PG No. 2 $C_{\rm i}$ $\bar{1}$ [triclinic] (polar, internal polar dipole)

* Harmonics for rank 0

$\vec{\mathbb{Q}}_0^{(1,1)}[q](A_g)$

** symmetry

$$1$$

** expression

$$\frac{\sqrt{3}Q_x x}{3} + \frac{\sqrt{3}Q_y y}{3} + \frac{\sqrt{3}Q_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](A_u, 1)$

** symmetry

$$x$$

** expression

$$Q_x$$

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](A_u, 2)$

** symmetry

$$y$$

** expression

$$Q_y$$

$\vec{\mathbb{Q}}_1^{(1,-1)}[q](A_u, 3)$

** symmetry

$$z$$

** expression

$$Q_z$$

$\vec{\mathbb{Q}}_1^{(1,1)}[q](A_u, 1)$

** symmetry

$$x$$

** expression

$$\frac{\sqrt{10}Q_x\left(2x^2 - y^2 - z^2\right)}{10} + \frac{3\sqrt{10}Q_y xy}{10} + \frac{3\sqrt{10}Q_z xz}{10}$$

$\vec{\mathbb{Q}}_1^{(1,1)}[q](A_u, 2)$

** symmetry

$$y$$

** expression

$$\frac{3\sqrt{10}Q_x xy}{10} - \frac{\sqrt{10}Q_y\left(x^2 - 2y^2 + z^2\right)}{10} + \frac{3\sqrt{10}Q_z yz}{10}$$

$\vec{\mathbb{Q}}_1^{(1,1)}[q](A_u, 3)$

** symmetry

$$z$$

** expression

$$\frac{3\sqrt{10}Q_x xz}{10} + \frac{3\sqrt{10}Q_y yz}{10} - \frac{\sqrt{10}Q_z\left(x^2 + y^2 - 2z^2\right)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A_g, 1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}Q_x x}{6} - \frac{\sqrt{6}Q_y y}{6} + \frac{\sqrt{6}Q_z z}{3}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x x}{2} - \frac{\sqrt{2}Q_y y}{2}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A_g, 3)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_y z}{2} + \frac{\sqrt{2}Q_z y}{2}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A_g, 4)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

$\vec{\mathbb{Q}}_2^{(1,-1)}[q](A_g, 5)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} + \frac{\sqrt{2}Q_y x}{2}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A_g, 1)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}Q_x x\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}Q_y y\left(x^2+y^2-4z^2\right)}{14} - \frac{\sqrt{21}Q_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x x\left(3x^2-7y^2-2z^2\right)}{14} + \frac{\sqrt{7}Q_y y\left(7x^2-3y^2+2z^2\right)}{14} + \frac{5\sqrt{7}Q_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A_g, 3)$
** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{7}Q_x xyz}{7} - \frac{\sqrt{7}Q_y z\left(x^2-4y^2+z^2\right)}{7} - \frac{\sqrt{7}Q_z y\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{Q}}_2^{(1,1)}[q](A_g, 4)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{7}Q_x z\left(4x^2-y^2-z^2\right)}{7} + \frac{5\sqrt{7}Q_y xyz}{7} - \frac{\sqrt{7}Q_z x\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{Q}}_{2}^{(1,1)}[q](A_g, 5)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}Q_x y \left(4x^2 - y^2 - z^2\right)}{7} - \frac{\sqrt{7}Q_y x \left(x^2 - 4y^2 + z^2\right)}{7} + \frac{5\sqrt{7}Q_z xyz}{7}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$Q_x yz + Q_y xz + Q_z xy$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 2)$
** symmetry

$$\frac{x \left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_x \left(2x^2 - y^2 - z^2\right)}{10} - \frac{\sqrt{15}Q_y xy}{5} - \frac{\sqrt{15}Q_z xz}{5}$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 3)$
** symmetry

$$-\frac{y \left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xy}{5} - \frac{\sqrt{15}Q_y \left(x^2 - 2y^2 + z^2\right)}{10} - \frac{\sqrt{15}Q_z yz}{5}$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 4)$
** symmetry

$$-\frac{z \left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_x xz}{5} - \frac{\sqrt{15}Q_y yz}{5} - \frac{\sqrt{15}Q_z \left(x^2 + y^2 - 2z^2\right)}{10}$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 5)$
** symmetry

$$\frac{\sqrt{15}x \left(y - z\right) \left(y + z\right)}{2}$$

** expression

$$\frac{Q_x \left(y - z\right) \left(y + z\right)}{2} + Q_y xy - Q_z xz$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 6)$
** symmetry

$$-\frac{\sqrt{15}y \left(x - z\right) \left(x + z\right)}{2}$$

** expression

$$-Q_x xy - \frac{Q_y \left(x - z\right) \left(x + z\right)}{2} + Q_z yz$$

$\vec{\mathbb{Q}}_{3}^{(1,-1)}[q](A_u, 7)$
** symmetry

$$\frac{\sqrt{15}z \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$Q_{x} x z - Q_{y} y z + \frac{Q_{z} \left(x - y\right) \left(x + y\right)}{2}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,1)$
** symmetry

$$\sqrt{15} x y z$$

** expression

$$\frac{\sqrt{15} Q_{x} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{6} - \frac{\sqrt{15} Q_{y} x z \left(x^{2} - 6 y^{2} + z^{2}\right)}{6} - \frac{\sqrt{15} Q_{z} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{6}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,2)$
** symmetry

$$\frac{x \left(2 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x} \left(8 x^{4} - 24 x^{2} y^{2} - 24 x^{2} z^{2} + 3 y^{4} + 6 y^{2} z^{2} + 3 z^{4}\right)}{12} + \frac{5 Q_{y} x y \left(4 x^{2} - 3 y^{2} - 3 z^{2}\right)}{12} + \frac{5 Q_{z} x z \left(4 x^{2} - 3 y^{2} - 3 z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,3)$
** symmetry

$$- \frac{y \left(3 x^{2} - 2 y^{2} + 3 z^{2}\right)}{2}$$

** expression

$$- \frac{5 Q_{x} x y \left(3 x^{2} - 4 y^{2} + 3 z^{2}\right)}{12} + \frac{Q_{y} \left(3 x^{4} - 24 x^{2} y^{2} + 6 x^{2} z^{2} + 8 y^{4} - 24 y^{2} z^{2} + 3 z^{4}\right)}{12} - \frac{5 Q_{z} y z \left(3 x^{2} - 4 y^{2} + 3 z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,4)$
** symmetry

$$- \frac{z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{2}$$

** expression

$$- \frac{5 Q_{x} x z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{12} - \frac{5 Q_{y} y z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{12} + \frac{Q_{z} \left(3 x^{4} + 6 x^{2} y^{2} - 24 x^{2} z^{2} + 3 y^{4} - 24 y^{2} z^{2} + 8 z^{4}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,5)$
** symmetry

$$\frac{\sqrt{15} x \left(y - z\right) \left(y + z\right)}{2}$$

** expression

$$\frac{\sqrt{15} Q_{x} \left(y - z\right) \left(y + z\right) \left(6 x^{2} - y^{2} - z^{2}\right)}{12} - \frac{\sqrt{15} Q_{y} x y \left(2 x^{2} - 5 y^{2} + 9 z^{2}\right)}{12} + \frac{\sqrt{15} Q_{z} x z \left(2 x^{2} + 9 y^{2} - 5 z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,6)$
** symmetry

$$- \frac{\sqrt{15} y \left(x - z\right) \left(x + z\right)}{2}$$

** expression

$$- \frac{\sqrt{15} Q_{x} x y \left(5 x^{2} - 2 y^{2} - 9 z^{2}\right)}{12} + \frac{\sqrt{15} Q_{y} \left(x - z\right) \left(x + z\right) \left(x^{2} - 6 y^{2} + z^{2}\right)}{12} - \frac{\sqrt{15} Q_{z} y z \left(9 x^{2} + 2 y^{2} - 5 z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(1,1)}[q](A_u,7)$
** symmetry

$$\frac{\sqrt{15} z \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{15} Q_{x} x z \left(5 x^{2} - 9 y^{2} - 2 z^{2}\right)}{12} + \frac{\sqrt{15} Q_{y} y z \left(9 x^{2} - 5 y^{2} + 2 z^{2}\right)}{12} - \frac{\sqrt{15} Q_{z} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}Q_x x\left(2x^2-3y^2-3z^2\right)}{6}-\frac{\sqrt{3}Q_y y\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}Q_z z\left(3x^2+3y^2-2z^2\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}Q_x x\left(x^2-6y^2+3z^2\right)}{42}+\frac{\sqrt{105}Q_y y\left(6x^2-y^2-3z^2\right)}{42}-\frac{\sqrt{105}Q_z z\left(3x^2+3y^2-2z^2\right)}{42}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,3)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}Q_x x\left(x^2-3z^2\right)}{14}-\frac{\sqrt{35}Q_y y\left(y^2-3z^2\right)}{14}-\frac{3\sqrt{35}Q_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,4)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_y z\left(3y^2-z^2\right)}{4}+\frac{\sqrt{5}Q_z y\left(y^2-3z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,5)$
** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{5}Q_x z\left(3x^2-z^2\right)}{4}-\frac{\sqrt{5}Q_z x\left(x^2-3z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,6)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x y\left(3x^2-y^2\right)}{4}+\frac{\sqrt{5}Q_y x\left(x^2-3y^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,7)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{35}Q_x xyz}{7}+\frac{\sqrt{35}Q_y z\left(6x^2-3y^2-z^2\right)}{28}+\frac{\sqrt{35}Q_z y\left(6x^2-y^2-3z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,8)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}Q_x z\left(3x^2-6y^2+z^2\right)}{28}+\frac{3\sqrt{35}Q_y xyz}{7}-\frac{\sqrt{35}Q_z x\left(x^2-6y^2+3z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(1,-1)}[q](A_g,9)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}Q_x y\left(3x^2+y^2-6z^2\right)}{28}-\frac{\sqrt{35}Q_y x\left(x^2+3y^2-6z^2\right)}{28}+\frac{3\sqrt{35}Q_z xyz}{7}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}Q_x x\left(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}Q_y y\left(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}Q_z z\left(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4\right)}{66}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}Q_x x\left(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4\right)}{132}-\frac{\sqrt{33}Q_y y\left(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4\right)}{132}+\frac{\sqrt{33}Q_z z\left(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4\right)}{132}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,3)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{11}Q_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}+\frac{\sqrt{11}Q_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}+\frac{21\sqrt{11}Q_z z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,4)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{9\sqrt{77}Q_x xyz\left(y-z\right)\left(y+z\right)}{22}-\frac{\sqrt{77}Q_y z\left(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4\right)}{22}-\frac{\sqrt{77}Q_z y\left(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,5)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_x z\left(6x^4-3x^2y^2-11x^2z^2+y^2z^2+z^4\right)}{22}-\frac{9\sqrt{77}Q_y xyz\left(x-z\right)\left(x+z\right)}{22}+\frac{\sqrt{77}Q_z x\left(x^4+x^2y^2-11x^2z^2-3y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g,6)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77} Q_x y \left(6 x^4 - 11 x^2 y^2 - 3 x^2 z^2 + y^4 + y^2 z^2\right)}{22} - \frac{\sqrt{77} Q_y x \left(x^4 - 11 x^2 y^2 + x^2 z^2 + 6 y^4 - 3 y^2 z^2\right)}{22} + \frac{9 \sqrt{77} Q_z x y z \left(x - y\right) \left(x + y\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g, 7)$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^2 - y^2 - z^2\right)}{2}$$

** expression

$$\frac{21 \sqrt{11} Q_x x y z \left(2 x^2 - y^2 - z^2\right)}{22} - \frac{\sqrt{11} Q_y z \left(6 x^4 - 51 x^2 y^2 + 5 x^2 z^2 + 6 y^4 + 5 y^2 z^2 - z^4\right)}{22} - \frac{\sqrt{11} Q_z y \left(6 x^4 + 5 x^2 y^2 - 51 x^2 z^2 - y^4 + 5 y^2 z^2 + 6 z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g, 8)$

** symmetry

$$- \frac{\sqrt{5} x z \left(x^2 - 6 y^2 + z^2\right)}{2}$$

** expression

$$- \frac{\sqrt{11} Q_x z \left(6 x^4 - 51 x^2 y^2 + 5 x^2 z^2 + 6 y^4 + 5 y^2 z^2 - z^4\right)}{22} - \frac{21 \sqrt{11} Q_y x y z \left(x^2 - 2 y^2 + z^2\right)}{22} + \frac{\sqrt{11} Q_z x \left(x^4 - 5 x^2 y^2 - 5 x^2 z^2 - 6 y^4 + 51 y^2 z^2 - 6 z^4\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(1,1)}[q](A_g, 9)$

** symmetry

$$- \frac{\sqrt{5} x y \left(x^2 + y^2 - 6 z^2\right)}{2}$$

** expression

$$- \frac{\sqrt{11} Q_x y \left(6 x^4 + 5 x^2 y^2 - 51 x^2 z^2 - y^4 + 5 y^2 z^2 + 6 z^4\right)}{22} + \frac{\sqrt{11} Q_y x \left(x^4 - 5 x^2 y^2 - 5 x^2 z^2 - 6 y^4 + 51 y^2 z^2 - 6 z^4\right)}{22} - \frac{21 \sqrt{11} Q_z x y z \left(x^2 + y^2 - 2 z^2\right)}{22}$$