# PG No. 7 $\quad C_{2\mathrm{v}} \quad mm2 \quad$ [ orthorhombic ] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,0)}[q](A_2)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B_1)$

** symmetry

$$y$$

** expression

$$-\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B_2)$

** symmetry

$$x$$

** expression

$$\frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_1)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{2}Q_x xz}{2} + \frac{\sqrt{2}Q_y yz}{2} + \frac{\sqrt{2}Q_z \left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_2, 1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_x yz}{2} - \frac{\sqrt{6}Q_y xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_2, 2)$

** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x yz}{2} + \frac{\sqrt{2}Q_y xz}{2} - \sqrt{2}Q_z xy$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B_1)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_x \left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}Q_y xy}{2} + \frac{\sqrt{2}Q_z xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B_2)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2} Q_{x} x y}{2} - \frac{\sqrt{2} Q_{y} \left(x - z\right) \left(x + z\right)}{2} - \frac{\sqrt{2} Q_{z} y z}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{1})$

** symmetry

$$\sqrt{15} x y z$$

** expression

$$\frac{\sqrt{5} Q_{x} x \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{5} Q_{y} y \left(x - z\right) \left(x + z\right)}{2} + \frac{\sqrt{5} Q_{z} z \left(x - y\right) \left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{2},1)$

** symmetry

$$- \frac{z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{3} Q_{x} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{4} + \frac{\sqrt{3} Q_{y} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{2},2)$

** symmetry

$$\frac{\sqrt{15} z \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{5} Q_{x} y \left(x^{2} - y^{2} + 2 z^{2}\right)}{4} - \frac{\sqrt{5} Q_{y} x \left(x^{2} - y^{2} - 2 z^{2}\right)}{4} - \sqrt{5} Q_{z} x y z$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](B_{1},1)$

** symmetry

$$- \frac{y \left(3 x^{2} - 2 y^{2} + 3 z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3} Q_{x} z \left(x^{2} - 4 y^{2} + z^{2}\right)}{4} - \frac{\sqrt{3} Q_{z} x \left(x^{2} - 4 y^{2} + z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](B_{1},2)$

** symmetry

$$- \frac{\sqrt{15} y \left(x - z\right) \left(x + z\right)}{2}$$

** expression

$$\frac{\sqrt{5} Q_{x} z \left(x^{2} + 2 y^{2} - z^{2}\right)}{4} - \sqrt{5} Q_{y} x y z - \frac{\sqrt{5} Q_{z} x \left(x^{2} - 2 y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](B_{2},1)$

** symmetry

$$\frac{x \left(2 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3} Q_{y} z \left(4 x^{2} - y^{2} - z^{2}\right)}{4} - \frac{\sqrt{3} Q_{z} y \left(4 x^{2} - y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](B_{2},2)$

** symmetry

$$\frac{\sqrt{15} x \left(y - z\right) \left(y + z\right)}{2}$$

** expression

$$- \sqrt{5} Q_{x} x y z + \frac{\sqrt{5} Q_{y} z \left(2 x^{2} + y^{2} - z^{2}\right)}{4} + \frac{\sqrt{5} Q_{z} y \left(2 x^{2} - y^{2} + z^{2}\right)}{4}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_1,1)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_{x}xz\left(x^{2}-3y^{2}\right)}{4}+\frac{\sqrt{7}Q_{y}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}Q_{z}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_1,2)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{4}-\frac{Q_{y}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{4}-\frac{Q_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_2,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_{x}yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}Q_{y}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}Q_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_2,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

** expression

$$-\frac{\sqrt{3}Q_{x}yz\left(9x^{2}+2y^{2}-5z^{2}\right)}{6}+\frac{\sqrt{3}Q_{y}xz\left(2x^{2}+9y^{2}-5z^{2}\right)}{6}+\frac{7\sqrt{3}Q_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_2,3)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{Q_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}+\frac{Q_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}-\frac{Q_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_1,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_{x}\left(y^{2}-2yz-z^{2}\right)\left(y^{2}+2yz-z^{2}\right)}{4}-\frac{\sqrt{7}Q_{y}xy\left(y^{2}-3z^{2}\right)}{4}+\frac{\sqrt{7}Q_{z}xz\left(3y^{2}-z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_1,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x}\left(y-z\right)\left(y+z\right)\left(6x^{2}-y^{2}-z^{2}\right)}{4}-\frac{Q_{y}xy\left(6x^{2}-y^{2}-15z^{2}\right)}{4}+\frac{Q_{z}xz\left(6x^{2}-15y^{2}-z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_2,1)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_x xy\left(x^2-3z^2\right)}{4}+\frac{\sqrt{7}Q_y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{4}+\frac{\sqrt{7}Q_z yz\left(3x^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_2,2)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{Q_x xy\left(x^2-6y^2+15z^2\right)}{4}+\frac{Q_y\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{4}+\frac{Q_z yz\left(15x^2-6y^2+z^2\right)}{4}$$