PG No. 36 $D_{3\mathrm{d}}(1)$ $\bar{3}m$ (-31m setting) [trigonal] (axial, internal polar octupole)

* Harmonics for rank 0
* Harmonics for rank 1
  $\vec{\mathbb{G}}_1^{(3,2)}[q](A_{2g})$
  ** symmetry

$$z$$

  ** expression

$$-\frac{3\sqrt{70}Q_1 x\left(x^2-3y^2\right)}{56}+\frac{3\sqrt{70}Q_2 y\left(3x^2-y^2\right)}{56}-\frac{\sqrt{105}Q_3 z\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}Q_{3x} y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{42}Q_{3y} x\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{105}Q_{bz} xyz}{7}$$

  $\vec{\mathbb{G}}_{1,1}^{(3,2)}[q](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(3,2)}[q](E_g)$
  ** symmetry

$$x$$

$$y$$

  ** expression

$$\frac{3\sqrt{70}Q_1 z\left(x-y\right)\left(x+y\right)}{56}-\frac{3\sqrt{70}Q_2 xyz}{28}-\frac{\sqrt{105}Q_3 x\left(y-z\right)\left(y+z\right)}{14}-\frac{5\sqrt{42}Q_{3x} xyz}{28}-\frac{\sqrt{42}Q_{3y} z\left(x^2+11y^2-4z^2\right)}{56}+\frac{3\sqrt{7}Q_{az} y\left(x^2+y^2-4z^2\right)}{28}-\frac{\sqrt{105}Q_{bz} y\left(x^2-y^2+2z^2\right)}{28}$$

$$-\frac{3\sqrt{70}Q_1 xyz}{28}-\frac{3\sqrt{70}Q_2 z\left(x-y\right)\left(x+y\right)}{56}+\frac{\sqrt{105}Q_3 y\left(x-z\right)\left(x+z\right)}{14}+\frac{\sqrt{42}Q_{3x} z\left(11x^2+y^2-4z^2\right)}{56}+\frac{5\sqrt{42}Q_{3y} xyz}{28}-\frac{3\sqrt{7}Q_{az} x\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{105}Q_{bz} x\left(x^2-y^2-2z^2\right)}{28}$$

* Harmonics for rank 2
  $\vec{\mathbb{G}}_2^{(3,0)}[q](A_{1u})$
  ** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

  ** expression

$$-\frac{\sqrt{210}Q_3\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{21}Q_{3x} yz}{7}-\frac{\sqrt{21}Q_{3y} xz}{7}+\frac{\sqrt{210}Q_{bz} xy}{14}$$

  $\vec{\mathbb{G}}_2^{(3,2)}[q](A_{1u})$
  ** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

  ** expression

$$-\frac{\sqrt{70}Q_1 xz\left(x^2-3y^2\right)}{8}+\frac{\sqrt{70}Q_2 yz\left(3x^2-y^2\right)}{8}+\frac{\sqrt{105}Q_3\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{42}-\frac{5\sqrt{42}Q_{3x} yz\left(3x^2+3y^2-4z^2\right)}{168}+\frac{5\sqrt{42}Q_{3y} xz\left(3x^2+3y^2-4z^2\right)}{168}-\frac{\sqrt{105}Q_{bz} xy\left(x^2+y^2-6z^2\right)}{21}$$

  $\vec{\mathbb{G}}_{2,1}^{(3,0)}[q](E_u,1)$, $\vec{\mathbb{G}}_{2,2}^{(3,0)}[q](E_u,1)$
  ** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

  ** expression

$$\frac{\sqrt{105}Q_1 xy}{14}+\frac{\sqrt{105}Q_2\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{7}Q_{3x}\left(5x^2-y^2-4z^2\right)}{28}+\frac{3\sqrt{7}Q_{3y} xy}{14}+\frac{\sqrt{42}Q_{az} xz}{7}$$

$$\frac{\sqrt{105}Q_1\left(x-y\right)\left(x+y\right)}{28}-\frac{\sqrt{105}Q_2 xy}{14}+\frac{3\sqrt{7}Q_{3x} xy}{14}-\frac{\sqrt{7}Q_{3y}\left(x^2-5y^2+4z^2\right)}{28}+\frac{\sqrt{42}Q_{az} yz}{7}$$

  $\vec{\mathbb{G}}_{2,1}^{(3,0)}[q](E_u,2)$, $\vec{\mathbb{G}}_{2,2}^{(3,0)}[q](E_u,2)$
  ** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$- \frac{\sqrt{105} Q_{1} y z}{14} - \frac{\sqrt{105} Q_{2} x z}{14} + \frac{3 \sqrt{7} Q_{3x} x z}{14} - \frac{3 \sqrt{7} Q_{3y} y z}{14} - \frac{\sqrt{42} Q_{az} \left(x - y\right) \left(x + y\right)}{28} + \frac{\sqrt{70} Q_{bz} \left(x^{2} + y^{2} - 2 z^{2}\right)}{28}$$

$$\frac{\sqrt{105} Q_{1} x z}{14} - \frac{\sqrt{105} Q_{2} y z}{14} - \frac{\sqrt{70} Q_{3} \left(x^{2} + y^{2} - 2 z^{2}\right)}{28} - \frac{3 \sqrt{7} Q_{3x} y z}{14} - \frac{3 \sqrt{7} Q_{3y} x z}{14} + \frac{\sqrt{42} Q_{az} x y}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(3,2)}[q](E_u,1)$, $\vec{\mathbb{G}}_{2,2}^{(3,2)}[q](E_u,1)$

** symmetry

$$\sqrt{3} y z$$

$$- \sqrt{3} x z$$

** expression

$$- \frac{\sqrt{210} Q_{1} x y \left(11 x^{2} - 17 y^{2} + 18 z^{2}\right)}{168} - \frac{\sqrt{210} Q_{2} \left(2 x^{4} - 21 x^{2} y^{2} + 9 x^{2} z^{2} + 5 y^{4} - 9 y^{2} z^{2}\right)}{168} + \frac{\sqrt{35} Q_{3} y z \left(y - z\right) \left(y + z\right)}{6} - \frac{5 \sqrt{14} Q_{3x} \left(2 x^{4} + 3 x^{2} y^{2} - 15 x^{2} z^{2} + y^{4} - 9 y^{2} z^{2} + 4 z^{4}\right)}{168} - \frac{5 \sqrt{14} Q_{3y} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{168} - \frac{5 \sqrt{21} Q_{az} x z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{84} + \frac{\sqrt{35} Q_{bz} x z \left(x^{2} + 3 y^{2} - 2 z^{2}\right)}{12}$$

$$\frac{\sqrt{210} Q_{1} \left(5 x^{4} - 21 x^{2} y^{2} - 9 x^{2} z^{2} + 2 y^{4} + 9 y^{2} z^{2}\right)}{168} - \frac{\sqrt{210} Q_{2} x y \left(17 x^{2} - 11 y^{2} - 18 z^{2}\right)}{168} + \frac{\sqrt{35} Q_{3} x z \left(x - z\right) \left(x + z\right)}{6} - \frac{5 \sqrt{14} Q_{3x} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{168} - \frac{5 \sqrt{14} Q_{3y} \left(x^{4} + 3 x^{2} y^{2} - 9 x^{2} z^{2} + 2 y^{4} - 15 y^{2} z^{2} + 4 z^{4}\right)}{168} - \frac{5 \sqrt{21} Q_{az} y z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{84} - \frac{\sqrt{35} Q_{bz} y z \left(3 x^{2} + y^{2} - 2 z^{2}\right)}{12}$$

$\vec{\mathbb{G}}_{2,1}^{(3,2)}[q](E_u,2)$, $\vec{\mathbb{G}}_{2,2}^{(3,2)}[q](E_u,2)$

** symmetry

$$\sqrt{3} x y$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$- \frac{\sqrt{210} Q_{1} y z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{168} - \frac{\sqrt{210} Q_{2} x z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{168} + \frac{\sqrt{35} Q_{3} x y \left(x - y\right) \left(x + y\right)}{6} + \frac{5 \sqrt{14} Q_{3x} x z \left(13 x^{2} - 15 y^{2} - 8 z^{2}\right)}{168} + \frac{5 \sqrt{14} Q_{3y} y z \left(15 x^{2} - 13 y^{2} + 8 z^{2}\right)}{168} - \frac{5 \sqrt{21} Q_{az} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{84} + \frac{\sqrt{35} Q_{bz} \left(5 x^{4} - 18 x^{2} y^{2} - 12 x^{2} z^{2} + 5 y^{4} - 12 y^{2} z^{2} + 4 z^{4}\right)}{84}$$

$$\frac{\sqrt{210} Q_{1} x z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{168} - \frac{\sqrt{210} Q_{2} y z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{168} + \frac{\sqrt{35} Q_{3} \left(x^{4} - 12 x^{2} y^{2} + 6 x^{2} z^{2} + y^{4} + 6 y^{2} z^{2} - 2 z^{4}\right)}{42} - \frac{5 \sqrt{14} Q_{3x} y z \left(27 x^{2} - y^{2} - 8 z^{2}\right)}{168} + \frac{5 \sqrt{14} Q_{3y} x z \left(x^{2} - 27 y^{2} + 8 z^{2}\right)}{168} + \frac{5 \sqrt{21} Q_{az} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{42} - \frac{\sqrt{35} Q_{bz} x y \left(x - y\right) \left(x + y\right)}{6}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(3,-2)}[q](A_{1g})$

** symmetry

$$\frac{\sqrt{10} y \left(3 x^{2} - y^{2}\right)}{4}$$

** expression

$$- \frac{\sqrt{3} Q_{2} z}{2} - \frac{\sqrt{2} Q_{3} y}{4} + \frac{\sqrt{2} Q_{bz} x}{4}$$

$\vec{\mathbb{G}}_{3}^{(3,0)}[q](A_{1g})$

** symmetry

$$\frac{\sqrt{10} y \left(3 x^{2} - y^{2}\right)}{4}$$

** expression

$$- \frac{\sqrt{6} Q_{2} z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{12} - \frac{Q_{3} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{4} + \frac{\sqrt{10} Q_{3x} z \left(x - y\right) \left(x + y\right)}{4} - \frac{\sqrt{10} Q_{3y} x y z}{2} - \frac{\sqrt{15} Q_{az} x \left(x^{2} - 3 y^{2}\right)}{12} + \frac{Q_{bz} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(3,2)}[q](A_{1g})$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{33}Q_2z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}+\frac{\sqrt{22}Q_3y\left(25x^4-55x^2y^2+15x^2z^2+4y^4+15y^2z^2-10z^4\right)}{88}+\frac{7\sqrt{55}Q_{3x}z\left(5x^4-18x^2y^2-4x^2z^2+y^4+4y^2z^2\right)}{176}+\frac{7\sqrt{55}Q_{3y}xyz\left(x^2-2y^2+z^2\right)}{22}-\frac{7\sqrt{330}Q_{az}x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{528}+\frac{\sqrt{22}Q_{bz}x\left(13x^4-100x^2y^2-30x^2z^2+55y^4-30y^2z^2+20z^4\right)}{176}$$

$\vec{\mathbb{G}}_3^{(3,-2)}[q](A_{2g},1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{2}Q_{3x}y}{2}-\frac{\sqrt{2}Q_{3y}x}{2}$$

$\vec{\mathbb{G}}_3^{(3,-2)}[q](A_{2g},2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{3}Q_1z}{2}-\frac{\sqrt{2}Q_3x}{4}-\frac{\sqrt{2}Q_{bz}y}{4}$$

$\vec{\mathbb{G}}_3^{(3,0)}[q](A_{2g},1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_1x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{15}Q_2y\left(3x^2-y^2\right)}{12}-\frac{\sqrt{10}Q_3z\left(x-y\right)\left(x+y\right)}{4}-\frac{Q_{3x}y\left(x^2+y^2-4z^2\right)}{4}+\frac{Q_{3y}x\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{10}Q_{bz}xyz}{2}$$

$\vec{\mathbb{G}}_3^{(3,0)}[q](A_{2g},2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{6}Q_1z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{Q_3x\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{10}Q_{3x}xyz}{2}-\frac{\sqrt{10}Q_{3y}z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{15}Q_{az}y\left(3x^2-y^2\right)}{12}-\frac{Q_{bz}y\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(3,2)}[q](A_{2g},1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{7\sqrt{330}Q_1x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{528}-\frac{7\sqrt{330}Q_2y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{528}+\frac{7\sqrt{55}Q_3z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}+\frac{5\sqrt{22}Q_{3x}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{5\sqrt{22}Q_{3y}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{7\sqrt{55}Q_{bz}xyz\left(x^2+y^2-2z^2\right)}{22}$$

$\vec{\mathbb{G}}_3^{(3,2)}[q](A_{2g},2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{33}Q_1z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}+\frac{\sqrt{22}Q_3x\left(4x^4-55x^2y^2+15x^2z^2+25y^4+15y^2z^2-10z^4\right)}{88}-\frac{7\sqrt{55}Q_{3x}xyz\left(2x^2-y^2-z^2\right)}{22}+\frac{7\sqrt{55}Q_{3y}z\left(x^4-18x^2y^2+4x^2z^2+5y^4-4y^2z^2\right)}{176}+\frac{7\sqrt{330}Q_{az}y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{528}-\frac{\sqrt{22}Q_{bz}y\left(55x^4-100x^2y^2-30x^2z^2+13y^4-30y^2z^2+20z^4\right)}{176}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](E_g,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_3x}{12}+\frac{\sqrt{3}Q_{3y}z}{6}-\frac{\sqrt{2}Q_{az}y}{2}+\frac{\sqrt{30}Q_{bz}y}{12}$$

$$\frac{\sqrt{30}Q_3y}{12}-\frac{\sqrt{3}Q_{3x}z}{6}+\frac{\sqrt{2}Q_{az}x}{2}+\frac{\sqrt{30}Q_{bz}x}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](E_g,2)$
** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{2}Q_1x}{4}-\frac{\sqrt{2}Q_2y}{4}-\frac{\sqrt{3}Q_3z}{3}+\frac{\sqrt{30}Q_{3x}y}{12}+\frac{\sqrt{30}Q_{3y}x}{12}$$

$$\frac{\sqrt{2}Q_1y}{4}+\frac{\sqrt{2}Q_2x}{4}+\frac{\sqrt{30}Q_{3x}x}{12}-\frac{\sqrt{30}Q_{3y}y}{12}-\frac{\sqrt{3}Q_{bz}z}{3}$$

$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(3,0)}[q](E_g,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{10}Q_1z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{10}Q_2xyz}{2}-\frac{\sqrt{15}Q_3x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{6}Q_{3y}z\left(3x^2+3y^2-2z^2\right)}{12}+\frac{Q_{az}y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{15}Q_{bz}y\left(3x^2-y^2\right)}{12}$$

$$\frac{\sqrt{10}Q_1xyz}{2}+\frac{\sqrt{10}Q_2z\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{15}Q_3y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{6}Q_{3x}z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{Q_{az}x\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{15}Q_{bz}x\left(x^2-3y^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(3,0)}[q](E_g,2)$
** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{Q_1x\left(x^2+y^2-4z^2\right)}{4}-\frac{Q_2y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{6}Q_3z\left(3x^2+3y^2-2z^2\right)}{12}+\frac{\sqrt{15}Q_{3x}y\left(3x^2-y^2\right)}{12}-\frac{\sqrt{15}Q_{3y}x\left(x^2-3y^2\right)}{12}+\frac{\sqrt{10}Q_{az}xyz}{2}$$

$$\frac{Q_1y\left(x^2+y^2-4z^2\right)}{4}+\frac{Q_2x\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{15}Q_{3x}x\left(x^2-3y^2\right)}{12}+\frac{\sqrt{15}Q_{3y}y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{10}Q_{az}z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{6}Q_{bz}z\left(3x^2+3y^2-2z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(3,2)}[q](E_g,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{55}Q_{1}z\left(5x^{4}-18x^{2}y^{2}-4x^{2}z^{2}+y^{4}+4y^{2}z^{2}\right)}{176}+\frac{7\sqrt{55}Q_{2}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{22}+\frac{\sqrt{330}Q_{3}x\left(4x^{4}+x^{2}y^{2}-41x^{2}z^{2}-3y^{4}+15y^{2}z^{2}+18z^{4}\right)}{264}+\frac{5\sqrt{33}Q_{3y}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{528}-\frac{5\sqrt{22}Q_{az}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{176}-\frac{\sqrt{330}Q_{bz}y\left(15x^{4}+16x^{2}y^{2}-138x^{2}z^{2}+y^{4}-26y^{2}z^{2}+36z^{4}\right)}{528}$$

$$-\frac{7\sqrt{55}Q_{1}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{22}-\frac{7\sqrt{55}Q_{2}z\left(x^{4}-18x^{2}y^{2}+4x^{2}z^{2}+5y^{4}-4y^{2}z^{2}\right)}{176}+\frac{\sqrt{330}Q_{3}y\left(3x^{4}-x^{2}y^{2}-15x^{2}z^{2}-4y^{4}+41y^{2}z^{2}-18z^{4}\right)}{264}-\frac{5\sqrt{33}Q_{3x}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{528}+\frac{5\sqrt{22}Q_{az}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{176}-\frac{\sqrt{330}Q_{bz}x\left(x^{4}+16x^{2}y^{2}-26x^{2}z^{2}+15y^{4}-138y^{2}z^{2}+36z^{4}\right)}{528}$$

$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(3,2)}[q](E_g,2)$

** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{22}Q_{1}x\left(13x^{4}-100x^{2}y^{2}-30x^{2}z^{2}+55y^{4}-30y^{2}z^{2}+20z^{4}\right)}{176}-\frac{\sqrt{22}Q_{2}y\left(55x^{4}-100x^{2}y^{2}-30x^{2}z^{2}+13y^{4}-30y^{2}z^{2}+20z^{4}\right)}{176}+\frac{\sqrt{33}Q_{3}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{132}-\frac{\sqrt{330}Q_{3x}y\left(15x^{4}+16x^{2}y^{2}-138x^{2}z^{2}+y^{4}-26y^{2}z^{2}+36z^{4}\right)}{528}-\frac{\sqrt{330}Q_{3y}x\left(x^{4}+16x^{2}y^{2}-26x^{2}z^{2}+15y^{4}-138y^{2}z^{2}+36z^{4}\right)}{528}-\frac{7\sqrt{55}Q_{az}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{22}$$

$$-\frac{\sqrt{22}Q_{1}y\left(25x^{4}-55x^{2}y^{2}+15x^{2}z^{2}+4y^{4}+15y^{2}z^{2}-10z^{4}\right)}{88}-\frac{\sqrt{22}Q_{2}x\left(4x^{4}-55x^{2}y^{2}+15x^{2}z^{2}+25y^{4}+15y^{2}z^{2}-10z^{4}\right)}{88}-\frac{\sqrt{330}Q_{3x}x\left(4x^{4}+x^{2}y^{2}-41x^{2}z^{2}-3y^{4}+15y^{2}z^{2}+18z^{4}\right)}{264}-\frac{\sqrt{330}Q_{3y}y\left(3x^{4}-x^{2}y^{2}-15x^{2}z^{2}-4y^{4}+41y^{2}z^{2}-18z^{4}\right)}{264}-\frac{7\sqrt{55}Q_{az}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{\sqrt{33}Q_{bz}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{132}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(3,-2)}[q](A_{1u},1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$\frac{\sqrt{21}Q_{3}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{210}Q_{3x}yz}{14}-\frac{\sqrt{210}Q_{3y}xz}{14}-\frac{\sqrt{21}Q_{bz}xy}{7}$$

$\vec{\mathbb{G}}_{4}^{(3,-2)}[q](A_{1u},2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{3\sqrt{5}Q_{1}\left(x^{2}+y^{2}-2z^{2}\right)}{20}+\frac{\sqrt{30}Q_{3}xz}{20}-\frac{\sqrt{3}Q_{3x}xy}{2}-\frac{\sqrt{3}Q_{3y}\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{30}Q_{bz}yz}{20}$$

$\vec{\mathbb{G}}_{4}^{(3,0)}[q](A_{1u},1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$\frac{5\sqrt{77}Q_{1}xz\left(x^{2}-3y^{2}\right)}{44}-\frac{5\sqrt{77}Q_{2}yz\left(3x^{2}-y^{2}\right)}{44}+\frac{5\sqrt{462}Q_{3}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{616}-\frac{3\sqrt{1155}Q_{3x}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}+\frac{3\sqrt{1155}Q_{3y}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}-\frac{5\sqrt{462}Q_{bz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{308}$$

$\vec{\mathbb{G}}_4^{(3,0)}[q](A_{1u},2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{110}Q_1\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{176}-\frac{\sqrt{165}Q_3xz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{66}Q_{3x}xy\left(3x^2-4y^2+3z^2\right)}{22}+\frac{\sqrt{66}Q_{3y}\left(9x^4-42x^2y^2-12x^2z^2+5y^4+12y^2z^2\right)}{176}-\frac{7\sqrt{11}Q_{az}yz\left(3x^2-y^2\right)}{44}-\frac{\sqrt{165}Q_{bz}yz\left(3x^2+3y^2-4z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,2)}[q](A_{1u},1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{9\sqrt{10010}Q_1xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{9\sqrt{10010}Q_2yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{\sqrt{15015}Q_3\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}+\frac{5\sqrt{6006}Q_{3x}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}-\frac{5\sqrt{6006}Q_{3y}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{\sqrt{15015}Q_{bz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{286}$$

$\vec{\mathbb{G}}_4^{(3,2)}[q](A_{1u},2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{143}Q_1\left(19x^6-240x^4y^2-45x^4z^2+255x^2y^4-90x^2y^2z^2+60x^2z^4-14y^6-45y^4z^2+60y^2z^4-8z^6\right)}{2288}+\frac{9\sqrt{143}Q_2xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{208}-\frac{\sqrt{858}Q_3xz\left(13x^4+125x^2y^2-85x^2z^2-20y^4-85y^2z^2+34z^4\right)}{1144}+\frac{\sqrt{2145}Q_{3x}xy\left(43x^4+26x^2y^2-508x^2z^2-17y^4+92y^2z^2+208z^4\right)}{2288}-\frac{\sqrt{2145}Q_{3y}\left(x^6-44x^4y^2+29x^4z^2-31x^2y^4+450x^2y^2z^2-104x^2z^4+14y^6-179y^4z^2+104y^2z^4\right)}{2288}+\frac{27\sqrt{1430}Q_{az}yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{\sqrt{858}Q_{bz}yz\left(125x^4-80x^2y^2-170x^2z^2+59y^4-170y^2z^2+68z^4\right)}{2288}$$

$\vec{\mathbb{G}}_4^{(3,-2)}[q](A_{2u})$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{5}Q_2\left(x^2+y^2-2z^2\right)}{20}+\frac{\sqrt{30}Q_3yz}{20}+\frac{\sqrt{3}Q_{3x}\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{3}Q_{3y}xy}{2}-\frac{\sqrt{30}Q_{bz}xz}{20}$$

$\vec{\mathbb{G}}_4^{(3,0)}[q](A_{2u})$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{110}Q_2\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{176}-\frac{\sqrt{165}Q_3yz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{66}Q_{3x}\left(5x^4-42x^2y^2+12x^2z^2+9y^4-12y^2z^2\right)}{176}+\frac{\sqrt{66}Q_{3y}xy\left(4x^2-3y^2-3z^2\right)}{22}+\frac{7\sqrt{11}Q_{az}xz\left(x^2-3y^2\right)}{44}+\frac{\sqrt{165}Q_{bz}xz\left(3x^2+3y^2-4z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,2)}[q](A_{2u})$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{143}Q_1xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{208}-\frac{3\sqrt{143}Q_2\left(14x^6-255x^4y^2+45x^4z^2+240x^2y^4+90x^2y^2z^2-60x^2z^4-19y^6+45y^4z^2-60y^2z^4+8z^6\right)}{2288}$$
$$+\frac{\sqrt{858}Q_3yz\left(20x^4-125x^2y^2+85x^2z^2-13y^4+85y^2z^2-34z^4\right)}{1144}$$
$$-\frac{\sqrt{2145}Q_{3x}\left(14x^6-31x^4y^2-179x^4z^2-44x^2y^4+450x^2y^2z^2+104x^2z^4+y^6+29y^4z^2-104y^2z^4\right)}{2288}$$
$$-\frac{\sqrt{2145}Q_{3y}xy\left(17x^4-26x^2y^2-92x^2z^2-43y^4+508y^2z^2-208z^4\right)}{2288}-\frac{27\sqrt{1430}Q_{az}xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}$$
$$+\frac{\sqrt{858}Q_{bz}xz\left(59x^4-80x^2y^2-170x^2z^2+125y^4-170y^2z^2+68z^4\right)}{2288}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E_u,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{35}Q_1xy}{70}-\frac{3\sqrt{35}Q_2\left(x-y\right)\left(x+y\right)}{140}+\frac{\sqrt{210}Q_3yz}{20}-\frac{\sqrt{21}Q_{3x}\left(x^2-3y^2+2z^2\right)}{28}-\frac{\sqrt{21}Q_{3y}xy}{7}+\frac{3\sqrt{14}Q_{az}xz}{14}+\frac{\sqrt{210}Q_{bz}xz}{20}$$

$$-\frac{3\sqrt{35}Q_1\left(x-y\right)\left(x+y\right)}{140}+\frac{3\sqrt{35}Q_2xy}{70}+\frac{\sqrt{210}Q_3xz}{20}-\frac{\sqrt{21}Q_{3x}xy}{7}+\frac{\sqrt{21}Q_{3y}\left(3x^2-y^2-2z^2\right)}{28}+\frac{3\sqrt{14}Q_{az}yz}{14}-\frac{\sqrt{210}Q_{bz}yz}{20}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E_u,2)$
** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

** expression

$$-\frac{3\sqrt{10}Q_1yz}{10}+\frac{3\sqrt{10}Q_2xz}{10}+\frac{\sqrt{15}Q_3xy}{5}-\frac{\sqrt{15}Q_{bz}\left(x-y\right)\left(x+y\right)}{10}$$

$$\frac{3\sqrt{10}Q_1xz}{10}+\frac{3\sqrt{10}Q_2yz}{10}-\frac{\sqrt{15}Q_3\left(x-y\right)\left(x+y\right)}{10}-\frac{\sqrt{15}Q_{bz}xy}{5}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E_u,3)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{9\sqrt{70}Q_1yz}{140}+\frac{9\sqrt{70}Q_2xz}{140}+\frac{\sqrt{42}Q_{3x}xz}{28}-\frac{\sqrt{42}Q_{3y}yz}{28}+\frac{3\sqrt{7}Q_{az}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}Q_{bz}\left(x^2+y^2-2z^2\right)}{35}$$

$$-\frac{9\sqrt{70}Q_1xz}{140}+\frac{9\sqrt{70}Q_2yz}{140}-\frac{\sqrt{105}Q_3\left(x^2+y^2-2z^2\right)}{35}-\frac{\sqrt{42}Q_{3x}yz}{28}-\frac{\sqrt{42}Q_{3y}xz}{28}-\frac{3\sqrt{7}Q_{az}xy}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E_u,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{\sqrt{770}Q_1xy\left(x^2-6y^2+15z^2\right)}{154}-\frac{\sqrt{770}Q_2\left(3x^4+42x^2y^2-60x^2z^2-17y^4+60y^2z^2\right)}{1232}-\frac{\sqrt{1155}Q_3yz\left(3x^2-y^2\right)}{44}-\frac{\sqrt{462}Q_{3x}\left(5x^4+18x^2y^2-48x^2z^2+13y^4-96y^2z^2+24z^4\right)}{1232}+\frac{\sqrt{462}Q_{3y}xy\left(x^2+y^2-6z^2\right)}{154}-\frac{9\sqrt{77}Q_{az}xz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{1155}Q_{bz}xz\left(x^2-3y^2\right)}{44}$$

$$-\frac{\sqrt{770}Q_1\left(17x^4-42x^2y^2-60x^2z^2-3y^4+60y^2z^2\right)}{1232}+\frac{\sqrt{770}Q_2xy\left(6x^2-y^2-15z^2\right)}{154}+\frac{\sqrt{1155}Q_3xz\left(x^2-3y^2\right)}{44}+\frac{\sqrt{462}Q_{3x}xy\left(x^2+y^2-6z^2\right)}{154}-\frac{\sqrt{462}Q_{3y}\left(13x^4+18x^2y^2-96x^2z^2+5y^4-48y^2z^2+24z^4\right)}{1232}-\frac{9\sqrt{77}Q_{az}yz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{1155}Q_{bz}yz\left(3x^2-y^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E_u,2)$

** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

** expression

$$-\frac{\sqrt{55}Q_1yz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{55}Q_2xz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{330}Q_3xy\left(x^2+y^2-6z^2\right)}{44}-\frac{7\sqrt{33}Q_{3x}xz\left(x^2-3y^2\right)}{44}+\frac{7\sqrt{33}Q_{3y}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{22}Q_{az}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}-\frac{\sqrt{330}Q_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{88}$$

$$\frac{\sqrt{55}Q_1xz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{55}Q_2yz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{330}Q_3\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{88}-\frac{7\sqrt{33}Q_{3x}yz\left(3x^2-y^2\right)}{44}-\frac{7\sqrt{33}Q_{3y}xz\left(x^2-3y^2\right)}{44}+\frac{7\sqrt{22}Q_{az}xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{330}Q_{bz}xy\left(x^2+y^2-6z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E_u,3)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{5\sqrt{385}Q_1yz\left(3x^2+3y^2-4z^2\right)}{308}+\frac{5\sqrt{385}Q_2xz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{2310}Q_3xy\left(x-y\right)\left(x+y\right)}{44}+\frac{\sqrt{231}Q_{3x}xz\left(13x^2-15y^2-8z^2\right)}{308}+\frac{\sqrt{231}Q_{3y}yz\left(15x^2-13y^2+8z^2\right)}{308}-\frac{13\sqrt{154}Q_{az}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{616}-\frac{\sqrt{2310}Q_{bz}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{616}$$

$$-\frac{5\sqrt{385}Q_1xz\left(3x^2+3y^2-4z^2\right)}{308}+\frac{5\sqrt{385}Q_2yz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{2310}Q_3\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{308}-\frac{\sqrt{231}Q_{3x}yz\left(27x^2-y^2-8z^2\right)}{308}+\frac{\sqrt{231}Q_{3y}xz\left(x^2-27y^2+8z^2\right)}{308}+\frac{13\sqrt{154}Q_{az}xy\left(x^2+y^2-6z^2\right)}{308}+\frac{\sqrt{2310}Q_{bz}xy\left(x-y\right)\left(x+y\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E_u,1)$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{1001}Q_1xy\left(13x^4-10x^2y^2-100x^2z^2-23y^4+260y^2z^2-80z^4\right)}{2288}+\frac{3\sqrt{1001}Q_2\left(2x^6-25x^4y^2-5x^4z^2-20x^2y^4+270x^2y^2z^2-40x^2z^4+7y^6-85y^4z^2+40y^2z^4\right)}{2288}+\frac{\sqrt{6006}Q_3yz\left(20x^4-5x^2y^2-35x^2z^2-25y^4+85y^2z^2-22z^4\right)}{1144}+\frac{\sqrt{15015}Q_{3x}\left(2x^6+7x^4y^2-37x^4z^2+8x^2y^4-90x^2y^2z^2+52x^2z^4+3y^6-53y^4z^2+68y^2z^4-8z^6\right)}{2288}-\frac{\sqrt{15015}Q_{3y}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}+\frac{3\sqrt{10010}Q_{az}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}-\frac{\sqrt{6006}Q_{bz}xz\left(5x^4+100x^2y^2-50x^2z^2+95y^4-290y^2z^2+44z^4\right)}{2288}$$

$$-\frac{3\sqrt{1001}Q_1\left(7x^6-20x^4y^2-85x^4z^2-25x^2y^4+270x^2y^2z^2+40x^2z^4+2y^6-5y^4z^2-40y^2z^4\right)}{2288}$$
$$+\frac{3\sqrt{1001}Q_2xy\left(23x^4+10x^2y^2-260x^2z^2-13y^4+100y^2z^2+80z^4\right)}{2288}$$
$$-\frac{\sqrt{6006}Q_3xz\left(25x^4+5x^2y^2-85x^2z^2-20y^4+35y^2z^2+22z^4\right)}{1144}-\frac{\sqrt{15015}Q_{3x}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}$$
$$+\frac{\sqrt{15015}Q_{3y}\left(3x^6+8x^4y^2-53x^4z^2+7x^2y^4-90x^2y^2z^2+68x^2z^4+2y^6-37y^4z^2+52y^2z^4-8z^6\right)}{2288}$$
$$+\frac{3\sqrt{10010}Q_{az}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{\sqrt{6006}Q_{bz}yz\left(95x^4+100x^2y^2-290x^2z^2+5y^4-50y^2z^2+44z^4\right)}{2288}$$

$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E_u,2)$
** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

** expression

$$-\frac{3\sqrt{286}Q_1yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{286}Q_2xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$-\frac{\sqrt{429}Q_3xy\left(47x^4-170x^2y^2+40x^2z^2+47y^4+40y^2z^2-40z^4\right)}{572}-\frac{3\sqrt{4290}Q_{3x}xz\left(17x^4-122x^2y^2-16x^2z^2+37y^4+48y^2z^2\right)}{2288}$$
$$-\frac{3\sqrt{4290}Q_{3y}yz\left(37x^4-122x^2y^2+48x^2z^2+17y^4-16y^2z^2\right)}{2288}+\frac{9\sqrt{715}Q_{az}\left(x^2+y^2-10z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{1144}$$
$$-\frac{\sqrt{429}Q_{bz}\left(x-y\right)\left(x+y\right)\left(19x^4-226x^2y^2-40x^2z^2+19y^4-40y^2z^2+40z^4\right)}{1144}$$

$$\frac{3\sqrt{286}Q_1xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{286}Q_2yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$+\frac{\sqrt{429}Q_3\left(x-y\right)\left(x+y\right)\left(7x^4-118x^2y^2+20x^2z^2+7y^4+20y^2z^2-20z^4\right)}{572}$$
$$-\frac{3\sqrt{4290}Q_{3x}yz\left(73x^4-98x^2y^2-48x^2z^2+5y^4+16y^2z^2\right)}{2288}+\frac{3\sqrt{4290}Q_{3y}xz\left(5x^4-98x^2y^2+16x^2z^2+73y^4-48y^2z^2\right)}{2288}$$
$$+\frac{9\sqrt{715}Q_{az}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{286}-\frac{\sqrt{429}Q_{bz}xy\left(13x^4-40x^2y^2-10x^2z^2+13y^4-10y^2z^2+10z^4\right)}{143}$$

$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E_u,3)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{2002}Q_1yz\left(25x^4-60x^2y^2+10x^2z^2+3y^4+10y^2z^2-4z^4\right)}{2288}-\frac{9\sqrt{2002}Q_2xz\left(3x^4-60x^2y^2+10x^2z^2+25y^4+10y^2z^2-4z^4\right)}{2288}$$
$$+\frac{3\sqrt{3003}Q_3xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}-\frac{\sqrt{30030}Q_{3x}xz\left(17x^4+16x^2y^2-62x^2z^2-y^4-14y^2z^2+20z^4\right)}{2288}$$
$$-\frac{\sqrt{30030}Q_{3y}yz\left(x^4-16x^2y^2+14x^2z^2-17y^4+62y^2z^2-20z^4\right)}{2288}+\frac{3\sqrt{5005}Q_{az}\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$-\frac{\sqrt{3003}Q_{bz}\left(x^6+15x^4y^2-30x^4z^2+15x^2y^4-180x^2y^2z^2+60x^2z^4+y^6-30y^4z^2+60y^2z^4-8z^6\right)}{1144}$$

$$-\frac{9\sqrt{2002}Q_1xz\left(4x^4-25x^2y^2-5x^2z^2+15y^4-5y^2z^2+2z^4\right)}{1144}+\frac{9\sqrt{2002}Q_2yz\left(15x^4-25x^2y^2-5x^2z^2+4y^4-5y^2z^2+2z^4\right)}{1144}$$
$$+\frac{\sqrt{3003}Q_3\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{286}$$
$$+\frac{\sqrt{30030}Q_{3x}yz\left(13x^4+17x^2y^2-43x^2z^2+4y^4-19y^2z^2+10z^4\right)}{1144}+\frac{\sqrt{30030}Q_{3y}xz\left(4x^4+17x^2y^2-19x^2z^2+13y^4-43y^2z^2+10z^4\right)}{1144}$$
$$-\frac{3\sqrt{5005}Q_{az}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{3\sqrt{3003}Q_{bz}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}$$