PG No. 17 $C_{3\mathrm{i}}$ $\bar{3}$ [trigonal] (polar, internal polar quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{Q}}_{0}^{(2,2)}[q](A_g)$

** symmetry

$$1$$

** expression

$$-\frac{\sqrt{5}Q_u\left(x^2+y^2-2z^2\right)}{10}+\frac{\sqrt{15}Q_v\left(x-y\right)\left(x+y\right)}{10}+\frac{\sqrt{15}Q_{xy}xy}{5}+\frac{\sqrt{15}Q_{xz}xz}{5}+\frac{\sqrt{15}Q_{yz}yz}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{Q}}_{1}^{(2,0)}[q](A_u)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{10}Q_uz}{5}+\frac{\sqrt{30}Q_{xz}x}{10}+\frac{\sqrt{30}Q_{yz}y}{10}$$

$\vec{\mathbb{Q}}_{1}^{(2,2)}[q](A_u)$

** symmetry

$$z$$

** expression

$$-\frac{3\sqrt{35}Q_uz\left(3x^2+3y^2-2z^2\right)}{70}+\frac{\sqrt{105}Q_vz\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}Q_{xy}xyz}{7}-\frac{\sqrt{105}Q_{xz}x\left(x^2+y^2-4z^2\right)}{35}-\frac{\sqrt{105}Q_{yz}y\left(x^2+y^2-4z^2\right)}{35}$$

$\vec{\mathbb{Q}}_{1,1}^{(2,0)}[q](E_u)$, $\vec{\mathbb{Q}}_{1,2}^{(2,0)}[q](E_u)$

** symmetry

$$x$$

$$y$$

** expression

$$-\frac{\sqrt{10}Q_ux}{10}+\frac{\sqrt{30}Q_vx}{10}+\frac{\sqrt{30}Q_{xy}y}{10}+\frac{\sqrt{30}Q_{xz}z}{10}$$

$$-\frac{\sqrt{10}Q_uy}{10}-\frac{\sqrt{30}Q_vy}{10}+\frac{\sqrt{30}Q_{xy}x}{10}+\frac{\sqrt{30}Q_{yz}z}{10}$$

$\vec{\mathbb{Q}}_{1,1}^{(2,2)}[q](E_u)$, $\vec{\mathbb{Q}}_{1,2}^{(2,2)}[q](E_u)$

** symmetry

$$x$$

$$y$$

** expression

$$-\frac{3\sqrt{35}Q_ux\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}Q_vx\left(3x^2-7y^2-2z^2\right)}{70}+\frac{\sqrt{105}Q_{xy}y\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}Q_{xz}z\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}Q_{yz}xyz}{7}$$

$$-\frac{3\sqrt{35}Q_uy\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}Q_vy\left(7x^2-3y^2+2z^2\right)}{70}-\frac{\sqrt{105}Q_{xy}x\left(x^2-4y^2+z^2\right)}{35}+\frac{\sqrt{105}Q_{xz}xyz}{7}-\frac{\sqrt{105}Q_{yz}z\left(x^2-4y^2+z^2\right)}{35}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_{2}^{(2,-2)}[q](A_g)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$Q_u$$

$\vec{\mathbb{Q}}_{2}^{(2,0)}[q](A_g)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{14}Q_u\left(x^2+y^2-2z^2\right)}{14}-\frac{\sqrt{42}Q_v\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}Q_{xy}xy}{7}+\frac{\sqrt{42}Q_{xz}xz}{14}+\frac{\sqrt{42}Q_{yz}yz}{14}$$

$\vec{\mathbb{Q}}_{2}^{(2,2)}[q](A_g)$
** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\frac{\sqrt{14}Q_u\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{56}-\frac{5\sqrt{42}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{168}-\frac{5\sqrt{42}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{84}-\frac{5\sqrt{42}Q_{xz}xz\left(3x^2+3y^2-4z^2\right)}{84}-\frac{5\sqrt{42}Q_{yz}yz\left(3x^2+3y^2-4z^2\right)}{84}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-2)}[q](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-2)}[q](E_g,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$Q_{yz}$$

$$-Q_{xz}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-2)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-2)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$Q_v$$

$$-Q_{xy}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,0)}[q](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(2,0)}[q](E_g,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}Q_uyz}{14}-\frac{3\sqrt{14}Q_vyz}{14}+\frac{3\sqrt{14}Q_{xy}xz}{14}+\frac{3\sqrt{14}Q_{xz}xy}{14}-\frac{\sqrt{14}Q_{yz}\left(2x^2-y^2-z^2\right)}{14}$$

$$-\frac{\sqrt{42}Q_uxz}{14}-\frac{3\sqrt{14}Q_vxz}{14}-\frac{3\sqrt{14}Q_{xy}yz}{14}-\frac{\sqrt{14}Q_{xz}\left(x^2-2y^2+z^2\right)}{14}-\frac{3\sqrt{14}Q_{yz}xy}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,0)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,0)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}Q_u\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{14}Q_v\left(x^2+y^2-2z^2\right)}{14}+\frac{3\sqrt{14}Q_{xz}xz}{14}-\frac{3\sqrt{14}Q_{yz}yz}{14}$$

$$\frac{\sqrt{42}Q_uxy}{7}-\frac{\sqrt{14}Q_{xy}\left(x^2+y^2-2z^2\right)}{14}-\frac{3\sqrt{14}Q_{xz}yz}{14}-\frac{3\sqrt{14}Q_{yz}xz}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,2)}[q](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(2,2)}[q](E_g,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{5\sqrt{42}Q_u yz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}Q_v yz\left(9x^2-5y^2+2z^2\right)}{84}-\frac{5\sqrt{14}Q_{xy}xz\left(x^2-6y^2+z^2\right)}{42}-\frac{5\sqrt{14}Q_{xz}xy\left(x^2+y^2-6z^2\right)}{42}+\frac{\sqrt{14}Q_{yz}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{42}$$

$$\frac{5\sqrt{42}Q_u xz\left(3x^2+3y^2-4z^2\right)}{84}-\frac{5\sqrt{14}Q_v xz\left(5x^2-9y^2-2z^2\right)}{84}-\frac{5\sqrt{14}Q_{xy}yz\left(6x^2-y^2-z^2\right)}{42}+\frac{\sqrt{14}Q_{xz}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{42}+\frac{5\sqrt{14}Q_{yz}xy\left(x^2+y^2-6z^2\right)}{42}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,2)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,2)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}Q_u\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{168}+\frac{\sqrt{14}Q_v\left(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4\right)}{168}+\frac{5\sqrt{14}Q_{xy}xy\left(x-y\right)\left(x+y\right)}{12}+\frac{5\sqrt{14}Q_{xz}xz\left(5x^2-9y^2-2z^2\right)}{84}+\frac{5\sqrt{14}Q_{yz}yz\left(9x^2-5y^2+2z^2\right)}{84}$$

$$\frac{5\sqrt{42}Q_u xy\left(x^2+y^2-6z^2\right)}{84}-\frac{5\sqrt{14}Q_v xy\left(x-y\right)\left(x+y\right)}{12}+\frac{\sqrt{14}Q_{xy}\left(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4\right)}{42}-\frac{5\sqrt{14}Q_{xz}yz\left(6x^2-y^2-z^2\right)}{42}+\frac{5\sqrt{14}Q_{yz}xz\left(x^2-6y^2+z^2\right)}{42}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_{3}^{(2,-2)}[q](A_u,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_u z}{5}-\frac{\sqrt{5}Q_{xz}x}{5}-\frac{\sqrt{5}Q_{yz}y}{5}$$

$\vec{\mathbb{Q}}_{3}^{(2,-2)}[q](A_u,2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{2}Q_v y}{2}+\frac{\sqrt{2}Q_{xy}x}{2}$$

$\vec{\mathbb{Q}}_{3}^{(2,-2)}[q](A_u,3)$
** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{2}Q_v x}{2}-\frac{\sqrt{2}Q_{xy}y}{2}$$

$\vec{\mathbb{Q}}_{3}^{(2,0)}[q](A_u,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_{u}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{15}-\frac{\sqrt{5}Q_{v}z\left(x-y\right)\left(x+y\right)}{2}-\sqrt{5}Q_{xy}xyz-\frac{\sqrt{5}Q_{xz}x\left(x^{2}+y^{2}-4z^{2}\right)}{20}-\frac{\sqrt{5}Q_{yz}y\left(x^{2}+y^{2}-4z^{2}\right)}{20}$$

$\vec{\mathbb{Q}}_{3}^{(2,0)}[q](A_{u},2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{5\sqrt{6}Q_{u}y\left(3x^{2}-y^{2}\right)}{24}+\frac{\sqrt{2}Q_{v}y\left(x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{\sqrt{2}Q_{xy}x\left(x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{5\sqrt{2}Q_{xz}xyz}{4}+\frac{5\sqrt{2}Q_{yz}z\left(x-y\right)\left(x+y\right)}{8}$$

$\vec{\mathbb{Q}}_{3}^{(2,0)}[q](A_{u},3)$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{5\sqrt{6}Q_{u}x\left(x^{2}-3y^{2}\right)}{24}+\frac{\sqrt{2}Q_{v}x\left(x^{2}+y^{2}-4z^{2}\right)}{8}-\frac{\sqrt{2}Q_{xy}y\left(x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{5\sqrt{2}Q_{xz}z\left(x-y\right)\left(x+y\right)}{8}-\frac{5\sqrt{2}Q_{yz}xyz}{4}$$

$\vec{\mathbb{Q}}_{3}^{(2,2)}[q](A_{u},1)$
** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{330}Q_{u}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{264}-\frac{7\sqrt{110}Q_{v}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}-\frac{7\sqrt{110}Q_{xy}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}$$
$$+\frac{\sqrt{110}Q_{xz}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}+\frac{\sqrt{110}Q_{yz}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}$$

$\vec{\mathbb{Q}}_{3}^{(2,2)}[q](A_{u},2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{7\sqrt{33}Q_{u}y\left(3x^{2}-y^{2}\right)\left(x^{2}+y^{2}-8z^{2}\right)}{264}+\frac{\sqrt{11}Q_{v}y\left(53x^{4}-104x^{2}y^{2}-6x^{2}z^{2}+11y^{4}-6y^{2}z^{2}+4z^{4}\right)}{88}$$
$$-\frac{\sqrt{11}Q_{xy}x\left(5x^{4}-53x^{2}y^{2}+3x^{2}z^{2}+26y^{4}+3y^{2}z^{2}-2z^{4}\right)}{44}+\frac{7\sqrt{11}Q_{xz}xyz\left(7x^{2}-5y^{2}-2z^{2}\right)}{44}-\frac{7\sqrt{11}Q_{yz}z\left(x^{4}-9x^{2}y^{2}+x^{2}z^{2}+2y^{4}-y^{2}z^{2}\right)}{44}$$

$\vec{\mathbb{Q}}_{3}^{(2,2)}[q](A_{u},3)$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{7\sqrt{33}Q_{u}x\left(x^{2}-3y^{2}\right)\left(x^{2}+y^{2}-8z^{2}\right)}{264}+\frac{\sqrt{11}Q_{v}x\left(11x^{4}-104x^{2}y^{2}-6x^{2}z^{2}+53y^{4}-6y^{2}z^{2}+4z^{4}\right)}{88}$$
$$+\frac{\sqrt{11}Q_{xy}y\left(26x^{4}-53x^{2}y^{2}+3x^{2}z^{2}+5y^{4}+3y^{2}z^{2}-2z^{4}\right)}{44}+\frac{7\sqrt{11}Q_{xz}z\left(2x^{4}-9x^{2}y^{2}-x^{2}z^{2}+y^{4}+y^{2}z^{2}\right)}{44}+\frac{7\sqrt{11}Q_{yz}xyz\left(5x^{2}-7y^{2}+2z^{2}\right)}{44}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-2)}[q](E_{u},1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-2)}[q](E_{u},1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{10}Q_{u}x}{5}-\frac{\sqrt{30}Q_{v}x}{30}-\frac{\sqrt{30}Q_{xy}y}{30}+\frac{2\sqrt{30}Q_{xz}z}{15}$$

$$\frac{\sqrt{10}Q_u y}{5} + \frac{\sqrt{30}Q_v y}{30} - \frac{\sqrt{30}Q_{xy} x}{30} + \frac{2\sqrt{30}Q_{yz} z}{15}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-2)}[q](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-2)}[q](E_u,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{xy}z}{3} + \frac{\sqrt{3}Q_{xz}y}{3} + \frac{\sqrt{3}Q_{yz}x}{3}$$

$$\frac{\sqrt{3}Q_v z}{3} + \frac{\sqrt{3}Q_{xz}x}{3} - \frac{\sqrt{3}Q_{yz}y}{3}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,0)}[q](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,0)}[q](E_u,1)$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{10}Q_u x\left(x^2+y^2-4z^2\right)}{40} - \frac{\sqrt{30}Q_v x\left(11x^2-9y^2-24z^2\right)}{120} - \frac{\sqrt{30}Q_{xy}y\left(21x^2+y^2-24z^2\right)}{120} + \frac{\sqrt{30}Q_{xz}z\left(9x^2-21y^2+4z^2\right)}{120} + \frac{\sqrt{30}Q_{yz}xyz}{4}$$

$$-\frac{3\sqrt{10}Q_u y\left(x^2+y^2-4z^2\right)}{40} - \frac{\sqrt{30}Q_v y\left(9x^2-11y^2+24z^2\right)}{120} - \frac{\sqrt{30}Q_{xy}x\left(x^2+21y^2-24z^2\right)}{120} + \frac{\sqrt{30}Q_{xz}xyz}{4} - \frac{\sqrt{30}Q_{yz}z\left(21x^2-9y^2-4z^2\right)}{120}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,0)}[q](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,0)}[q](E_u,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{xy}z\left(3x^2+3y^2-2z^2\right)}{6} + \frac{\sqrt{3}Q_{xz}y\left(3x^2-2y^2+3z^2\right)}{6} - \frac{\sqrt{3}Q_{yz}x\left(2x^2-3y^2-3z^2\right)}{6}$$

$$\frac{\sqrt{3}Q_v z\left(3x^2+3y^2-2z^2\right)}{6} + \frac{\sqrt{3}Q_{xz}x\left(x^2-9y^2+6z^2\right)}{12} + \frac{\sqrt{3}Q_{yz}y\left(9x^2-y^2-6z^2\right)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,2)}[q](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,2)}[q](E_u,1)$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{55}Q_u x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88} - \frac{\sqrt{165}Q_v x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{264}$$
$$- \frac{\sqrt{165}Q_{xy}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{132}$$
$$- \frac{\sqrt{165}Q_{xz}z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{132} - \frac{7\sqrt{165}Q_{yz}xyz\left(x^2+y^2-2z^2\right)}{44}$$

$$\frac{3\sqrt{55}Q_u y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88} - \frac{\sqrt{165}Q_v y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{264}$$
$$+ \frac{\sqrt{165}Q_{xy}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{132} - \frac{7\sqrt{165}Q_{xz}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+ \frac{\sqrt{165}Q_{yz}z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{132}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,2)}[q](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,2)}[q](E_u,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{21\sqrt{22}Q_{u}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{21\sqrt{66}Q_{v}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xy}z\left(6x^{4}-51x^{2}y^{2}+5x^{2}z^{2}+6y^{4}+5y^{2}z^{2}-z^{4}\right)}{66}-\frac{\sqrt{66}Q_{xz}y\left(6x^{4}+5x^{2}y^{2}-51x^{2}z^{2}-y^{4}+5y^{2}z^{2}+6z^{4}\right)}{66}+\frac{\sqrt{66}Q_{yz}x\left(x^{4}-5x^{2}y^{2}-5x^{2}z^{2}-6y^{4}+51y^{2}z^{2}-6z^{4}\right)}{66}$$

$$-\frac{21\sqrt{22}Q_{u}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{66}Q_{v}z\left(39x^{4}-174x^{2}y^{2}-20x^{2}z^{2}+39y^{4}-20y^{2}z^{2}+4z^{4}\right)}{264}+\frac{21\sqrt{66}Q_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xz}x\left(5x^{4}-4x^{2}y^{2}-46x^{2}z^{2}-9y^{4}+66y^{2}z^{2}+12z^{4}\right)}{132}-\frac{\sqrt{66}Q_{yz}y\left(9x^{4}+4x^{2}y^{2}-66x^{2}z^{2}-5y^{4}+46y^{2}z^{2}-12z^{4}\right)}{132}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_{4}^{(2,-2)}[q](A_g,1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$-\frac{3\sqrt{70}Q_{u}\left(x^{2}+y^{2}-2z^{2}\right)}{70}+\frac{\sqrt{210}Q_{v}\left(x-y\right)\left(x+y\right)}{140}+\frac{\sqrt{210}Q_{xy}xy}{70}-\frac{2\sqrt{210}Q_{xz}xz}{35}-\frac{2\sqrt{210}Q_{yz}yz}{35}$$

$\vec{\mathbb{Q}}_{4}^{(2,-2)}[q](A_g,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{3}Q_{v}xz}{2}-\frac{\sqrt{3}Q_{xy}yz}{2}+\frac{\sqrt{3}Q_{xz}\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{3}Q_{yz}xy}{2}$$

$\vec{\mathbb{Q}}_{4}^{(2,-2)}[q](A_g,3)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{3}Q_{v}yz}{2}+\frac{\sqrt{3}Q_{xy}xz}{2}+\frac{\sqrt{3}Q_{xz}xy}{2}+\frac{\sqrt{3}Q_{yz}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{Q}}_{4}^{(2,0)}[q](A_g,1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$\frac{\sqrt{385}Q_{u}\left(3x^{4}+6x^{2}y^{2}-24x^{2}z^{2}+3y^{4}-24y^{2}z^{2}+8z^{4}\right)}{308}+\frac{3\sqrt{1155}Q_{v}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{308}+\frac{3\sqrt{1155}Q_{xy}xy\left(x^{2}+y^{2}-6z^{2}\right)}{154}-\frac{\sqrt{1155}Q_{xz}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}-\frac{\sqrt{1155}Q_{yz}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}$$

$\vec{\mathbb{Q}}_{4}^{(2,0)}[q](A_g,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{7\sqrt{22}Q_{u}xz\left(x^{2}-3y^{2}\right)}{88}+\frac{3\sqrt{66}Q_{v}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{88}-\frac{3\sqrt{66}Q_{xy}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{88}+\frac{\sqrt{66}Q_{xz}\left(x^{4}-21x^{2}y^{2}+15x^{2}z^{2}+6y^{4}-15y^{2}z^{2}\right)}{88}+\frac{\sqrt{66}Q_{yz}xy\left(19x^{2}-9y^{2}-30z^{2}\right)}{88}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g, 3)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{22}Q_u yz\left(3x^2-y^2\right)}{88}+\frac{3\sqrt{66}Q_v yz\left(3x^2+3y^2-4z^2\right)}{88}+\frac{3\sqrt{66}Q_{xy}xz\left(3x^2+3y^2-4z^2\right)}{88}+\frac{\sqrt{66}Q_{xz}xy\left(9x^2-19y^2+30z^2\right)}{88}-\frac{\sqrt{66}Q_{yz}\left(6x^4-21x^2y^2-15x^2z^2+y^4+15y^2z^2\right)}{88}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g, 1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{3\sqrt{715}Q_u\left(5x^6+15x^4y^2-90x^4z^2+15x^2y^4-180x^2y^2z^2+120x^2z^4+5y^6-90y^4z^2+120y^2z^4-16z^6\right)}{2288}+\frac{7\sqrt{2145}Q_v\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}+\frac{7\sqrt{2145}Q_{xy}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}+\frac{7\sqrt{2145}Q_{xz}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}+\frac{7\sqrt{2145}Q_{yz}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{2002}Q_u xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{1144}+\frac{\sqrt{6006}Q_v xz\left(19x^4-160x^2y^2-10x^2z^2+85y^4-10y^2z^2+4z^4\right)}{1144}+\frac{\sqrt{6006}Q_{xy}yz\left(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4\right)}{572}-\frac{\sqrt{6006}Q_{xz}\left(2x^6-7x^4y^2-23x^4z^2-8x^2y^4+90x^2y^2z^2+8x^2z^4+y^6-7y^4z^2-8y^2z^4\right)}{572}-\frac{\sqrt{6006}Q_{yz}xy\left(5x^4-2x^2y^2-44x^2z^2-7y^4+76y^2z^2-16z^4\right)}{572}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g, 3)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{2002}Q_u yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{1144}+\frac{\sqrt{6006}Q_v yz\left(85x^4-160x^2y^2-10x^2z^2+19y^4-10y^2z^2+4z^4\right)}{1144}-\frac{\sqrt{6006}Q_{xy}xz\left(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4\right)}{572}-\frac{\sqrt{6006}Q_{xz}xy\left(7x^4+2x^2y^2-76x^2z^2-5y^4+44y^2z^2+16z^4\right)}{572}+\frac{\sqrt{6006}Q_{yz}\left(x^6-8x^4y^2-7x^4z^2-7x^2y^4+90x^2y^2z^2-8x^2z^4+2y^6-23y^4z^2+8y^2z^4\right)}{572}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](E_g, 1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](E_g, 1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{7}Q_u yz}{7}+\frac{\sqrt{21}Q_v yz}{14}-\frac{\sqrt{21}Q_{xy}xz}{14}-\frac{\sqrt{21}Q_{xz}xy}{14}-\frac{\sqrt{21}Q_{yz}\left(x^2+3y^2-4z^2\right)}{28}$$

$$-\frac{3\sqrt{7}Q_u xz}{7}+\frac{\sqrt{21}Q_v xz}{14}+\frac{\sqrt{21}Q_{xy}yz}{14}+\frac{\sqrt{21}Q_{xz}\left(3x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{21}Q_{yz}xy}{14}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{6}Q_v\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{6}Q_{xy}xy}{2}$$

$$\frac{\sqrt{6}Q_vxy}{2}+\frac{\sqrt{6}Q_{xy}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](E_g,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{14}Q_u\left(x-y\right)\left(x+y\right)}{28}-\frac{\sqrt{42}Q_v\left(x^2+y^2-2z^2\right)}{28}+\frac{\sqrt{42}Q_{xz}xz}{7}-\frac{\sqrt{42}Q_{yz}yz}{7}$$

$$-\frac{3\sqrt{14}Q_uxy}{14}+\frac{\sqrt{42}Q_{xy}\left(x^2+y^2-2z^2\right)}{28}-\frac{\sqrt{42}Q_{xz}yz}{7}-\frac{\sqrt{42}Q_{yz}xz}{7}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{17\sqrt{154}Q_uyz\left(3x^2+3y^2-4z^2\right)}{616}-\frac{\sqrt{462}Q_vyz\left(33x^2-51y^2+40z^2\right)}{616}-\frac{\sqrt{462}Q_{xy}xz\left(9x^2+93y^2-40z^2\right)}{616}$$
$$-\frac{9\sqrt{462}Q_{xz}xy\left(x^2+y^2-6z^2\right)}{616}+\frac{\sqrt{462}Q_{yz}\left(6x^4+3x^2y^2-39x^2z^2-3y^4+15y^2z^2+4z^4\right)}{616}$$

$$\frac{17\sqrt{154}Q_uxz\left(3x^2+3y^2-4z^2\right)}{616}+\frac{\sqrt{462}Q_vxz\left(51x^2-33y^2-40z^2\right)}{616}+\frac{\sqrt{462}Q_{xy}yz\left(93x^2+9y^2-40z^2\right)}{616}$$
$$+\frac{\sqrt{462}Q_{xz}\left(3x^4-3x^2y^2-15x^2z^2-6y^4+39y^2z^2-4z^4\right)}{616}+\frac{9\sqrt{462}Q_{yz}xy\left(x^2+y^2-6z^2\right)}{616}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{7\sqrt{11}Q_u\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{44}+\frac{\sqrt{33}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{44}$$
$$-\frac{\sqrt{33}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{22}+\frac{7\sqrt{33}Q_{xz}xz\left(x^2-3y^2\right)}{44}-\frac{7\sqrt{33}Q_{yz}yz\left(3x^2-y^2\right)}{44}$$

$$-\frac{7\sqrt{11}Q_uxy\left(x-y\right)\left(x+y\right)}{11}+\frac{\sqrt{33}Q_vxy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}Q_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}Q_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}Q_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](E_g,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_u\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}Q_v\left(2x^4-3x^2y^2-9x^2z^2+2y^4-9y^2z^2+3z^4\right)}{77}-\frac{\sqrt{231}Q_{xy}xy\left(x-y\right)\left(x+y\right)}{22}+\frac{\sqrt{231}Q_{xz}xz\left(2x^2-33y^2+9z^2\right)}{154}+\frac{\sqrt{231}Q_{yz}yz\left(33x^2-2y^2-9z^2\right)}{154}$$

$$\frac{2\sqrt{77}Q_uxy\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{231}Q_vxy\left(x-y\right)\left(x+y\right)}{22}+\frac{\sqrt{231}Q_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}-\frac{\sqrt{231}Q_{xz}yz\left(39x^2-31y^2+18z^2\right)}{308}+\frac{\sqrt{231}Q_{yz}xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{21\sqrt{286}Q_uyz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}-\frac{7\sqrt{858}Q_vyz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{1144}+\frac{7\sqrt{858}Q_{xy}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}+\frac{7\sqrt{858}Q_{xz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{\sqrt{858}Q_{yz}\left(x^6-4x^4y^2-11x^4z^2-11x^2y^4+90x^2y^2z^2-4x^2z^4-6y^6+101y^4z^2-116y^2z^4+8z^6\right)}{572}$$

$$-\frac{21\sqrt{286}Q_uxz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}+\frac{7\sqrt{858}Q_vxz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{1144}+\frac{7\sqrt{858}Q_{xy}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{572}-\frac{\sqrt{858}Q_{xz}\left(6x^6+11x^4y^2-101x^4z^2+4x^2y^4-90x^2y^2z^2+116x^2z^4-y^6+11y^4z^2+4y^2z^4-8z^6\right)}{572}-\frac{7\sqrt{858}Q_{yz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{9\sqrt{1001}Q_u\left(x^2+y^2-10z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{2288}+\frac{\sqrt{3003}Q_v\left(x-y\right)\left(x+y\right)\left(17x^4-230x^2y^2-8x^2z^2+17y^4-8y^2z^2+8z^4\right)}{2288}+\frac{\sqrt{3003}Q_{xy}xy\left(49x^4-166x^2y^2+8x^2z^2+49y^4+8y^2z^2-8z^4\right)}{1144}+\frac{3\sqrt{3003}Q_{xz}xz\left(7x^4-58x^2y^2-4x^2z^2+23y^4+12y^2z^2\right)}{1144}+\frac{3\sqrt{3003}Q_{yz}yz\left(23x^4-58x^2y^2+12x^2z^2+7y^4-4y^2z^2\right)}{1144}$$

$$-\frac{9\sqrt{1001}Q_uxy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{3003}Q_vxy\left(25x^4-82x^2y^2-4x^2z^2+25y^4-4y^2z^2+4z^4\right)}{572}-\frac{\sqrt{3003}Q_{xy}\left(x-y\right)\left(x+y\right)\left(2x^4-29x^2y^2+x^2z^2+2y^4+y^2z^2-z^4\right)}{286}+\frac{3\sqrt{3003}Q_{xz}yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{286}-\frac{3\sqrt{3003}Q_{yz}xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{286}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](E_g,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{21\sqrt{143}Q_{u}\left(x-y\right)\left(x+y\right)\left(x^{4}+2x^{2}y^{2}-16x^{2}z^{2}+y^{4}-16y^{2}z^{2}+16z^{4}\right)}{1144} - \frac{\sqrt{429}Q_{v}\left(13x^{6}-45x^{4}y^{2}-150x^{4}z^{2}-45x^{2}y^{4}+540x^{2}y^{2}z^{2}+60x^{2}z^{4}+13y^{6}-150y^{4}z^{2}+60y^{2}z^{4}-8z^{6}\right)}{1144} - \frac{21\sqrt{429}Q_{xy}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572} - \frac{7\sqrt{429}Q_{xz}xz\left(7x^{4}-4x^{2}y^{2}-22x^{2}z^{2}-11y^{4}+26y^{2}z^{2}+4z^{4}\right)}{572} - \frac{7\sqrt{429}Q_{yz}yz\left(11x^{4}+4x^{2}y^{2}-26x^{2}z^{2}-7y^{4}+22y^{2}z^{2}-4z^{4}\right)}{572}$$

$$-\frac{21\sqrt{143}Q_{u}xy\left(x^{4}+2x^{2}y^{2}-16x^{2}z^{2}+y^{4}-16y^{2}z^{2}+16z^{4}\right)}{572} + \frac{21\sqrt{429}Q_{v}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572} - \frac{\sqrt{429}Q_{xy}\left(2x^{6}-15x^{4}y^{2}-15x^{4}z^{2}-15x^{2}y^{4}+180x^{2}y^{2}z^{2}-15x^{2}z^{4}+2y^{6}-15y^{4}z^{2}-15y^{2}z^{4}+2z^{6}\right)}{286} + \frac{7\sqrt{429}Q_{xz}yz\left(8x^{4}+7x^{2}y^{2}-23x^{2}z^{2}-y^{4}+y^{2}z^{2}+2z^{4}\right)}{286} - \frac{7\sqrt{429}Q_{yz}xz\left(x^{4}-7x^{2}y^{2}-x^{2}z^{2}-8y^{4}+23y^{2}z^{2}-2z^{4}\right)}{286}$$