+3y^2-4z^2\right)}{140}-\frac{3\sqrt{35}G_{az}yz}{35}-\frac{\sqrt{21}G_{bz}yz}{7}$$

$\vec{\mathbb{G}}_1^{(3,3)}[g](A_g)$

** symmetry

y

** expression

$$-\frac{\sqrt{210}G_1\left(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2\right)}{168}+\frac{\sqrt{210}G_2xy\left(13x^2-15y^2+6z^2\right)}{168}-\frac{\sqrt{35}G_3xz\left(x^2-6y^2+z^2\right)}{14}-\frac{5\sqrt{14}G_{3x}xy\left(x^2+y^2-6z^2\right)}{56}$$
$$+\frac{\sqrt{14}G_{3y}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{56}-\frac{5\sqrt{21}G_{az}yz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{\sqrt{35}G_{bz}yz\left(9x^2-5y^2+2z^2\right)}{28}$$

$\vec{\mathbb{G}}_1^{(3,1)}[g](B_g,1)$

** symmetry

x

** expression

$$\frac{3\sqrt{14}G_1xy}{14}+\frac{3\sqrt{14}G_2\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{21}G_3yz}{7}-\frac{\sqrt{210}G_{3x}\left(3x^2+y^2-4z^2\right)}{140}-\frac{\sqrt{210}G_{3y}xy}{70}-\frac{3\sqrt{35}G_{az}xz}{35}+\frac{\sqrt{21}G_{bz}xz}{7}$$

$\vec{\mathbb{G}}_1^{(3,1)}[g](B_g,2)$

** symmetry

z

** expression

$$\frac{\sqrt{21}G_3xy}{7}+\frac{2\sqrt{210}G_{3x}xz}{35}+\frac{2\sqrt{210}G_{3y}yz}{35}-\frac{3\sqrt{35}G_{az}\left(x^2+y^2-2z^2\right)}{70}+\frac{\sqrt{21}G_{bz}\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_1^{(3,3)}[g](B_g,1)$

** symmetry

x

** expression

$$\frac{\sqrt{210}G_1xy\left(15x^2-13y^2-6z^2\right)}{168}+\frac{\sqrt{210}G_2\left(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2\right)}{168}$$
$$+\frac{\sqrt{35}G_3yz\left(6x^2-y^2-z^2\right)}{14}-\frac{\sqrt{14}G_{3x}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{56}$$
$$-\frac{5\sqrt{14}G_{3y}xy\left(x^2+y^2-6z^2\right)}{56}-\frac{5\sqrt{21}G_{az}xz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{\sqrt{35}G_{bz}xz\left(5x^2-9y^2-2z^2\right)}{28}$$

$\vec{\mathbb{G}}_1^{(3,3)}[g](B_g,2)$

** symmetry

z

** expression

$$\frac{\sqrt{210}G_1yz\left(3x^2-y^2\right)}{24}+\frac{\sqrt{210}G_2xz\left(x^2-3y^2\right)}{24}-\frac{\sqrt{35}G_3xy\left(x^2+y^2-6z^2\right)}{14}-\frac{5\sqrt{14}G_{3x}xz\left(3x^2+3y^2-4z^2\right)}{56}$$
$$-\frac{5\sqrt{14}G_{3y}yz\left(3x^2+3y^2-4z^2\right)}{56}+\frac{\sqrt{21}G_{az}\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{84}-\frac{\sqrt{35}G_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{28}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_u,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\frac{\sqrt{14}G_{3x}x}{7}+\frac{\sqrt{14}G_{3y}y}{7}+\frac{\sqrt{21}G_{az}z}{7}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{70}G_1y}{14}+\frac{\sqrt{70}G_2x}{14}-\frac{\sqrt{42}G_{3x}x}{42}+\frac{\sqrt{42}G_{3y}y}{42}+\frac{\sqrt{105}G_{bz}z}{21}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_u,3)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{105}G_3y}{21}+\frac{2\sqrt{42}G_{3x}z}{21}-\frac{\sqrt{7}G_{az}x}{7}+\frac{\sqrt{105}G_{bz}x}{21}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_u,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{5\sqrt{210}G_1y\left(3x^2-y^2\right)}{168}-\frac{5\sqrt{210}G_2x\left(x^2-3y^2\right)}{168}-\frac{3\sqrt{14}G_{3x}x\left(x^2+y^2-4z^2\right)}{56}-\frac{3\sqrt{14}G_{3y}y\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{21}G_{az}z\left(3x^2+3y^2-2z^2\right)}{21}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{70}G_1y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{70}G_2x\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{42}G_{3x}x\left(11x^2-9y^2-24z^2\right)}{168}$$
$$-\frac{\sqrt{42}G_{3y}y\left(9x^2-11y^2+24z^2\right)}{168}-\frac{5\sqrt{7}G_{az}z\,(x-y)\,(x+y)}{14}+\frac{\sqrt{105}G_{bz}z\left(3x^2+3y^2-2z^2\right)}{42}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_u,3)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{70}G_1xyz}{28}+\frac{5\sqrt{70}G_2z\,(x-y)\,(x+y)}{56}+\frac{\sqrt{105}G_3y\left(3x^2-2y^2+3z^2\right)}{42}+\frac{\sqrt{42}G_{3x}z\left(9x^2-21y^2+4z^2\right)}{168}$$
$$+\frac{5\sqrt{42}G_{3y}xyz}{28}-\frac{\sqrt{7}G_{az}x\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{105}G_{bz}x\left(x^2-9y^2+6z^2\right)}{84}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_u,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{1155}G_1y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{264}-\frac{\sqrt{1155}G_2x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{264}-\frac{3\sqrt{770}G_3xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{15\sqrt{77}G_{3x}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{616}+\frac{15\sqrt{77}G_{3y}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{616}$$
$$+\frac{5\sqrt{462}G_{az}z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{1848}-\frac{3\sqrt{770}G_{bz}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_u, 2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{385}G_1 y\left(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4\right)}{616}+\frac{\sqrt{385}G_2 x\left(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4\right)}{616}$$
$$+\frac{3\sqrt{2310}G_3 xyz\,(x-y)\,(x+y)}{44}-\frac{5\sqrt{231}G_{3x}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{1848}$$
$$-\frac{5\sqrt{231}G_{3y}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{1848}-\frac{5\sqrt{154}G_{az}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}$$
$$+\frac{\sqrt{2310}G_{bz}z\left(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4\right)}{1848}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_u, 3)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{385}G_1 xyz\left(7x^2-5y^2-2z^2\right)}{44}+\frac{\sqrt{385}G_2 z\left(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2\right)}{44}-\frac{\sqrt{2310}G_3 y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{462}$$
$$-\frac{5\sqrt{231}G_{3x}z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{924}-\frac{5\sqrt{231}G_{3y}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{5\sqrt{154}G_{az}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{308}-\frac{\sqrt{2310}G_{bz}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{924}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](B_u, 1)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{105}G_3 x}{21}+\frac{2\sqrt{42}G_{3y}z}{21}-\frac{\sqrt{7}G_{az}y}{7}-\frac{\sqrt{105}G_{bz}y}{21}$$

$\vec{\mathbb{G}}_2^{(3,-1)}[g](B_u, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}G_1 x}{14}-\frac{\sqrt{70}G_2 y}{14}+\frac{\sqrt{105}G_3 z}{21}-\frac{\sqrt{42}G_{3x}y}{42}-\frac{\sqrt{42}G_{3y}x}{42}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](B_u, 1)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{70}G_1 z\,(x-y)\,(x+y)}{56}-\frac{5\sqrt{70}G_2 xyz}{28}-\frac{\sqrt{105}G_3 x\left(2x^2-3y^2-3z^2\right)}{42}+\frac{5\sqrt{42}G_{3x}xyz}{28}$$
$$-\frac{\sqrt{42}G_{3y}z\left(21x^2-9y^2-4z^2\right)}{168}-\frac{\sqrt{7}G_{az}y\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{105}G_{bz}y\left(9x^2-y^2-6z^2\right)}{84}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](B_u, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}G_1 x\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{70}G_2 y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{105}G_3 z\left(3x^2+3y^2-2z^2\right)}{42}$$
$$-\frac{\sqrt{42}G_{3x}y\left(21x^2+y^2-24z^2\right)}{168}-\frac{\sqrt{42}G_{3y}x\left(x^2+21y^2-24z^2\right)}{168}-\frac{5\sqrt{7}G_{az}xyz}{7}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](B_u, 1)$

** symmetry

$\sqrt{3}yz$

** expression

$$-\frac{\sqrt{385}G_1z\left(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2\right)}{44}+\frac{\sqrt{385}G_2xyz\left(5x^2-7y^2+2z^2\right)}{44}+\frac{\sqrt{2310}G_3x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{462}$$
$$-\frac{5\sqrt{231}G_{3x}xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{5\sqrt{231}G_{3y}z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{924}$$
$$+\frac{5\sqrt{154}G_{az}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{308}-\frac{\sqrt{2310}G_{bz}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{924}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](B_u,2)$

** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{\sqrt{385}G_1x\left(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4\right)}{308}+\frac{\sqrt{385}G_2y\left(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4\right)}{308}$$
$$-\frac{\sqrt{2310}G_3z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{462}-\frac{5\sqrt{231}G_{3x}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{924}$$
$$+\frac{5\sqrt{231}G_{3y}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{924}-\frac{5\sqrt{154}G_{az}xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{3\sqrt{2310}G_{bz}xyz\left(x-y\right)\left(x+y\right)}{44}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_g,1)$

** symmetry

$\sqrt{15}xyz$

** expression

G_3

$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_g,2)$

** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{10}G_1}{4}-\frac{\sqrt{6}G_{3y}}{4}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_g,3)$

** symmetry

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{6}G_1}{4}+\frac{\sqrt{10}G_{3y}}{4}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_g,1)$

** symmetry

$\sqrt{15}xyz$

** expression

$$\frac{\sqrt{10}G_1xz}{4}-\frac{\sqrt{10}G_2yz}{4}+\frac{\sqrt{6}G_{3x}yz}{4}+\frac{\sqrt{6}G_{3y}xz}{4}-G_{az}xy$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_g,2)$

** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{6}G_1\left(x^2+4y^2-5z^2\right)}{24}-\frac{\sqrt{6}G_2xy}{8}-G_3xz-\frac{\sqrt{10}G_{3x}xy}{40}+\frac{\sqrt{10}G_{3y}\left(7x^2-4y^2-3z^2\right)}{40}-\frac{\sqrt{15}G_{az}yz}{20}-\frac{G_{bz}yz}{4}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_g,3)$
** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{10}G_1\,(x-z)\,(x+z)}{8}+\frac{\sqrt{10}G_2xy}{8}+\frac{3\sqrt{6}G_{3x}xy}{8}-\frac{\sqrt{6}G_{3y}\,(x-z)\,(x+z)}{8}+\frac{G_{az}yz}{4}-\frac{\sqrt{15}G_{bz}yz}{4}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](A_g,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{5\sqrt{33}G_1xz\left(3x^2+3y^2-4z^2\right)}{132}-\frac{5\sqrt{33}G_2yz\left(3x^2+3y^2-4z^2\right)}{132}-\frac{7\sqrt{22}G_3\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{66}$$
$$+\frac{\sqrt{55}G_{3x}yz\left(9x^2-19y^2+16z^2\right)}{132}-\frac{\sqrt{55}G_{3y}xz\left(19x^2-9y^2-16z^2\right)}{132}+\frac{\sqrt{330}G_{az}xy\left(x^2+y^2-6z^2\right)}{132}+\frac{35\sqrt{22}G_{bz}xy\,(x-y)\,(x+y)}{132}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](A_g,2)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{55}G_1\left(x^4-9x^2y^2+3x^2z^2+4y^4-15y^2z^2+2z^4\right)}{88}+\frac{\sqrt{55}G_2xy\left(2x^2-5y^2+9z^2\right)}{44}+\frac{\sqrt{330}G_3xz\left(x^2-6y^2+z^2\right)}{132}+\frac{5\sqrt{33}G_{3x}xy\left(6x^2-y^2-15z^2\right)}{132}$$
$$-\frac{\sqrt{33}G_{3y}\left(7x^4-51x^2y^2+9x^2z^2+12y^4-21y^2z^2+2z^4\right)}{264}+\frac{5\sqrt{22}G_{az}yz\left(6x^2-y^2-z^2\right)}{44}-\frac{\sqrt{330}G_{bz}yz\left(12x^2+5y^2-9z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](A_g,3)$
** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{33}G_1\left(x^4+57x^2y^2-63x^2z^2-14y^4+27y^2z^2+6z^4\right)}{264}-\frac{5\sqrt{33}G_2xy\left(2x^2-5y^2+9z^2\right)}{132}-\frac{35\sqrt{22}G_3xz\,(x-z)\,(x+z)}{132}$$
$$+\frac{\sqrt{55}G_{3x}xy\left(10x^2-11y^2+3z^2\right)}{132}+\frac{\sqrt{55}G_{3y}\left(5x^4+33x^2y^2-63x^2z^2-14y^4+51y^2z^2+2z^4\right)}{264}$$
$$+\frac{\sqrt{330}G_{az}yz\left(12x^2-9y^2+5z^2\right)}{132}+\frac{5\sqrt{22}G_{bz}yz\left(6x^2-y^2-z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](A_g,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{7\sqrt{286}G_1xz\left(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4\right)}{1144}+\frac{7\sqrt{286}G_2yz\left(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4\right)}{1144}$$
$$+\frac{\sqrt{429}G_3\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$
$$-\frac{7\sqrt{4290}G_{3x}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{1144}+\frac{7\sqrt{4290}G_{3y}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{1144}$$
$$+\frac{7\sqrt{715}G_{az}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{21\sqrt{429}G_{bz}xy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](A_g,2)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{4290}G_1\left(23x^6-393x^4y^2+48x^4z^2+468x^2y^4-450x^2y^2z^2+27x^2z^4-40y^6+132y^4z^2-57y^2z^4+2z^6\right)}{6864}$$
$$-\frac{7\sqrt{4290}G_2xy\left(7x^4-27x^2y^2+11x^2z^2+10y^4-19y^2z^2+4z^4\right)}{2288}$$
$$+\frac{7\sqrt{715}G_3xz\left(x^4-16x^2y^2+2x^2z^2+16y^4-16y^2z^2+z^4\right)}{572}+\frac{35\sqrt{286}G_{3x}xy\left(x^4-x^2y^2-7x^2z^2-2y^4+23y^2z^2-8z^4\right)}{2288}$$
$$-\frac{5\sqrt{286}G_{3y}\left(x^6-11x^4y^2-4x^4z^2-4x^2y^4+90x^2y^2z^2-11x^2z^4+8y^6-116y^4z^2+101y^2z^4-6z^6\right)}{2288}$$
$$+\frac{35\sqrt{429}G_{az}yz\left(x^4-x^2y^2-x^2z^2-2y^4+7y^2z^2-2z^4\right)}{1144}-\frac{7\sqrt{715}G_{bz}yz\left(13x^4-43x^2y^2+17x^2z^2+10y^4-19y^2z^2+4z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](A_g,3)$
** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{\sqrt{286}G_1\left(37x^6-505x^4y^2-50x^4z^2+370x^2y^4+810x^2y^2z^2-85x^2z^4-12y^6-190y^4z^2+55y^2z^4+2z^6\right)}{2288}$$
$$-\frac{7\sqrt{286}G_2xy\left(31x^4-85x^2y^2-55x^2z^2+16y^4+95y^2z^2-20z^4\right)}{2288}$$
$$+\frac{21\sqrt{429}G_3xz\,(x-z)\,(x+z)\left(x^2-10y^2+z^2\right)}{572}+\frac{21\sqrt{4290}G_{3x}xy\left(x^4+x^2y^2-13x^2z^2-3y^2z^2+8z^4\right)}{2288}$$
$$-\frac{\sqrt{4290}G_{3y}\left(3x^6-19x^4y^2-26x^4z^2-26x^2y^4+270x^2y^2z^2-19x^2z^4-4y^6+86y^4z^2-131y^2z^4+10z^6\right)}{2288}$$
$$+\frac{7\sqrt{715}G_{az}yz\left(13x^4+17x^2y^2-43x^2z^2+4y^4-19y^2z^2+10z^4\right)}{1144}-\frac{7\sqrt{429}G_{bz}yz\left(35x^4-35x^2y^2-35x^2z^2-4y^4+25y^2z^2-4z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{10}G_2}{4}-\frac{\sqrt{6}G_{3x}}{4}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g,2)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$G_{az}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g,3)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{\sqrt{6}G_2}{4}-\frac{\sqrt{10}G_{3x}}{4}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g,4)$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$G_{bz}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{6}G_1xy}{8}+\frac{\sqrt{6}G_2\left(4x^2+y^2-5z^2\right)}{24}-G_3yz-\frac{\sqrt{10}G_{3x}\left(4x^2-7y^2+3z^2\right)}{40}-\frac{\sqrt{10}G_{3y}xy}{40}-\frac{\sqrt{15}G_{az}xz}{20}+\frac{G_{bz}xz}{4}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g,2)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-G_3xy+\frac{\sqrt{10}G_{3x}xz}{10}+\frac{\sqrt{10}G_{3y}yz}{10}-\frac{\sqrt{15}G_{az}\left(x^2+y^2-2z^2\right)}{15}-\frac{G_{bz}\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g,3)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{10}G_1xy}{8}-\frac{\sqrt{10}G_2\left(y-z\right)\left(y+z\right)}{8}+\frac{\sqrt{6}G_{3x}\left(y-z\right)\left(y+z\right)}{8}-\frac{3\sqrt{6}G_{3y}xy}{8}-\frac{G_{az}xz}{4}-\frac{\sqrt{15}G_{bz}xz}{4}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g,4)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{10}G_1yz}{4}+\frac{\sqrt{10}G_2xz}{4}+\frac{\sqrt{6}G_{3x}xz}{4}-\frac{\sqrt{6}G_{3y}yz}{4}-\frac{G_{az}\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{55}G_1xy\left(5x^2-2y^2-9z^2\right)}{44}+\frac{\sqrt{55}G_2\left(4x^4-9x^2y^2-15x^2z^2+y^4+3y^2z^2+2z^4\right)}{88}$$
$$-\frac{\sqrt{330}G_3yz\left(6x^2-y^2-z^2\right)}{132}-\frac{\sqrt{33}G_{3x}\left(12x^4-51x^2y^2-21x^2z^2+7y^4+9y^2z^2+2z^4\right)}{264}$$
$$-\frac{5\sqrt{33}G_{3y}xy\left(x^2-6y^2+15z^2\right)}{132}-\frac{5\sqrt{22}G_{az}xz\left(x^2-6y^2+z^2\right)}{44}+\frac{\sqrt{330}G_{bz}xz\left(5x^2+12y^2-9z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g,2)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{7\sqrt{55}G_1yz\left(3x^2-y^2\right)}{44}-\frac{7\sqrt{55}G_2xz\left(x^2-3y^2\right)}{44}+\frac{\sqrt{330}G_3xy\left(x^2+y^2-6z^2\right)}{132}-\frac{5\sqrt{33}G_{3x}xz\left(3x^2+3y^2-4z^2\right)}{132}$$
$$-\frac{5\sqrt{33}G_{3y}yz\left(3x^2+3y^2-4z^2\right)}{132}+\frac{\sqrt{22}G_{az}\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{88}+\frac{\sqrt{330}G_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{264}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g,3)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{5\sqrt{33}G_1xy\left(5x^2-2y^2-9z^2\right)}{132}+\frac{\sqrt{33}G_2\left(14x^4-57x^2y^2-27x^2z^2-y^4+63y^2z^2-6z^4\right)}{264}$$
$$+\frac{35\sqrt{22}G_3yz\left(y-z\right)\left(y+z\right)}{132}+\frac{\sqrt{55}G_{3x}\left(14x^4-33x^2y^2-51x^2z^2-5y^4+63y^2z^2-2z^4\right)}{264}$$
$$+\frac{\sqrt{55}G_{3y}xy\left(11x^2-10y^2-3z^2\right)}{132}+\frac{\sqrt{330}G_{az}xz\left(9x^2-12y^2-5z^2\right)}{132}-\frac{5\sqrt{22}G_{bz}xz\left(x^2-6y^2+z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g,4)$

** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{5\sqrt{33}G_1yz\left(3x^2+3y^2-4z^2\right)}{132}+\frac{5\sqrt{33}G_2xz\left(3x^2+3y^2-4z^2\right)}{132}+\frac{35\sqrt{22}G_3xy\,(x-y)\,(x+y)}{132}-\frac{\sqrt{55}G_{3x}xz\left(5x^2+33y^2-16z^2\right)}{132}$$
$$+\frac{\sqrt{55}G_{3y}yz\left(33x^2+5y^2-16z^2\right)}{132}+\frac{\sqrt{330}G_{az}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{264}+\frac{7\sqrt{22}G_{bz}\left(x^4-18x^2y^2+12x^2z^2+y^4+12y^2z^2-4z^4\right)}{264}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{7\sqrt{4290}G_1xy\left(10x^4-27x^2y^2-19x^2z^2+7y^4+11y^2z^2+4z^4\right)}{2288}$$
$$+\frac{\sqrt{4290}G_2\left(40x^6-468x^4y^2-132x^4z^2+393x^2y^4+450x^2y^2z^2+57x^2z^4-23y^6-48y^4z^2-27y^2z^4-2z^6\right)}{6864}$$
$$+\frac{7\sqrt{715}G_3yz\left(16x^4-16x^2y^2-16x^2z^2+y^4+2y^2z^2+z^4\right)}{572}$$
$$-\frac{5\sqrt{286}G_{3x}\left(8x^6-4x^4y^2-116x^4z^2-11x^2y^4+90x^2y^2z^2+101x^2z^4+y^6-4y^4z^2-11y^2z^4-6z^6\right)}{2288}$$
$$-\frac{35\sqrt{286}G_{3y}xy\left(2x^4+x^2y^2-23x^2z^2-y^4+7y^2z^2+8z^4\right)}{2288}-\frac{35\sqrt{429}G_{az}xz\left(2x^4+x^2y^2-7x^2z^2-y^4+y^2z^2+2z^4\right)}{1144}$$
$$+\frac{7\sqrt{715}G_{bz}xz\left(10x^4-43x^2y^2-19x^2z^2+13y^4+17y^2z^2+4z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{7\sqrt{4290}G_1yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{7\sqrt{4290}G_2xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}$$
$$+\frac{7\sqrt{715}G_3xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}+\frac{35\sqrt{286}G_{3x}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$+\frac{35\sqrt{286}G_{3y}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$-\frac{5\sqrt{429}G_{az}\left(5x^6+15x^4y^2-90x^4z^2+15x^2y^4-180x^2y^2z^2+120x^2z^4+5y^6-90y^4z^2+120y^2z^4-16z^6\right)}{3432}$$
$$+\frac{7\sqrt{715}G_{bz}\,(x-y)\,(x+y)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g,3)$

** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{7\sqrt{286}G_1xy\left(16x^4-85x^2y^2+95x^2z^2+31y^4-55y^2z^2-20z^4\right)}{2288}$$
$$-\frac{\sqrt{286}G_2\left(12x^6-370x^4y^2+190x^4z^2+505x^2y^4-810x^2y^2z^2-55x^2z^4-37y^6+50y^4z^2+85y^2z^4-2z^6\right)}{2288}$$
$$+\frac{21\sqrt{429}G_3yz\,(y-z)\,(y+z)\left(10x^2-y^2-z^2\right)}{572}$$
$$-\frac{\sqrt{4290}G_{3x}\left(4x^6+26x^4y^2-86x^4z^2+19x^2y^4-270x^2y^2z^2+131x^2z^4-3y^6+26y^4z^2+19y^2z^4-10z^6\right)}{2288}$$
$$-\frac{21\sqrt{4290}G_{3y}xy\left(x^2y^2-3x^2z^2+y^4-13y^2z^2+8z^4\right)}{2288}-\frac{7\sqrt{715}G_{az}xz\left(4x^4+17x^2y^2-19x^2z^2+13y^4-43y^2z^2+10z^4\right)}{1144}$$
$$+\frac{7\sqrt{429}G_{bz}xz\left(4x^4+35x^2y^2-25x^2z^2-35y^4+35y^2z^2+4z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g,4)$

** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{7\sqrt{286}G_{1}yz\left(85x^{4}-160x^{2}y^{2}-10x^{2}z^{2}+19y^{4}-10y^{2}z^{2}+4z^{4}\right)}{2288}+\frac{7\sqrt{286}G_{2}xz\left(19x^{4}-160x^{2}y^{2}-10x^{2}z^{2}+85y^{4}-10y^{2}z^{2}+4z^{4}\right)}{2288}$$
$$-\frac{21\sqrt{429}G_{3}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}-\frac{7\sqrt{4290}G_{3x}xz\left(7x^{4}-4x^{2}y^{2}-22x^{2}z^{2}-11y^{4}+26y^{2}z^{2}+4z^{4}\right)}{2288}$$
$$-\frac{7\sqrt{4290}G_{3y}yz\left(11x^{4}+4x^{2}y^{2}-26x^{2}z^{2}-7y^{4}+22y^{2}z^{2}-4z^{4}\right)}{2288}+\frac{7\sqrt{715}G_{az}\left(x-y\right)\left(x+y\right)\left(x^{4}+2x^{2}y^{2}-16x^{2}z^{2}+y^{4}-16y^{2}z^{2}+16z^{4}\right)}{1144}$$
$$-\frac{\sqrt{429}G_{bz}\left(13x^{6}-45x^{4}y^{2}-150x^{4}z^{2}-45x^{2}y^{4}+540x^{2}y^{2}z^{2}+60x^{2}z^{4}+13y^{6}-150y^{4}z^{2}+60y^{2}z^{4}-8z^{6}\right)}{1144}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(3,-3)}[g](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{30}G_{1}y}{12}+\frac{\sqrt{30}G_{2}x}{12}-\frac{\sqrt{2}G_{3x}x}{4}-\frac{\sqrt{2}G_{3y}y}{4}+\frac{\sqrt{3}G_{az}z}{3}$$

$\vec{\mathbb{G}}_{4}^{(3,-3)}[g](A_u,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

** expression

$$\frac{\sqrt{42}G_{1}y}{12}-\frac{\sqrt{42}G_{2}x}{12}-\frac{\sqrt{70}G_{3x}x}{28}-\frac{\sqrt{70}G_{3y}y}{28}+\frac{\sqrt{105}G_{az}z}{21}$$

$\vec{\mathbb{G}}_{4}^{(3,-3)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{14}G_{1}y}{28}+\frac{\sqrt{14}G_{2}x}{28}-\frac{\sqrt{210}G_{3x}x}{28}+\frac{\sqrt{210}G_{3y}y}{28}-\frac{\sqrt{21}G_{bz}z}{7}$$

$\vec{\mathbb{G}}_{4}^{(3,-3)}[g](A_u,4)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{2}G_{2}z}{8}+\frac{\sqrt{30}G_{3x}z}{8}+\frac{\sqrt{5}G_{az}x}{4}-\frac{\sqrt{3}G_{bz}x}{4}$$

$\vec{\mathbb{G}}_{4}^{(3,-3)}[g](A_u,5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{14}G_{2}z}{8}+\frac{\sqrt{21}G_{3}y}{7}-\frac{\sqrt{210}G_{3x}z}{56}-\frac{\sqrt{35}G_{az}x}{28}-\frac{3\sqrt{21}G_{bz}x}{28}$$

$\vec{\mathbb{G}}_{4}^{(3,-1)}[g](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$\frac{\sqrt{165}G_{1}y\left(x^{2}-y^{2}+2z^{2}\right)}{44}+\frac{\sqrt{165}G_{2}x\left(x^{2}-y^{2}-2z^{2}\right)}{44}-\frac{3\sqrt{110}G_{3}xyz}{11}-\frac{\sqrt{11}G_{3x}x\left(3x^{2}-7y^{2}-2z^{2}\right)}{44}$$
$$+\frac{\sqrt{11}G_{3y}y\left(7x^{2}-3y^{2}+2z^{2}\right)}{44}-\frac{\sqrt{66}G_{az}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{44}-\frac{\sqrt{110}G_{bz}z\left(x-y\right)\left(x+y\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_u, 2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$\frac{\sqrt{231}G_1 y\left(11x^2 + y^2 - 14z^2\right)}{308} - \frac{\sqrt{231}G_2 x\left(x^2 + 11y^2 - 14z^2\right)}{308} + \frac{\sqrt{385}G_{3x} x\left(3x^2 - 11y^2 + 2z^2\right)}{308}$$
$$- \frac{\sqrt{385}G_{3y} y\left(11x^2 - 3y^2 - 2z^2\right)}{308} - \frac{\sqrt{2310}G_{az} z\left(3x^2 + 3y^2 - 2z^2\right)}{308} - \frac{5\sqrt{154}G_{bz} z\,(x-y)\,(x+y)}{44}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_u, 3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$\frac{9\sqrt{77}G_1 y\left(x^2 + y^2 - 4z^2\right)}{308} + \frac{9\sqrt{77}G_2 x\left(x^2 + y^2 - 4z^2\right)}{308} - \frac{\sqrt{1155}G_{3x} x\left(x^2 - 11y^2 + 8z^2\right)}{308}$$
$$- \frac{\sqrt{1155}G_{3y} y\left(11x^2 - y^2 - 8z^2\right)}{308} + \frac{3\sqrt{770}G_{az} z\,(x-y)\,(x+y)}{308} - \frac{\sqrt{462}G_{bz} z\left(3x^2 + 3y^2 - 2z^2\right)}{308}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_u, 4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{15\sqrt{11}G_1 xyz}{44} - \frac{3\sqrt{11}G_2 z\left(4x^2 - y^2 - z^2\right)}{44} - \frac{5\sqrt{66}G_3 y\,(x-z)\,(x+z)}{44} - \frac{\sqrt{165}G_{3x} z\left(3y^2 - z^2\right)}{44}$$
$$+ \frac{5\sqrt{165}G_{3y} xyz}{44} - \frac{3\sqrt{110}G_{az} x\,(y-z)\,(y+z)}{44} - \frac{\sqrt{66}G_{bz} x\left(2x^2 - 3y^2 - 3z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_u, 5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$\frac{15\sqrt{77}G_1 xyz}{308} - \frac{3\sqrt{77}G_2 z\left(8x^2 + 13y^2 - 7z^2\right)}{308} + \frac{\sqrt{462}G_3 y\left(3x^2 - 2y^2 + 3z^2\right)}{308} - \frac{\sqrt{1155}G_{3x} z\left(4x^2 - 7y^2 + z^2\right)}{308}$$
$$+ \frac{3\sqrt{1155}G_{3y} xyz}{308} + \frac{3\sqrt{770}G_{az} x\left(2x^2 - 5y^2 - z^2\right)}{308} + \frac{\sqrt{462}G_{bz} x\left(4x^2 - y^2 - 11z^2\right)}{308}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u, 1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$\frac{5\sqrt{858}G_1 y\left(5x^4 + 3x^2y^2 - 39x^2z^2 - 2y^4 + 17y^2z^2 - 2z^4\right)}{1144} + \frac{5\sqrt{858}G_2 x\left(2x^4 - 3x^2y^2 - 17x^2z^2 - 5y^4 + 39y^2z^2 + 2z^4\right)}{1144}$$
$$- \frac{\sqrt{1430}G_{3x} x\left(6x^4 - 65x^2y^2 + 5x^2z^2 + 13y^4 + 117y^2z^2 - 22z^4\right)}{1144} - \frac{\sqrt{1430}G_{3y} y\left(13x^4 - 65x^2y^2 + 117x^2z^2 + 6y^4 + 5y^2z^2 - 22z^4\right)}{1144}$$
$$- \frac{\sqrt{2145}G_{az} z\left(3x^4 - 78x^2y^2 + 20x^2z^2 + 3y^4 + 20y^2z^2 - 4z^4\right)}{572} + \frac{35\sqrt{143}G_{bz} z\,(x-y)\,(x+y)\left(x^2 + y^2 - 2z^2\right)}{572}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u, 2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$\frac{\sqrt{30030}G_1 y\left(4x^4 + 3x^2y^2 - 33x^2z^2 - y^4 + 7y^2z^2 + 2z^4\right)}{1144} + \frac{\sqrt{30030}G_2 x\left(x^4 - 3x^2y^2 - 7x^2z^2 - 4y^4 + 33y^2z^2 - 2z^4\right)}{1144} + \frac{3\sqrt{5005}G_3 xyz\left(x^2 + y^2 - 2z^2\right)}{143}$$
$$+ \frac{\sqrt{2002}G_{3x} x\left(75x^4 - 389x^2y^2 - 361x^2z^2 + 124y^4 + 423y^2z^2 + 110z^4\right)}{8008} + \frac{\sqrt{2002}G_{3y} y\left(124x^4 - 389x^2y^2 + 423x^2z^2 + 75y^4 - 361y^2z^2 + 110z^4\right)}{8008}$$
$$+ \frac{\sqrt{3003}G_{az} z\left(111x^4 - 366x^2y^2 - 100x^2z^2 + 111y^4 - 100y^2z^2 + 20z^4\right)}{4004} - \frac{\sqrt{5005}G_{bz} z\,(x-y)\,(x+y)\left(x^2 + y^2 - 2z^2\right)}{572}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u, 3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{9\sqrt{10010}G_1 y\left(10x^4-15x^2y^2-15x^2z^2+3y^4-15y^2z^2+10z^4\right)}{8008}+\frac{9\sqrt{10010}G_2 x\left(3x^4-15x^2y^2-15x^2z^2+10y^4-15y^2z^2+10z^4\right)}{8008}$$
$$-\frac{3\sqrt{15015}G_3 xyz\,(x-y)\,(x+y)}{143}+\frac{\sqrt{6006}G_{3x}x\left(5x^4-53x^2y^2+3x^2z^2-58y^4+507y^2z^2-86z^4\right)}{8008}$$
$$+\frac{\sqrt{6006}G_{3y}y\left(58x^4+53x^2y^2-507x^2z^2-5y^4-3y^2z^2+86z^4\right)}{8008}+\frac{3\sqrt{1001}G_{az}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{572}$$
$$+\frac{\sqrt{15015}G_{bz}z\left(9x^4+186x^2y^2-80x^2z^2+9y^4-80y^2z^2+16z^4\right)}{4004}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u, 4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{21\sqrt{1430}G_1 xyz\left(7x^2-2y^2-5z^2\right)}{1144}-\frac{3\sqrt{1430}G_2 z\left(37x^4-87x^2y^2-45x^2z^2+2y^4+25y^2z^2+2z^4\right)}{2288}-\frac{7\sqrt{2145}G_3 y\,(x-z)\,(x+z)\left(x^2-2y^2+z^2\right)}{572}$$
$$-\frac{\sqrt{858}G_{3x}z\left(33x^4-39x^2y^2-53x^2z^2-30y^4+73y^2z^2-2z^4\right)}{2288}-\frac{7\sqrt{858}G_{3y}xyz\left(13x^2+10y^2-23z^2\right)}{1144}$$
$$+\frac{3\sqrt{143}G_{az}x\left(3x^4+13x^2y^2-43x^2z^2+10y^4-99y^2z^2+38z^4\right)}{1144}+\frac{\sqrt{2145}G_{bz}x\left(x^4+37x^2y^2-47x^2z^2-6y^4-75y^2z^2+36z^4\right)}{1144}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u, 5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{10010}G_1 xyz\left(7x^2-2y^2-5z^2\right)}{1144}+\frac{3\sqrt{10010}G_2 z\left(x^4-21x^2y^2+5x^2z^2-4y^4+15y^2z^2-2z^4\right)}{2288}$$
$$-\frac{\sqrt{15015}G_3 y\left(51x^4-80x^2y^2-66x^2z^2+16y^4-80y^2z^2+51z^4\right)}{4004}+\frac{\sqrt{6006}G_{3x}z\left(187x^4+213x^2y^2-445x^2z^2-268y^4+465y^2z^2-2z^4\right)}{16016}$$
$$-\frac{3\sqrt{6006}G_{3y}xyz\left(9x^2+2y^2-11z^2\right)}{1144}-\frac{3\sqrt{1001}G_{az}x\left(17x^4-15x^2y^2-155x^2z^2-32y^4+237y^2z^2+38z^4\right)}{8008}$$
$$-\frac{\sqrt{15015}G_{bz}x\left(11x^4-195x^2y^2+85x^2z^2+88y^4+57y^2z^2-52z^4\right)}{8008}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u, 1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{858}G_1 y\left(211x^6-900x^4y^2-465x^4z^2+570x^2y^4-300x^2y^2z^2+615x^2z^4-35y^6+165y^4z^2-225y^2z^4+4z^6\right)}{3432}$$
$$+\frac{\sqrt{858}G_2 x\left(35x^6-570x^4y^2-165x^4z^2+900x^2y^4+300x^2y^2z^2+225x^2z^4-211y^6+465y^4z^2-615y^2z^4-4z^6\right)}{3432}$$
$$+\frac{3\sqrt{143}G_3 xyz\left(3x^4-5x^2y^2-5x^2z^2+3y^4-5y^2z^2+3z^4\right)}{22}$$
$$-\frac{\sqrt{1430}G_{3x}x\left(7x^6-6x^4y^2-141x^4z^2+60x^2y^2z^2+225x^2z^4+13y^6-195y^4z^2+165y^2z^4-56z^6\right)}{1144}$$
$$-\frac{\sqrt{1430}G_{3y}y\left(13x^6-195x^4z^2-6x^2y^4+60x^2y^2z^2+165x^2z^4+7y^6-141y^4z^2+225y^2z^4-56z^6\right)}{1144}$$
$$-\frac{\sqrt{2145}G_{az}z\left(43x^6+30x^4y^2-225x^4z^2+30x^2y^4-120x^2y^2z^2+147x^2z^4+43y^6-225y^4z^2+147y^2z^4-14z^6\right)}{1716}$$
$$+\frac{3\sqrt{143}G_{bz}z\,(x-y)\,(x+y)\left(23x^4-97x^2y^2-75x^2z^2+23y^4-75y^2z^2+45z^4\right)}{572}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u, 2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{30030}G_1 y\left(341x^6-1935x^4y^2+690x^4z^2+1113x^2y^4+480x^2y^2z^2-930x^2z^4-43y^6-210y^4z^2+270y^2z^4+8z^6\right)}{34320}$$
$$-\frac{\sqrt{30030}G_2 x\left(43x^6-1113x^4y^2+210x^4z^2+1935x^2y^4-480x^2y^2z^2-270x^2z^4-341y^6-690y^4z^2+930y^2z^4-8z^6\right)}{34320}$$
$$-\frac{3\sqrt{5005}G_3 xyz\left(3x^4-20x^2y^2+10x^2z^2+3y^4+10y^2z^2-6z^4\right)}{260}$$
$$-\frac{\sqrt{2002}G_{3x} x\left(x^6+57x^4y^2-78x^4z^2+45x^2y^4-840x^2y^2z^2+270x^2z^4-11y^6+30y^4z^2+390y^2z^4-80z^6\right)}{2288}$$
$$+\frac{\sqrt{2002}G_{3y} y\left(11x^6-45x^4y^2-30x^4z^2-57x^2y^4+840x^2y^2z^2-390x^2z^4-y^6+78y^4z^2-270y^2z^4+80z^6\right)}{2288}$$
$$-\frac{\sqrt{3003}G_{az} z\left(19x^6+255x^4y^2-180x^4z^2+255x^2y^4-1020x^2y^2z^2+210x^2z^4+19y^6-180y^4z^2+210y^2z^4-20z^6\right)}{3432}$$
$$-\frac{3\sqrt{5005}G_{bz} z\left(x-y\right)\left(x+y\right)\left(x^4-284x^2y^2+90x^2z^2+y^4+90y^2z^2-54z^4\right)}{5720}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u,3)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{10010}G_1 y\left(85x^6-75x^4y^2-1050x^4z^2-141x^2y^4+1860x^2y^2z^2+120x^2z^4+19y^6-258y^4z^2+120y^2z^4-32z^6\right)}{11440}$$
$$+\frac{\sqrt{10010}G_2 x\left(19x^6-141x^4y^2-258x^4z^2-75x^2y^4+1860x^2y^2z^2+120x^2z^4+85y^6-1050y^4z^2+120y^2z^4-32z^6\right)}{11440}$$
$$+\frac{3\sqrt{15015}G_3 xyz\left(x-y\right)\left(x+y\right)\left(3x^2+3y^2-10z^2\right)}{260}$$
$$-\frac{\sqrt{6006}G_{3x} x\left(7x^6+3x^4y^2-150x^4z^2-15x^2y^4+60x^2y^2z^2+240x^2z^4-11y^6+210y^4z^2-240y^2z^4-32z^6\right)}{2288}$$
$$-\frac{\sqrt{6006}G_{3y} y\left(11x^6+15x^4y^2-210x^4z^2-3x^2y^4-60x^2y^2z^2+240x^2z^4-7y^6+150y^4z^2-240y^2z^4+32z^6\right)}{2288}$$
$$-\frac{3\sqrt{1001}G_{az} z\left(x-y\right)\left(x+y\right)\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{1144}$$
$$+\frac{\sqrt{15015}G_{bz} z\left(67x^6-195x^4y^2-270x^4z^2-195x^2y^4+780x^2y^2z^2+84x^2z^4+67y^6-270y^4z^2+84y^2z^4-8z^6\right)}{5720}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u,4)$
** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{3\sqrt{1430}G_1 xyz\left(249x^4-305x^2y^2-525x^2z^2+18y^4+245y^2z^2+84z^4\right)}{5720}$$
$$-\frac{\sqrt{1430}G_2 z\left(184x^6-1185x^4y^2-525x^4z^2+345x^2y^4+1680x^2y^2z^2+147x^2z^4-2y^6-105y^4z^2-105y^2z^4-2z^6\right)}{5720}$$
$$+\frac{3\sqrt{2145}G_3 y\left(x-z\right)\left(x+z\right)\left(8x^4+5x^2y^2-127x^2z^2-3y^4+5y^2z^2+8z^4\right)}{1430}$$
$$+\frac{\sqrt{858}G_{3x} z\left(88x^6+75x^4y^2-465x^4z^2-15x^2y^4-120x^2y^2z^2+291x^2z^4-2y^6+15y^4z^2+3y^2z^4-14z^6\right)}{1144}$$
$$+\frac{3\sqrt{858}G_{3y} xyz\left(39x^4+45x^2y^2-175x^2z^2+6y^4-65y^2z^2+72z^4\right)}{1144}$$
$$-\frac{\sqrt{143}G_{az} x\left(11x^6+24x^4y^2-255x^4z^2+15x^2y^4-330x^2y^2z^2+480x^2z^4+2y^6-75y^4z^2+240y^2z^4-112z^6\right)}{572}$$
$$+\frac{\sqrt{2145}G_{bz} x\left(17x^6-30x^4y^2-327x^4z^2-45x^2y^4+570x^2y^2z^2+450x^2z^4+2y^6+105y^4z^2-390y^2z^4-64z^6\right)}{2860}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u,5)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{10010}G_1xyz\left(87x^4-365x^2y^2+75x^2z^2+120y^4-35y^2z^2-12z^4\right)}{5720}$$
$$-\frac{\sqrt{10010}G_2z\left(40x^6-735x^4y^2+45x^4z^2+885x^2y^4-300x^2y^2z^2+3x^2z^4-56y^6-15y^4z^2+39y^2z^4-2z^6\right)}{5720}$$
$$+\frac{\sqrt{15015}G_3y\left(8x^6-15x^4y^2-75x^4z^2-21x^2y^4+300x^2y^2z^2-75x^2z^4+2y^6-21y^4z^2-15y^2z^4+8z^6\right)}{1430}$$
$$+\frac{\sqrt{6006}G_{3x}z\left(8x^6-75x^4y^2-15x^4z^2-75x^2y^4+300x^2y^2z^2-21x^2z^4+8y^6-15y^4z^2-21y^2z^4+2z^6\right)}{1144}$$
$$+\frac{3\sqrt{6006}G_{3y}xyz\left(9x^4-15x^2y^2-15x^2z^2-24y^4+95y^2z^2-24z^4\right)}{1144}$$
$$-\frac{\sqrt{1001}G_{az}x\left(x^6-6x^4y^2-15x^4z^2-15x^2y^4+150x^2y^2z^2-8y^6+165y^4z^2-240y^2z^4+16z^6\right)}{572}$$
$$+\frac{3\sqrt{15015}G_{bz}x\left(x^6-12x^4y^2-9x^4z^2-5x^2y^4+150x^2y^2z^2-10x^2z^4+8y^6-105y^4z^2+30y^2z^4\right)}{2860}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$-\frac{\sqrt{2}G_1z}{8}-\frac{\sqrt{30}G_{3y}z}{8}-\frac{\sqrt{5}G_{az}y}{4}-\frac{\sqrt{3}G_{bz}y}{4}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u,2)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{2}G_1x}{2}+\frac{\sqrt{2}G_2y}{2}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u,3)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{\sqrt{14}G_1z}{8}+\frac{\sqrt{21}G_3x}{7}-\frac{\sqrt{210}G_{3y}z}{56}-\frac{\sqrt{35}G_{az}y}{28}+\frac{3\sqrt{21}G_{bz}y}{28}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u,4)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{14}G_1x}{28}+\frac{\sqrt{14}G_2y}{28}+\frac{\sqrt{21}G_3z}{7}+\frac{\sqrt{210}G_{3x}y}{28}+\frac{\sqrt{210}G_{3y}x}{28}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{3\sqrt{11}G_1z\left(x^2-4y^2+z^2\right)}{44}-\frac{15\sqrt{11}G_2xyz}{44}+\frac{5\sqrt{66}G_3x\left(y-z\right)\left(y+z\right)}{44}-\frac{5\sqrt{165}G_{3x}xyz}{44}$$
$$+\frac{\sqrt{165}G_{3y}z\left(3x^2-z^2\right)}{44}+\frac{3\sqrt{110}G_{az}y\left(x-z\right)\left(x+z\right)}{44}+\frac{\sqrt{66}G_{bz}y\left(3x^2-2y^2+3z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u,2)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{11}G_1x\left(x^2+y^2-4z^2\right)}{44}+\frac{3\sqrt{11}G_2y\left(x^2+y^2-4z^2\right)}{44}+\frac{5\sqrt{66}G_3z\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{165}G_{3x}y\left(3x^2-y^2\right)}{44}-\frac{\sqrt{165}G_{3y}x\left(x^2-3y^2\right)}{44}+\frac{5\sqrt{66}G_{bz}xyz}{22}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u,3)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{77}G_1z\left(13x^2+8y^2-7z^2\right)}{308}-\frac{15\sqrt{77}G_2xyz}{308}-\frac{\sqrt{462}G_3x\left(2x^2-3y^2-3z^2\right)}{308}+\frac{3\sqrt{1155}G_{3x}xyz}{308}$$
$$+\frac{\sqrt{1155}G_{3y}z\left(7x^2-4y^2-z^2\right)}{308}-\frac{3\sqrt{770}G_{az}y\left(5x^2-2y^2+z^2\right)}{308}+\frac{\sqrt{462}G_{bz}y\left(x^2-4y^2+11z^2\right)}{308}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u,4)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{9\sqrt{77}G_1x\left(x^2+y^2-4z^2\right)}{308}+\frac{9\sqrt{77}G_2y\left(x^2+y^2-4z^2\right)}{308}+\frac{\sqrt{462}G_3z\left(3x^2+3y^2-2z^2\right)}{308}$$
$$+\frac{\sqrt{1155}G_{3x}y\left(7x^2-5y^2+8z^2\right)}{308}-\frac{\sqrt{1155}G_{3y}x\left(5x^2-7y^2-8z^2\right)}{308}-\frac{3\sqrt{770}G_{az}xyz}{154}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$-\frac{3\sqrt{1430}G_1z\left(2x^4-87x^2y^2+25x^2z^2+37y^4-45y^2z^2+2z^4\right)}{2288}+\frac{21\sqrt{1430}G_2xyz\left(2x^2-7y^2+5z^2\right)}{1144}-\frac{7\sqrt{2145}G_3x\left(y-z\right)\left(y+z\right)\left(2x^2-y^2-z^2\right)}{572}$$
$$+\frac{7\sqrt{858}G_{3x}xyz\left(10x^2+13y^2-23z^2\right)}{1144}-\frac{\sqrt{858}G_{3y}z\left(30x^4+39x^2y^2-73x^2z^2-33y^4+53y^2z^2+2z^4\right)}{2288}$$
$$-\frac{3\sqrt{143}G_{az}y\left(10x^4+13x^2y^2-99x^2z^2+3y^4-43y^2z^2+38z^4\right)}{1144}-\frac{\sqrt{2145}G_{bz}y\left(6x^4-37x^2y^2+75x^2z^2-y^4+47y^2z^2-36z^4\right)}{1144}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u,2)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{1430}G_1x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{1430}G_2y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}$$
$$+\frac{7\sqrt{2145}G_3z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{572}-\frac{7\sqrt{858}G_{3x}y\left(21x^4-26x^2y^2-48x^2z^2+y^4+16y^2z^2\right)}{2288}$$
$$+\frac{7\sqrt{858}G_{3y}x\left(x^4-26x^2y^2+16x^2z^2+21y^4-48y^2z^2\right)}{2288}-\frac{63\sqrt{143}G_{az}xyz\left(x-y\right)\left(x+y\right)}{143}+\frac{7\sqrt{2145}G_{bz}xyz\left(x^2+y^2-2z^2\right)}{286}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u,3)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{10010}G_1z\left(4x^4+21x^2y^2-15x^2z^2-y^4-5y^2z^2+2z^4\right)}{2288}+\frac{3\sqrt{10010}G_2xyz\left(2x^2-7y^2+5z^2\right)}{1144}$$
$$-\frac{\sqrt{15015}G_3x\left(16x^4-80x^2y^2-80x^2z^2+51y^4-66y^2z^2+51z^4\right)}{4004}-\frac{3\sqrt{6006}G_{3x}xyz\left(2x^2+9y^2-11z^2\right)}{1144}$$
$$-\frac{\sqrt{6006}G_{3y}z\left(268x^4-213x^2y^2-465x^2z^2-187y^4+445y^2z^2+2z^4\right)}{16016}$$
$$+\frac{3\sqrt{1001}G_{az}y\left(32x^4+15x^2y^2-237x^2z^2-17y^4+155y^2z^2-38z^4\right)}{8008}+\frac{\sqrt{15015}G_{bz}y\left(88x^4-195x^2y^2+57x^2z^2+11y^4+85y^2z^2-52z^4\right)}{8008}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u,4)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{9\sqrt{10010}G_1x\left(x^4-40x^2y^2+30x^2z^2+15y^4+30y^2z^2-20z^4\right)}{16016}-\frac{9\sqrt{10010}G_2y\left(15x^4-40x^2y^2+30x^2z^2+y^4+30y^2z^2-20z^4\right)}{16016}$$
$$-\frac{\sqrt{15015}G_3z\left(51x^4-66x^2y^2-80x^2z^2+51y^4-80y^2z^2+16z^4\right)}{4004}-\frac{\sqrt{6006}G_{3x}y\left(73x^4+20x^2y^2-498x^2z^2-53y^4+510y^2z^2-172z^4\right)}{16016}$$
$$+\frac{\sqrt{6006}G_{3y}x\left(53x^4-20x^2y^2-510x^2z^2-73y^4+498y^2z^2+172z^4\right)}{16016}-\frac{3\sqrt{1001}G_{az}xyz\left(x^2+y^2-2z^2\right)}{286}+\frac{3\sqrt{15015}G_{bz}xyz\left(x-y\right)\left(x+y\right)}{143}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{1430}G_1z\left(2x^6-345x^4y^2+105x^4z^2+1185x^2y^4-1680x^2y^2z^2+105x^2z^4-184y^6+525y^4z^2-147y^2z^4+2z^6\right)}{5720}$$
$$-\frac{3\sqrt{1430}G_2xyz\left(18x^4-305x^2y^2+245x^2z^2+249y^4-525y^2z^2+84z^4\right)}{5720}$$
$$+\frac{3\sqrt{2145}G_3x\left(y-z\right)\left(y+z\right)\left(3x^4-5x^2y^2-5x^2z^2-8y^4+127y^2z^2-8z^4\right)}{1430}-\frac{3\sqrt{858}G_{3x}xyz\left(6x^4+45x^2y^2-65x^2z^2+39y^4-175y^2z^2+72z^4\right)}{1144}$$
$$+\frac{\sqrt{858}G_{3y}z\left(2x^6+15x^4y^2-15x^4z^2-75x^2y^4+120x^2y^2z^2-3x^2z^4-88y^6+465y^4z^2-291y^2z^4+14z^6\right)}{1144}$$
$$+\frac{\sqrt{143}G_{az}y\left(2x^6+15x^4y^2-75x^4z^2+24x^2y^4-330x^2y^2z^2+240x^2z^4+11y^6-255y^4z^2+480y^2z^4-112z^6\right)}{572}$$
$$+\frac{\sqrt{2145}G_{bz}y\left(2x^6-45x^4y^2+105x^4z^2-30x^2y^4+570x^2y^2z^2-390x^2z^4+17y^6-327y^4z^2+450y^2z^4-64z^6\right)}{2860}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u,2)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{1430}G_1x\left(53x^6-1128x^4y^2+15x^4z^2+1875x^2y^4+30x^2y^2z^2-30x^2z^4-376y^6+15y^4z^2-30y^2z^4+8z^6\right)}{5720}$$
$$+\frac{\sqrt{1430}G_2y\left(376x^6-1875x^4y^2-15x^4z^2+1128x^2y^4-30x^2y^2z^2+30x^2z^4-53y^6-15y^4z^2+30y^2z^4-8z^6\right)}{5720}$$
$$-\frac{3\sqrt{2145}G_3z\left(x-y\right)\left(x+y\right)\left(8x^4-127x^2y^2+5x^2z^2+8y^4+5y^2z^2-3z^4\right)}{1430}$$
$$-\frac{3\sqrt{858}G_{3x}y\left(8x^6-5x^4y^2-105x^4z^2-12x^2y^4+150x^2y^2z^2+30x^2z^4+y^6-9y^4z^2-10y^2z^4\right)}{1144}$$
$$+\frac{3\sqrt{858}G_{3y}x\left(x^6-12x^4y^2-9x^4z^2-5x^2y^4+150x^2y^2z^2-10x^2z^4+8y^6-105y^4z^2+30y^2z^4\right)}{1144}$$
$$-\frac{3\sqrt{143}G_{az}xyz\left(x-y\right)\left(x+y\right)\left(3x^2+3y^2-10z^2\right)}{52}+\frac{3\sqrt{2145}G_{bz}xyz\left(111x^4-350x^2y^2-20x^2z^2+111y^4-20y^2z^2+12z^4\right)}{2860}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u,3)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{10010}G_1z\left(56x^6-885x^4y^2+15x^4z^2+735x^2y^4+300x^2y^2z^2-39x^2z^4-40y^6-45y^4z^2-3y^2z^4+2z^6\right)}{5720}$$
$$+\frac{3\sqrt{10010}G_2xyz\left(120x^4-365x^2y^2-35x^2z^2+87y^4+75y^2z^2-12z^4\right)}{5720}$$
$$+\frac{\sqrt{15015}G_3x\left(2x^6-21x^4y^2-21x^4z^2-15x^2y^4+300x^2y^2z^2-15x^2z^4+8y^6-75y^4z^2-75y^2z^4+8z^6\right)}{1430}$$
$$-\frac{3\sqrt{6006}G_{3x}xyz\left(24x^4+15x^2y^2-95x^2z^2-9y^4+15y^2z^2+24z^4\right)}{1144}$$
$$+\frac{\sqrt{6006}G_{3y}z\left(8x^6-75x^4y^2-15x^4z^2-75x^2y^4+300x^2y^2z^2-21x^2z^4+8y^6-15y^4z^2-21y^2z^4+2z^6\right)}{1144}$$
$$+\frac{\sqrt{1001}G_{az}y\left(8x^6+15x^4y^2-165x^4z^2+6x^2y^4-150x^2y^2z^2+240x^2z^4-y^6+15y^4z^2-16z^6\right)}{572}$$
$$-\frac{3\sqrt{15015}G_{bz}y\left(8x^6-5x^4y^2-105x^4z^2-12x^2y^4+150x^2y^2z^2+30x^2z^4+y^6-9y^4z^2-10y^2z^4\right)}{2860}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u, 4)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{\sqrt{10010}G_1x\left(7x^6-78x^4y^2-69x^4z^2-45x^2y^4+1050x^2y^2z^2-60x^2z^4+40y^6-465y^4z^2-60y^2z^4+16z^6\right)}{5720}$$
$$-\frac{\sqrt{10010}G_2y\left(40x^6-45x^4y^2-465x^4z^2-78x^2y^4+1050x^2y^2z^2-60x^2z^4+7y^6-69y^4z^2-60y^2z^4+16z^6\right)}{5720}$$
$$+\frac{\sqrt{15015}G_3z\left(8x^6-75x^4y^2-15x^4z^2-75x^2y^4+300x^2y^2z^2-21x^2z^4+8y^6-15y^4z^2-21y^2z^4+2z^6\right)}{1430}$$
$$+\frac{\sqrt{6006}G_{3x}y\left(8x^6+15x^4y^2-165x^4z^2+6x^2y^4-150x^2y^2z^2+240x^2z^4-y^6+15y^4z^2-16z^6\right)}{1144}$$
$$-\frac{\sqrt{6006}G_{3y}x\left(x^6-6x^4y^2-15x^4z^2-15x^2y^4+150x^2y^2z^2-8y^6+165y^4z^2-240y^2z^4+16z^6\right)}{1144}$$
$$+\frac{3\sqrt{1001}G_{az}xyz\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{572}-\frac{3\sqrt{15015}G_{bz}xyz\left(x-y\right)\left(x+y\right)\left(3x^2+3y^2-10z^2\right)}{260}$$