* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(3,3)}[g](A_u)$

** symmetry

     1

** expression

$$\frac{\sqrt{70}G_1 y\left(3x^2-y^2\right)}{28} + \frac{\sqrt{70}G_2 x\left(x^2-3y^2\right)}{28} + \frac{\sqrt{105}G_3 xyz}{7} - \frac{\sqrt{42}G_{3x} x\left(x^2+y^2-4z^2\right)}{28}$$

$$- \frac{\sqrt{42}G_{3y} y\left(x^2+y^2-4z^2\right)}{28} - \frac{\sqrt{7}G_{az} z\left(3x^2+3y^2-2z^2\right)}{14} + \frac{\sqrt{105}G_{bz} z\left(x-y\right)\left(x+y\right)}{14}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(3,1)}[g](A_g)$

** symmetry

     z

** expression

$$\frac{\sqrt{21}G_3 xy}{7} + \frac{2\sqrt{210}G_{3x} xz}{35} + \frac{2\sqrt{210}G_{3y} yz}{35} - \frac{3\sqrt{35}G_{az}\left(x^2+y^2-2z^2\right)}{70} + \frac{\sqrt{21}G_{bz}\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_1^{(3,3)}[g](A_g)$

** symmetry

     z

** expression

$$\frac{\sqrt{210}G_1 yz\left(3x^2-y^2\right)}{24} + \frac{\sqrt{210}G_2 xz\left(x^2-3y^2\right)}{24} - \frac{\sqrt{35}G_3 xy\left(x^2+y^2-6z^2\right)}{14} - \frac{5\sqrt{14}G_{3x} xz\left(3x^2+3y^2-4z^2\right)}{56}$$

$$- \frac{5\sqrt{14}G_{3y} yz\left(3x^2+3y^2-4z^2\right)}{56} + \frac{\sqrt{21}G_{az}\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{84} - \frac{\sqrt{35}G_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{28}$$

$\vec{\mathbb{G}}_{1,1}^{(3,1)}[g](E_{1g}),\ \vec{\mathbb{G}}_{1,2}^{(3,1)}[g](E_{1g})$

** symmetry

     x

     y

** expression

$$\frac{3\sqrt{14}G_1 xy}{14} + \frac{3\sqrt{14}G_2\left(x-y\right)\left(x+y\right)}{28} + \frac{\sqrt{21}G_3 yz}{7} - \frac{\sqrt{210}G_{3x}\left(3x^2+y^2-4z^2\right)}{140} - \frac{\sqrt{210}G_{3y} xy}{70} - \frac{3\sqrt{35}G_{az} xz}{35} + \frac{\sqrt{21}G_{bz} xz}{7}$$

$$\frac{3\sqrt{14}G_1\left(x-y\right)\left(x+y\right)}{28} - \frac{3\sqrt{14}G_2 xy}{14} + \frac{\sqrt{21}G_3 xz}{7} - \frac{\sqrt{210}G_{3x} xy}{70} - \frac{\sqrt{210}G_{3y}\left(x^2+3y^2-4z^2\right)}{140} - \frac{3\sqrt{35}G_{az} yz}{35} - \frac{\sqrt{21}G_{bz} yz}{7}$$

$\vec{\mathbb{G}}_{1,1}^{(3,3)}[g](E_{1g}),\ \vec{\mathbb{G}}_{1,2}^{(3,3)}[g](E_{1g})$

** symmetry

     x

     y

** expression

$$\frac{\sqrt{210}G_1 xy\left(15x^2-13y^2-6z^2\right)}{168} + \frac{\sqrt{210}G_2\left(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2\right)}{168}$$

$$+ \frac{\sqrt{35}G_3 yz\left(6x^2-y^2-z^2\right)}{14} - \frac{\sqrt{14}G_{3x}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{56}$$

$$- \frac{5\sqrt{14}G_{3y} xy\left(x^2+y^2-6z^2\right)}{56} - \frac{5\sqrt{21}G_{az} xz\left(3x^2+3y^2-4z^2\right)}{84} + \frac{\sqrt{35}G_{bz} xz\left(5x^2-9y^2-2z^2\right)}{28}$$

$$- \frac{\sqrt{210}G_1\left(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2\right)}{168} + \frac{\sqrt{210}G_2 xy\left(13x^2-15y^2+6z^2\right)}{168} - \frac{\sqrt{35}G_3 xz\left(x^2-6y^2+z^2\right)}{14} - \frac{5\sqrt{14}G_{3x} xy\left(x^2+y^2-6z^2\right)}{56}$$

$$+ \frac{\sqrt{14}G_{3y}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{56} - \frac{5\sqrt{21}G_{az} yz\left(3x^2+3y^2-4z^2\right)}{84} + \frac{\sqrt{35}G_{bz} yz\left(9x^2-5y^2+2z^2\right)}{28}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(3,-1)}[g](A_u)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}G_{3x}x}{7} + \frac{\sqrt{14}G_{3y}y}{7} + \frac{\sqrt{21}G_{az}z}{7}$$

$\vec{\mathbb{G}}_2^{(3,1)}[g](A_u)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{5\sqrt{210}G_1 y\left(3x^2 - y^2\right)}{168} - \frac{5\sqrt{210}G_2 x\left(x^2 - 3y^2\right)}{168} - \frac{3\sqrt{14}G_{3x}x\left(x^2 + y^2 - 4z^2\right)}{56} - \frac{3\sqrt{14}G_{3y}y\left(x^2 + y^2 - 4z^2\right)}{56} - \frac{\sqrt{21}G_{az}z\left(3x^2 + 3y^2 - 2z^2\right)}{21}$$

$\vec{\mathbb{G}}_2^{(3,3)}[g](A_u)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{1155}G_1 y\left(3x^2 - y^2\right)\left(x^2 + y^2 - 8z^2\right)}{264} - \frac{\sqrt{1155}G_2 x\left(x^2 - 3y^2\right)\left(x^2 + y^2 - 8z^2\right)}{264} - \frac{3\sqrt{770}G_3 xyz\left(x^2 + y^2 - 2z^2\right)}{44}$$
$$+ \frac{15\sqrt{77}G_{3x}x\left(x^4 + 2x^2 y^2 - 12x^2 z^2 + y^4 - 12y^2 z^2 + 8z^4\right)}{616} + \frac{15\sqrt{77}G_{3y}y\left(x^4 + 2x^2 y^2 - 12x^2 z^2 + y^4 - 12y^2 z^2 + 8z^4\right)}{616}$$
$$+ \frac{5\sqrt{462}G_{az}z\left(15x^4 + 30x^2 y^2 - 40x^2 z^2 + 15y^4 - 40y^2 z^2 + 8z^4\right)}{1848} - \frac{3\sqrt{770}G_{bz}z\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 2z^2\right)}{88}$$

$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E_{1u}),\ \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E_{1u})$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{105}G_3 x}{21} + \frac{2\sqrt{42}G_{3y}z}{21} - \frac{\sqrt{7}G_{az}y}{7} - \frac{\sqrt{105}G_{bz}y}{21}$$

$$-\frac{\sqrt{105}G_3 y}{21} - \frac{2\sqrt{42}G_{3x}z}{21} + \frac{\sqrt{7}G_{az}x}{7} - \frac{\sqrt{105}G_{bz}x}{21}$$

$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E_{1u}),\ \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E_{1u})$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{70}G_1 z\left(x - y\right)\left(x + y\right)}{56} - \frac{5\sqrt{70}G_2 xyz}{28} - \frac{\sqrt{105}G_3 x\left(2x^2 - 3y^2 - 3z^2\right)}{42} + \frac{5\sqrt{42}G_{3x}xyz}{28}$$
$$- \frac{\sqrt{42}G_{3y}z\left(21x^2 - 9y^2 - 4z^2\right)}{168} - \frac{\sqrt{7}G_{az}y\left(x^2 + y^2 - 4z^2\right)}{28} + \frac{\sqrt{105}G_{bz}y\left(9x^2 - y^2 - 6z^2\right)}{84}$$

$$-\frac{5\sqrt{70}G_1 xyz}{28} - \frac{5\sqrt{70}G_2 z\left(x - y\right)\left(x + y\right)}{56} - \frac{\sqrt{105}G_3 y\left(3x^2 - 2y^2 + 3z^2\right)}{42} - \frac{\sqrt{42}G_{3x}z\left(9x^2 - 21y^2 + 4z^2\right)}{168}$$
$$- \frac{5\sqrt{42}G_{3y}xyz}{28} + \frac{\sqrt{7}G_{az}x\left(x^2 + y^2 - 4z^2\right)}{28} - \frac{\sqrt{105}G_{bz}x\left(x^2 - 9y^2 + 6z^2\right)}{84}$$

$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E_{1u}),\ \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E_{1u})$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

2

$$-\frac{\sqrt{385}G_1 z\left(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2\right)}{44}+\frac{\sqrt{385}G_2 xyz\left(5x^2-7y^2+2z^2\right)}{44}+\frac{\sqrt{2310}G_3 x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{462}$$

$$-\frac{5\sqrt{231}G_{3x}xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{5\sqrt{231}G_{3y}z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{924}$$

$$+\frac{5\sqrt{154}G_{az}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{308}-\frac{\sqrt{2310}G_{bz}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{924}$$

$$-\frac{\sqrt{385}G_1 xyz\left(7x^2-5y^2-2z^2\right)}{44}-\frac{\sqrt{385}G_2 z\left(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2\right)}{44}+\frac{\sqrt{2310}G_3 y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{462}$$

$$+\frac{5\sqrt{231}G_{3x}z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{924}+\frac{5\sqrt{231}G_{3y}xyz\left(x^2+y^2-2z^2\right)}{44}$$

$$-\frac{5\sqrt{154}G_{az}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{308}+\frac{\sqrt{2310}G_{bz}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{924}$$

$\vec{\mathbb{G}}_{2,1}^{(3,-1)}[g](E_{2u}),\ \vec{\mathbb{G}}_{2,2}^{(3,-1)}[g](E_{2u})$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}G_1 y}{14}+\frac{\sqrt{70}G_2 x}{14}-\frac{\sqrt{42}G_{3x}x}{42}+\frac{\sqrt{42}G_{3y}y}{42}+\frac{\sqrt{105}G_{bz}z}{21}$$

$$-\frac{\sqrt{70}G_1 x}{14}+\frac{\sqrt{70}G_2 y}{14}-\frac{\sqrt{105}G_3 z}{21}+\frac{\sqrt{42}G_{3x}y}{42}+\frac{\sqrt{42}G_{3y}x}{42}$$

$\vec{\mathbb{G}}_{2,1}^{(3,1)}[g](E_{2u}),\ \vec{\mathbb{G}}_{2,2}^{(3,1)}[g](E_{2u})$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{70}G_1 y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{70}G_2 x\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{42}G_{3x}x\left(11x^2-9y^2-24z^2\right)}{168}$$

$$-\frac{\sqrt{42}G_{3y}y\left(9x^2-11y^2+24z^2\right)}{168}-\frac{5\sqrt{7}G_{az}z\,(x-y)\,(x+y)}{14}+\frac{\sqrt{105}G_{bz}z\left(3x^2+3y^2-2z^2\right)}{42}$$

$$-\frac{\sqrt{70}G_1 x\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{70}G_2 y\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{105}G_3 z\left(3x^2+3y^2-2z^2\right)}{42}$$

$$+\frac{\sqrt{42}G_{3x}y\left(21x^2+y^2-24z^2\right)}{168}+\frac{\sqrt{42}G_{3y}x\left(x^2+21y^2-24z^2\right)}{168}+\frac{5\sqrt{7}G_{az}xyz}{7}$$

$\vec{\mathbb{G}}_{2,1}^{(3,3)}[g](E_{2u}),\ \vec{\mathbb{G}}_{2,2}^{(3,3)}[g](E_{2u})$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{385}G_1 y\left(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4\right)}{616}+\frac{\sqrt{385}G_2 x\left(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4\right)}{616}$$

$$+\frac{3\sqrt{2310}G_3 xyz\,(x-y)\,(x+y)}{44}-\frac{5\sqrt{231}G_{3x}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{1848}$$

$$-\frac{5\sqrt{231}G_{3y}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{1848}-\frac{5\sqrt{154}G_{az}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}$$

$$+\frac{\sqrt{2310}G_{bz}z\left(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4\right)}{1848}$$

$$\frac{\sqrt{385}G_1 x \left(5x^4 - 53x^2 y^2 + 3x^2 z^2 + 26y^4 + 3y^2 z^2 - 2z^4\right)}{308} - \frac{\sqrt{385}G_2 y \left(26x^4 - 53x^2 y^2 + 3x^2 z^2 + 5y^4 + 3y^2 z^2 - 2z^4\right)}{308}$$

$$+ \frac{\sqrt{2310}G_3 z \left(6x^4 - 51x^2 y^2 + 5x^2 z^2 + 6y^4 + 5y^2 z^2 - z^4\right)}{462} + \frac{5\sqrt{231}G_{3x} y \left(6x^4 + 5x^2 y^2 - 51x^2 z^2 - y^4 + 5y^2 z^2 + 6z^4\right)}{924}$$

$$- \frac{5\sqrt{231}G_{3y} x \left(x^4 - 5x^2 y^2 - 5x^2 z^2 - 6y^4 + 51y^2 z^2 - 6z^4\right)}{924} + \frac{5\sqrt{154}G_{az} xyz \left(x^2 + y^2 - 2z^2\right)}{44} - \frac{3\sqrt{2310}G_{bz} xyz \left(x - y\right)\left(x + y\right)}{44}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(3,-3)}[g](A_g)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$G_{az}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](A_g)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-G_3 xy + \frac{\sqrt{10}G_{3x} xz}{10} + \frac{\sqrt{10}G_{3y} yz}{10} - \frac{\sqrt{15}G_{az}\left(x^2 + y^2 - 2z^2\right)}{15} - \frac{G_{bz}\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](A_g)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{7\sqrt{55}G_1 yz\left(3x^2 - y^2\right)}{44} - \frac{7\sqrt{55}G_2 xz\left(x^2 - 3y^2\right)}{44} + \frac{\sqrt{330}G_3 xy\left(x^2 + y^2 - 6z^2\right)}{132} - \frac{5\sqrt{33}G_{3x} xz\left(3x^2 + 3y^2 - 4z^2\right)}{132}$$

$$- \frac{5\sqrt{33}G_{3y} yz\left(3x^2 + 3y^2 - 4z^2\right)}{132} + \frac{\sqrt{22}G_{az}\left(3x^4 + 6x^2 y^2 - 24x^2 z^2 + 3y^4 - 24y^2 z^2 + 8z^4\right)}{88} + \frac{\sqrt{330}G_{bz}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{264}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](A_g)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{7\sqrt{4290}G_1 yz\left(3x^2 - y^2\right)\left(3x^2 + 3y^2 - 8z^2\right)}{2288} - \frac{7\sqrt{4290}G_2 xz\left(x^2 - 3y^2\right)\left(3x^2 + 3y^2 - 8z^2\right)}{2288}$$

$$+ \frac{7\sqrt{715}G_3 xy\left(x^4 + 2x^2 y^2 - 16x^2 z^2 + y^4 - 16y^2 z^2 + 16z^4\right)}{572} + \frac{35\sqrt{286}G_{3x} xz\left(5x^4 + 10x^2 y^2 - 20x^2 z^2 + 5y^4 - 20y^2 z^2 + 8z^4\right)}{2288}$$

$$+ \frac{35\sqrt{286}G_{3y} yz\left(5x^4 + 10x^2 y^2 - 20x^2 z^2 + 5y^4 - 20y^2 z^2 + 8z^4\right)}{2288}$$

$$- \frac{5\sqrt{429}G_{az}\left(5x^6 + 15x^4 y^2 - 90x^4 z^2 + 15x^2 y^4 - 180x^2 y^2 z^2 + 120x^2 z^4 + 5y^6 - 90y^4 z^2 + 120y^2 z^4 - 16z^6\right)}{3432}$$

$$+ \frac{7\sqrt{715}G_{bz}\left(x - y\right)\left(x + y\right)\left(x^4 + 2x^2 y^2 - 16x^2 z^2 + y^4 - 16y^2 z^2 + 16z^4\right)}{1144}$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g, 1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2 - y^2\right)}{4}$$

** expression

$$G_1$$

$\vec{\mathbb{G}}_3^{(3,-3)}[g](B_g, 2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2 - 3y^2\right)}{4}$$

** expression

$$G_2$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g, 1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2 - y^2\right)}{4}$$

** expression

$$\frac{\sqrt{15}G_1\left(x^2 + y^2 - 2z^2\right)}{12} + \frac{\sqrt{10}G_3xz}{4} - \frac{G_{3x}xy}{2} - \frac{G_{3y}\left(x - y\right)\left(x + y\right)}{4} + \frac{\sqrt{10}G_{bz}yz}{4}$$

$\vec{\mathbb{G}}_3^{(3,-1)}[g](B_g, 2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2 - 3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{15}G_2\left(x^2 + y^2 - 2z^2\right)}{12} - \frac{\sqrt{10}G_3yz}{4} - \frac{G_{3x}\left(x - y\right)\left(x + y\right)}{4} + \frac{G_{3y}xy}{2} + \frac{\sqrt{10}G_{bz}xz}{4}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g, 1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2 - y^2\right)}{4}$$

** expression

$$\frac{\sqrt{22}G_1\left(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4\right)}{176} + \frac{5\sqrt{33}G_3xz\left(3x^2 + 3y^2 - 4z^2\right)}{132} - \frac{\sqrt{330}G_{3x}xy\left(5x^2 - 2y^2 - 9z^2\right)}{66}$$
$$+ \frac{\sqrt{330}G_{3y}\left(x^4 - 42x^2y^2 + 36x^2z^2 + 13y^4 - 36y^2z^2\right)}{528} - \frac{7\sqrt{55}G_{az}yz\left(3x^2 - y^2\right)}{44} + \frac{5\sqrt{33}G_{bz}yz\left(3x^2 + 3y^2 - 4z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,1)}[g](B_g, 2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2 - 3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{22}G_2\left(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4\right)}{176} - \frac{5\sqrt{33}G_3yz\left(3x^2 + 3y^2 - 4z^2\right)}{132} - \frac{\sqrt{330}G_{3x}\left(13x^4 - 42x^2y^2 - 36x^2z^2 + y^4 + 36y^2z^2\right)}{528}$$
$$- \frac{\sqrt{330}G_{3y}xy\left(2x^2 - 5y^2 + 9z^2\right)}{66} - \frac{7\sqrt{55}G_{az}xz\left(x^2 - 3y^2\right)}{44} + \frac{5\sqrt{33}G_{bz}xz\left(3x^2 + 3y^2 - 4z^2\right)}{132}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g, 1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2 - y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{429}G_1\left(113x^6 - 1740x^4y^2 + 45x^4z^2 + 1725x^2y^4 + 90x^2y^2z^2 - 60x^2z^4 - 118y^6 + 45y^4z^2 - 60y^2z^4 + 8z^6\right)}{6864} + \frac{7\sqrt{429}G_2xy\left(x^2 - 3y^2\right)\left(3x^2 - y^2\right)}{208}$$
$$- \frac{7\sqrt{286}G_3xz\left(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4\right)}{1144} - \frac{7\sqrt{715}G_{3x}xy\left(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4\right)}{2288}$$
$$+ \frac{7\sqrt{715}G_{3y}\left(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4\right)}{2288}$$
$$- \frac{7\sqrt{4290}G_{az}yz\left(3x^2 - y^2\right)\left(3x^2 + 3y^2 - 8z^2\right)}{2288} + \frac{7\sqrt{286}G_{bz}yz\left(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4\right)}{2288}$$

$\vec{\mathbb{G}}_3^{(3,3)}[g](B_g, 2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2 - 3y^2\right)}{4}$$

** expression

$$\frac{7\sqrt{429}G_1xy\left(x^2 - 3y^2\right)\left(3x^2 - y^2\right)}{208} + \frac{\sqrt{429}G_2\left(118x^6 - 1725x^4y^2 - 45x^4z^2 + 1740x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 113y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6\right)}{6864}$$
$$+ \frac{7\sqrt{286}G_3yz\left(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4\right)}{1144}$$
$$- \frac{7\sqrt{715}G_{3x}\left(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4\right)}{2288}$$
$$- \frac{7\sqrt{715}G_{3y}xy\left(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4\right)}{2288} - \frac{7\sqrt{4290}G_{az}xz\left(x^2 - 3y^2\right)\left(3x^2 + 3y^2 - 8z^2\right)}{2288}$$
$$+ \frac{7\sqrt{286}G_{bz}xz\left(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4\right)}{2288}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E_{1g})$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$G_{3x}$$

$$G_{3y}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E_{1g})$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{G_1 xy}{2}-\frac{G_2\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{6}G_3 yz}{4}+\frac{\sqrt{15}G_{3x}\left(x^2-3y^2+2z^2\right)}{20}+\frac{\sqrt{15}G_{3y}xy}{5}+\frac{\sqrt{10}G_{az}xz}{10}+\frac{\sqrt{6}G_{bz}xz}{4}$$

$$-\frac{G_1\left(x-y\right)\left(x+y\right)}{4}+\frac{G_2 xy}{2}+\frac{\sqrt{6}G_3 xz}{4}+\frac{\sqrt{15}G_{3x}xy}{5}-\frac{\sqrt{15}G_{3y}\left(3x^2-y^2-2z^2\right)}{20}+\frac{\sqrt{10}G_{az}yz}{10}-\frac{\sqrt{6}G_{bz}yz}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E_{1g})$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{330}G_1 xy\left(5x^2-2y^2-9z^2\right)}{66}-\frac{\sqrt{330}G_2\left(13x^4-42x^2y^2-36x^2z^2+y^4+36y^2z^2\right)}{528}$$
$$+\frac{\sqrt{55}G_3 yz\left(9x^2-19y^2+16z^2\right)}{132}-\frac{\sqrt{22}G_{3x}\left(17x^4-6x^2y^2-96x^2z^2-23y^4+144y^2z^2-8z^4\right)}{528}$$
$$-\frac{5\sqrt{22}G_{3y}xy\left(x^2+y^2-6z^2\right)}{66}-\frac{5\sqrt{33}G_{az}xz\left(3x^2+3y^2-4z^2\right)}{132}-\frac{\sqrt{55}G_{bz}xz\left(5x^2+33y^2-16z^2\right)}{132}$$

$$\frac{\sqrt{330}G_1\left(x^4-42x^2y^2+36x^2z^2+13y^4-36y^2z^2\right)}{528}-\frac{\sqrt{330}G_2 xy\left(2x^2-5y^2+9z^2\right)}{66}-\frac{\sqrt{55}G_{3}xz\left(19x^2-9y^2-16z^2\right)}{132}-\frac{5\sqrt{22}G_{3x}xy\left(x^2+y^2-6z^2\right)}{66}$$
$$+\frac{\sqrt{22}G_{3y}\left(23x^4+6x^2y^2-144x^2z^2-17y^4+96y^2z^2+8z^4\right)}{528}-\frac{5\sqrt{33}G_{az}yz\left(3x^2+3y^2-4z^2\right)}{132}+\frac{\sqrt{55}G_{bz}yz\left(33x^2+5y^2-16z^2\right)}{132}$$

$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E_{1g})$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{715}G_1 xy\left(7x^4+2x^2y^2-76x^2z^2-5y^4+44y^2z^2+16z^4\right)}{2288}$$
$$-\frac{7\sqrt{715}G_2\left(2x^6-7x^4y^2-23x^4z^2-8x^2y^4+90x^2y^2z^2+8x^2z^4+y^6-7y^4z^2-8y^2z^4\right)}{2288}-\frac{7\sqrt{4290}G_3 yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{1144}$$
$$+\frac{5\sqrt{429}G_{3x}\left(6x^6+11x^4y^2-101x^4z^2+4x^2y^4-90x^2y^2z^2+116x^2z^4-y^6+11y^4z^2+4y^2z^4-8z^6\right)}{2288}$$
$$+\frac{35\sqrt{429}G_{3y}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}+\frac{35\sqrt{286}G_{az}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$-\frac{7\sqrt{4290}G_{bz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{2288}$$

$$\frac{7\sqrt{715}G_1\left(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4\right)}{2288}$$

$$-\frac{7\sqrt{715}G_2xy\left(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4\right)}{2288}$$

$$+\frac{7\sqrt{4290}G_{3}xz\left(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4\right)}{1144} + \frac{35\sqrt{429}G_{3x}xy\left(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4\right)}{2288}$$

$$-\frac{5\sqrt{429}G_{3y}\left(x^6 - 4x^4y^2 - 11x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 - 4x^2z^4 - 6y^6 + 101y^4z^2 - 116y^2z^4 + 8z^6\right)}{2288}$$

$$+\frac{35\sqrt{286}G_{az}yz\left(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4\right)}{2288} - \frac{7\sqrt{4290}G_{bz}yz\left(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4\right)}{2288}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-3)}[g](E_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-3)}[g](E_{2g})$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$G_3$$

$$G_{bz}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-1)}[g](E_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-1)}[g](E_{2g})$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{10}G_1xz}{4} - \frac{\sqrt{10}G_2yz}{4} + \frac{\sqrt{6}G_{3x}yz}{4} + \frac{\sqrt{6}G_{3y}xz}{4} - G_{az}xy$$

$$\frac{\sqrt{10}G_1yz}{4} + \frac{\sqrt{10}G_2xz}{4} + \frac{\sqrt{6}G_{3x}xz}{4} - \frac{\sqrt{6}G_{3y}yz}{4} - \frac{G_{az}\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(3,1)}[g](E_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,1)}[g](E_{2g})$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{5\sqrt{33}G_1xz\left(3x^2 + 3y^2 - 4z^2\right)}{132} - \frac{5\sqrt{33}G_2yz\left(3x^2 + 3y^2 - 4z^2\right)}{132} - \frac{7\sqrt{22}G_3\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{66}$$

$$+\frac{\sqrt{55}G_{3x}yz\left(9x^2 - 19y^2 + 16z^2\right)}{132} - \frac{\sqrt{55}G_{3y}xz\left(19x^2 - 9y^2 - 16z^2\right)}{132} + \frac{\sqrt{330}G_{az}xy\left(x^2 + y^2 - 6z^2\right)}{132} + \frac{35\sqrt{22}G_{bz}xy\left(x - y\right)\left(x + y\right)}{132}$$

$$\frac{5\sqrt{33}G_1yz\left(3x^2 + 3y^2 - 4z^2\right)}{132} + \frac{5\sqrt{33}G_2xz\left(3x^2 + 3y^2 - 4z^2\right)}{132} + \frac{35\sqrt{22}G_3xy\left(x - y\right)\left(x + y\right)}{132} - \frac{\sqrt{55}G_{3x}xz\left(5x^2 + 33y^2 - 16z^2\right)}{132}$$

$$+\frac{\sqrt{55}G_{3y}yz\left(33x^2 + 5y^2 - 16z^2\right)}{132} + \frac{\sqrt{330}G_{az}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{264} + \frac{7\sqrt{22}G_{bz}\left(x^4 - 18x^2y^2 + 12x^2z^2 + y^4 + 12y^2z^2 - 4z^4\right)}{264}$$

$\vec{\mathbb{G}}_{3,1}^{(3,3)}[g](E_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,3)}[g](E_{2g})$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$-\frac{7\sqrt{286}G_1xz\left(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4\right)}{1144} + \frac{7\sqrt{286}G_2yz\left(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4\right)}{1144}$$

$$+\frac{\sqrt{429}G_3\left(2x^6 - 15x^4y^2 - 15x^4z^2 - 15x^2y^4 + 180x^2y^2z^2 - 15x^2z^4 + 2y^6 - 15y^4z^2 - 15y^2z^4 + 2z^6\right)}{286}$$

$$-\frac{7\sqrt{4290}G_{3x}yz\left(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4\right)}{1144} + \frac{7\sqrt{4290}G_{3y}xz\left(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4\right)}{1144}$$

$$+\frac{7\sqrt{715}G_{az}xy\left(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4\right)}{572} - \frac{21\sqrt{429}G_{bz}xy\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 10z^2\right)}{572}$$

$$\frac{7\sqrt{286}G_1yz\left(85x^4-160x^2y^2-10x^2z^2+19y^4-10y^2z^2+4z^4\right)}{2288}+\frac{7\sqrt{286}G_2xz\left(19x^4-160x^2y^2-10x^2z^2+85y^4-10y^2z^2+4z^4\right)}{2288}$$

$$-\frac{21\sqrt{429}G_{3}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}-\frac{7\sqrt{4290}G_{3x}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{2288}$$

$$-\frac{7\sqrt{4290}G_{3y}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{2288}+\frac{7\sqrt{715}G_{az}\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$

$$-\frac{\sqrt{429}G_{bz}\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(3,-3)}[g](A_u)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{\sqrt{42}G_{3x}x}{14}-\frac{\sqrt{42}G_{3y}y}{14}+\frac{2\sqrt{7}G_{az}z}{7}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](A_u)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{3\sqrt{385}G_1y\left(3x^2-y^2\right)}{308}+\frac{3\sqrt{385}G_2x\left(x^2-3y^2\right)}{308}-\frac{\sqrt{2310}G_3xyz}{22}-\frac{\sqrt{231}G_{3x}x\left(x^2+y^2-4z^2\right)}{308}$$

$$-\frac{\sqrt{231}G_{3y}y\left(x^2+y^2-4z^2\right)}{308}-\frac{3\sqrt{154}G_{az}z\left(3x^2+3y^2-2z^2\right)}{154}-\frac{\sqrt{2310}G_{bz}z\left(x-y\right)\left(x+y\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](A_u)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{15\sqrt{2002}G_1y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{2288}+\frac{15\sqrt{2002}G_2x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{2288}+\frac{5\sqrt{3003}G_3xyz\left(x^2+y^2-2z^2\right)}{286}$$

$$+\frac{\sqrt{30030}G_{3x}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{1456}+\frac{\sqrt{30030}G_{3y}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{1456}$$

$$+\frac{3\sqrt{5005}G_{az}z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{4004}+\frac{5\sqrt{3003}G_{bz}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{572}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](A_u)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{3\sqrt{2002}G_1y\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{4576}$$

$$+\frac{3\sqrt{2002}G_2x\left(x^2-3y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{4576}+\frac{3\sqrt{3003}G_3xyz\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{1144}$$

$$-\frac{\sqrt{30030}G_{3x}x\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{4576}$$

$$-\frac{\sqrt{30030}G_{3y}y\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{4576}$$

$$-\frac{\sqrt{5005}G_{az}z\left(35x^6+105x^4y^2-210x^4z^2+105x^2y^4-420x^2y^2z^2+168x^2z^4+35y^6-210y^4z^2+168y^2z^4-16z^6\right)}{2288}$$

$$+\frac{3\sqrt{3003}G_{bz}z\left(x-y\right)\left(x+y\right)\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{2288}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u,1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{G_2z}{2}-\frac{\sqrt{6}G_3y}{4}+\frac{\sqrt{6}G_{bz}x}{4}$$

$\vec{\mathbb{G}}_4^{(3,-3)}[g](B_u, 2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2 - y^2\right)}{4}$$

** expression

$$\frac{G_1 z}{2} + \frac{\sqrt{6}G_3 x}{4} + \frac{\sqrt{6}G_{bz}y}{4}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u, 1)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2 - 3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{22}G_2 z\left(3x^2 + 3y^2 - 2z^2\right)}{44} + \frac{\sqrt{33}G_3 y\left(x^2 + y^2 - 4z^2\right)}{44} + \frac{\sqrt{330}G_{3x}z\left(x - y\right)\left(x + y\right)}{44}$$
$$- \frac{\sqrt{330}G_{3y}xyz}{22} - \frac{3\sqrt{55}G_{az}x\left(x^2 - 3y^2\right)}{44} - \frac{\sqrt{33}G_{bz}x\left(x^2 + y^2 - 4z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,-1)}[g](B_u, 2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2 - y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{22}G_1 z\left(3x^2 + 3y^2 - 2z^2\right)}{44} - \frac{\sqrt{33}G_3 x\left(x^2 + y^2 - 4z^2\right)}{44} + \frac{\sqrt{330}G_{3x}xyz}{22}$$
$$+ \frac{\sqrt{330}G_{3y}z\left(x - y\right)\left(x + y\right)}{44} - \frac{3\sqrt{55}G_{az}y\left(3x^2 - y^2\right)}{44} - \frac{\sqrt{33}G_{bz}y\left(x^2 + y^2 - 4z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u, 1)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2 - 3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{715}G_2 z\left(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4\right)}{2288} + \frac{\sqrt{4290}G_3 y\left(29x^4 - 47x^2y^2 - 33x^2z^2 + 8y^4 - 33y^2z^2 + 22z^4\right)}{1144}$$
$$- \frac{7\sqrt{429}G_{3x}z\left(11x^4 + 18x^2y^2 - 28x^2z^2 - 17y^4 + 28y^2z^2\right)}{2288} + \frac{7\sqrt{429}G_{3y}xyz\left(5x^2 + 2y^2 - 7z^2\right)}{286}$$
$$+ \frac{21\sqrt{286}G_{az}x\left(x^2 - 3y^2\right)\left(x^2 + y^2 - 8z^2\right)}{2288} + \frac{\sqrt{4290}G_{bz}x\left(5x^4 - 116x^2y^2 + 66x^2z^2 + 47y^4 + 66y^2z^2 - 44z^4\right)}{2288}$$

$\vec{\mathbb{G}}_4^{(3,1)}[g](B_u, 2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2 - y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{715}G_1 z\left(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4\right)}{2288} - \frac{\sqrt{4290}G_3 x\left(8x^4 - 47x^2y^2 - 33x^2z^2 + 29y^4 - 33y^2z^2 + 22z^4\right)}{1144}$$
$$- \frac{7\sqrt{429}G_{3x}xyz\left(2x^2 + 5y^2 - 7z^2\right)}{286} - \frac{7\sqrt{429}G_{3y}z\left(17x^4 - 18x^2y^2 - 28x^2z^2 - 11y^4 + 28y^2z^2\right)}{2288}$$
$$+ \frac{21\sqrt{286}G_{az}y\left(3x^2 - y^2\right)\left(x^2 + y^2 - 8z^2\right)}{2288} + \frac{\sqrt{4290}G_{bz}y\left(47x^4 - 116x^2y^2 + 66x^2z^2 + 5y^4 + 66y^2z^2 - 44z^4\right)}{2288}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u, 1)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2 - 3y^2\right)}{4}$$

$$\frac{3\sqrt{715}G_1 xyz\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{40}$$

$$+\frac{\sqrt{715}G_2 z\left(232x^6-3165x^4y^2-105x^4z^2+3270x^2y^4-210x^2y^2z^2+84x^2z^4-197y^6-105y^4z^2+84y^2z^4-8z^6\right)}{5720}$$

$$-\frac{\sqrt{4290}G_3 y\left(40x^6-45x^4y^2-465x^4z^2-78x^2y^4+1050x^2y^2z^2-60x^2z^4+7y^6-69y^4z^2-60y^2z^4+16z^6\right)}{2860}$$

$$-\frac{3\sqrt{429}G_{3x} z\left(24x^6-75x^4y^2-95x^4z^2-90x^2y^4+330x^2y^2z^2+24x^2z^4+9y^6-15y^4z^2-24y^2z^4\right)}{1144}$$

$$-\frac{3\sqrt{429}G_{3y} xyz\left(51x^4-30x^2y^2-140x^2z^2-81y^4+300y^2z^2-48z^4\right)}{1144}+\frac{3\sqrt{286}G_{az} x\left(x^2-3y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{1144}$$

$$-\frac{\sqrt{4290}G_{bz} x\left(19x^6-141x^4y^2-258x^4z^2-75x^2y^4+1860x^2y^2z^2+120x^2z^4+85y^6-1050y^4z^2+120y^2z^4-32z^6\right)}{5720}$$

$\vec{\mathbb{G}}_4^{(3,3)}[g](B_u,2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{715}G_1 z\left(197x^6-3270x^4y^2+105x^4z^2+3165x^2y^4+210x^2y^2z^2-84x^2z^4-232y^6+105y^4z^2-84y^2z^4+8z^6\right)}{5720}$$

$$+\frac{3\sqrt{715}G_2 xyz\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{40}$$

$$+\frac{\sqrt{4290}G_3 x\left(7x^6-78x^4y^2-69x^4z^2-45x^2y^4+1050x^2y^2z^2-60x^2z^4+40y^6-465y^4z^2-60y^2z^4+16z^6\right)}{2860}$$

$$-\frac{3\sqrt{429}G_{3x} xyz\left(81x^4+30x^2y^2-300x^2z^2-51y^4+140y^2z^2+48z^4\right)}{1144}$$

$$+\frac{3\sqrt{429}G_{3y} z\left(9x^6-90x^4y^2-15x^4z^2-75x^2y^4+330x^2y^2z^2-24x^2z^4+24y^6-95y^4z^2+24y^2z^4\right)}{1144}$$

$$+\frac{3\sqrt{286}G_{az} y\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{1144}$$

$$-\frac{\sqrt{4290}G_{bz} y\left(85x^6-75x^4y^2-1050x^4z^2-141x^2y^4+1860x^2y^2z^2+120x^2z^4+19y^6-258y^4z^2+120y^2z^4-32z^6\right)}{5720}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E_{1u}),\ \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E_{1u})$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{42}G_3 x}{28}+\frac{\sqrt{105}G_{3y} z}{14}+\frac{\sqrt{70}G_{az} y}{14}+\frac{\sqrt{42}G_{bz} y}{28}$$

$$\frac{\sqrt{42}G_3 y}{28}-\frac{\sqrt{105}G_{3x} z}{14}-\frac{\sqrt{70}G_{az} x}{14}+\frac{\sqrt{42}G_{bz} x}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E_{1u}),\ \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E_{1u})$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{15\sqrt{154}G_1 z\left(x-y\right)\left(x+y\right)}{308}+\frac{15\sqrt{154}G_2 xyz}{154}+\frac{\sqrt{231}G_3 x\left(x^2-19y^2+16z^2\right)}{308}+\frac{2\sqrt{2310}G_{3x} xyz}{77}$$

$$-\frac{\sqrt{2310}G_{3y} z\left(7x^2-y^2-2z^2\right)}{308}-\frac{3\sqrt{385}G_{az} y\left(x^2+y^2-4z^2\right)}{308}-\frac{\sqrt{231}G_{bz} y\left(11x^2-9y^2+16z^2\right)}{308}$$

$$\frac{15\sqrt{154}G_1 xyz}{154}+\frac{15\sqrt{154}G_2 z\left(x-y\right)\left(x+y\right)}{308}+\frac{\sqrt{231}G_3 y\left(19x^2-y^2-16z^2\right)}{308}-\frac{\sqrt{2310}G_{3x} z\left(x^2-7y^2+2z^2\right)}{308}$$

$$-\frac{2\sqrt{2310}G_{3y} xyz}{77}+\frac{3\sqrt{385}G_{az} x\left(x^2+y^2-4z^2\right)}{308}+\frac{\sqrt{231}G_{bz} x\left(9x^2-11y^2-16z^2\right)}{308}$$

$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E_{1u})$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{5005}G_1z\left(x^4+54x^2y^2-20x^2z^2-19y^4+20y^2z^2\right)}{2288}-\frac{3\sqrt{5005}G_2xyz\left(2x^2-7y^2+5z^2\right)}{286}$$
$$+\frac{\sqrt{30030}G_3x\left(8x^4+9x^2y^2-89x^2z^2+y^4-33y^2z^2+50z^4\right)}{8008}-\frac{4\sqrt{3003}G_{3x}xyz\left(x^2+y^2-2z^2\right)}{143}$$
$$+\frac{\sqrt{3003}G_{3y}z\left(239x^4+30x^2y^2-488x^2z^2-209y^4+408y^2z^2+8z^4\right)}{16016}$$
$$+\frac{57\sqrt{2002}G_{az}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{16016}-\frac{\sqrt{30030}G_{bz}y\left(23x^4+32x^2y^2-234x^2z^2+9y^4-122y^2z^2+100z^4\right)}{16016}$$

$$\frac{3\sqrt{5005}G_1xyz\left(7x^2-2y^2-5z^2\right)}{286}+\frac{3\sqrt{5005}G_2z\left(19x^4-54x^2y^2-20x^2z^2-y^4+20y^2z^2\right)}{2288}$$
$$-\frac{\sqrt{30030}G_3y\left(x^4+9x^2y^2-33x^2z^2+8y^4-89y^2z^2+50z^4\right)}{8008}+\frac{\sqrt{3003}G_{3x}z\left(209x^4-30x^2y^2-408x^2z^2-239y^4+488y^2z^2-8z^4\right)}{16016}$$
$$+\frac{4\sqrt{3003}G_{3y}xyz\left(x^2+y^2-2z^2\right)}{143}-\frac{57\sqrt{2002}G_{az}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{16016}$$
$$-\frac{\sqrt{30030}G_{bz}x\left(9x^4+32x^2y^2-122x^2z^2+23y^4-234y^2z^2+100z^4\right)}{16016}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E_{1u})$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{5005}G_1z\left(9x^6-90x^4y^2-15x^4z^2-75x^2y^4+330x^2y^2z^2-24x^2z^4+24y^6-95y^4z^2+24y^2z^4\right)}{5720}$$
$$-\frac{3\sqrt{5005}G_2xyz\left(51x^4-30x^2y^2-140x^2z^2-81y^4+300y^2z^2-48z^4\right)}{5720}$$
$$-\frac{\sqrt{30030}G_3x\left(x^6-6x^4y^2-15x^4z^2-15x^2y^4+150x^2y^2z^2-8y^6+165y^4z^2-240y^2z^4+16z^6\right)}{2860}$$
$$+\frac{3\sqrt{3003}G_{3x}xyz\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{1144}$$
$$-\frac{\sqrt{3003}G_{3y}z\left(5x^6-30x^4y^2-15x^4z^2-75x^2y^4+210x^2y^2z^2-12x^2z^4-40y^6+225y^4z^2-156y^2z^4+8z^6\right)}{1144}$$
$$-\frac{\sqrt{2002}G_{az}y\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{1144}$$
$$+\frac{\sqrt{30030}G_{bz}y\left(11x^6+15x^4y^2-210x^4z^2-3x^2y^4-60x^2y^2z^2+240x^2z^4-7y^6+150y^4z^2-240y^2z^4+32z^6\right)}{5720}$$

$$\frac{3\sqrt{5005}G_1xyz\left(81x^4+30x^2y^2-300x^2z^2-51y^4+140y^2z^2+48z^4\right)}{5720}$$
$$+\frac{3\sqrt{5005}G_2z\left(24x^6-75x^4y^2-95x^4z^2-90x^2y^4+330x^2y^2z^2+24x^2z^4+9y^6-15y^4z^2-24y^2z^4\right)}{5720}$$
$$-\frac{\sqrt{30030}G_3y\left(8x^6+15x^4y^2-165x^4z^2+6x^2y^4-150x^2y^2z^2+240x^2z^4-y^6+15y^4z^2-16z^6\right)}{2860}$$
$$-\frac{\sqrt{3003}G_{3x}z\left(40x^6+75x^4y^2-225x^4z^2+30x^2y^4-210x^2y^2z^2+156x^2z^4-5y^6+15y^4z^2+12y^2z^4-8z^6\right)}{1144}$$
$$-\frac{3\sqrt{3003}G_{3y}xyz\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{1144}$$
$$+\frac{\sqrt{2002}G_{az}x\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{1144}$$
$$-\frac{\sqrt{30030}G_{bz}x\left(7x^6+3x^4y^2-150x^4z^2-15x^2y^4+60x^2y^2z^2+240x^2z^4-11y^6+210y^4z^2-240y^2z^4-32z^6\right)}{5720}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E_{2u},1)$, $\vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E_{2u},1)$

** symmetry

$$\frac{\sqrt{35}\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$-\frac{\sqrt{2}G_1 y}{2} + \frac{\sqrt{2}G_2 x}{2}$$

$$\frac{\sqrt{2}G_1 x}{2} + \frac{\sqrt{2}G_2 y}{2}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-3)}[g](E_{2u}, 2),\ \vec{\mathbb{G}}_{4,2}^{(3,-3)}[g](E_{2u}, 2)$

** symmetry

$$-\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{14}G_1 y}{28} - \frac{\sqrt{14}G_2 x}{28} + \frac{\sqrt{210}G_{3x} x}{28} - \frac{\sqrt{210}G_{3y} y}{28} + \frac{\sqrt{21}G_{bz} z}{7}$$

$$\frac{\sqrt{14}G_1 x}{28} - \frac{\sqrt{14}G_2 y}{28} - \frac{\sqrt{21}G_3 z}{7} - \frac{\sqrt{210}G_{3x} y}{28} - \frac{\sqrt{210}G_{3y} x}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E_{2u}, 1),\ \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E_{2u}, 1)$

** symmetry

$$\frac{\sqrt{35}\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$-\frac{3\sqrt{11}G_1 y\left(x^2 + y^2 - 4z^2\right)}{44} + \frac{3\sqrt{11}G_2 x\left(x^2 + y^2 - 4z^2\right)}{44} - \frac{5\sqrt{66}G_3 xyz}{22} - \frac{\sqrt{165}G_{3x} x\left(x^2 - 3y^2\right)}{44} + \frac{\sqrt{165}G_{3y} y\left(3x^2 - y^2\right)}{44} + \frac{5\sqrt{66}G_{bz} z\left(x - y\right)\left(x + y\right)}{44}$$

$$\frac{3\sqrt{11}G_1 x\left(x^2 + y^2 - 4z^2\right)}{44} + \frac{3\sqrt{11}G_2 y\left(x^2 + y^2 - 4z^2\right)}{44} + \frac{5\sqrt{66}G_3 z\left(x - y\right)\left(x + y\right)}{44} - \frac{\sqrt{165}G_{3x} y\left(3x^2 - y^2\right)}{44} - \frac{\sqrt{165}G_{3y} x\left(x^2 - 3y^2\right)}{44} + \frac{5\sqrt{66}G_{bz} xyz}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-1)}[g](E_{2u}, 2),\ \vec{\mathbb{G}}_{4,2}^{(3,-1)}[g](E_{2u}, 2)$

** symmetry

$$-\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$-\frac{9\sqrt{77}G_1 y\left(x^2 + y^2 - 4z^2\right)}{308} - \frac{9\sqrt{77}G_2 x\left(x^2 + y^2 - 4z^2\right)}{308} + \frac{\sqrt{1155}G_{3x} x\left(x^2 - 11y^2 + 8z^2\right)}{308}$$
$$+ \frac{\sqrt{1155}G_{3y} y\left(11x^2 - y^2 - 8z^2\right)}{308} - \frac{3\sqrt{770}G_{az} z\left(x - y\right)\left(x + y\right)}{308} + \frac{\sqrt{462}G_{bz} z\left(3x^2 + 3y^2 - 2z^2\right)}{308}$$

$$\frac{9\sqrt{77}G_1 x\left(x^2 + y^2 - 4z^2\right)}{308} - \frac{9\sqrt{77}G_2 y\left(x^2 + y^2 - 4z^2\right)}{308} - \frac{\sqrt{462}G_3 z\left(3x^2 + 3y^2 - 2z^2\right)}{308}$$
$$- \frac{\sqrt{1155}G_{3x} y\left(7x^2 - 5y^2 + 8z^2\right)}{308} + \frac{\sqrt{1155}G_{3y} x\left(5x^2 - 7y^2 - 8z^2\right)}{308} + \frac{3\sqrt{770}G_{az} xyz}{154}$$

$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E_{2u}, 1),\ \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E_{2u}, 1)$

** symmetry

$$\frac{\sqrt{35}\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{3\sqrt{1430}G_1y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{1430}G_2x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}$$

$$-\frac{7\sqrt{2145}G_3xyz\left(x^2+y^2-2z^2\right)}{286}-\frac{7\sqrt{858}G_{3x}x\left(5x^4-34x^2y^2-16x^2z^2+9y^4+48y^2z^2\right)}{2288}-\frac{7\sqrt{858}G_{3y}y\left(9x^4-34x^2y^2+48x^2z^2+5y^4-16y^2z^2\right)}{2288}$$

$$-\frac{63\sqrt{143}G_{az}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{572}+\frac{7\sqrt{2145}G_{bz}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{572}$$

$$\frac{3\sqrt{1430}G_1x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{1430}G_2y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{2288}$$

$$+\frac{7\sqrt{2145}G_3z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{572}-\frac{7\sqrt{858}G_{3x}y\left(21x^4-26x^2y^2-48x^2z^2+y^4+16y^2z^2\right)}{2288}$$

$$+\frac{7\sqrt{858}G_{3y}x\left(x^4-26x^2y^2+16x^2z^2+21y^4-48y^2z^2\right)}{2288}-\frac{63\sqrt{143}G_{az}xyz\,(x-y)\,(x+y)}{143}+\frac{7\sqrt{2145}G_{bz}xyz\left(x^2+y^2-2z^2\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(3,1)}[g](E_{2u},2),\ \vec{\mathbb{G}}_{4,2}^{(3,1)}[g](E_{2u},2)$
** symmetry

$$-\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{9\sqrt{10010}G_1y\left(10x^4-15x^2y^2-15x^2z^2+3y^4-15y^2z^2+10z^4\right)}{8008}-\frac{9\sqrt{10010}G_2x\left(3x^4-15x^2y^2-15x^2z^2+10y^4-15y^2z^2+10z^4\right)}{8008}$$

$$+\frac{3\sqrt{15015}G_3xyz\,(x-y)\,(x+y)}{143}-\frac{\sqrt{6006}G_{3x}x\left(5x^4-53x^2y^2+3x^2z^2-58y^4+507y^2z^2-86z^4\right)}{8008}$$

$$-\frac{\sqrt{6006}G_{3y}y\left(58x^4+53x^2y^2-507x^2z^2-5y^4-3y^2z^2+86z^4\right)}{8008}-\frac{3\sqrt{1001}G_{az}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{572}$$

$$-\frac{\sqrt{15015}G_{bz}z\left(9x^4+186x^2y^2-80x^2z^2+9y^4-80y^2z^2+16z^4\right)}{4004}$$

$$-\frac{9\sqrt{10010}G_1x\left(x^4-40x^2y^2+30x^2z^2+15y^4+30y^2z^2-20z^4\right)}{16016}+\frac{9\sqrt{10010}G_2y\left(15x^4-40x^2y^2+30x^2z^2+y^4+30y^2z^2-20z^4\right)}{16016}$$

$$+\frac{\sqrt{15015}G_3z\left(51x^4-66x^2y^2-80x^2z^2+51y^4-80y^2z^2+16z^4\right)}{4004}+\frac{\sqrt{6006}G_{3x}y\left(73x^4+20x^2y^2-498x^2z^2-53y^4+510y^2z^2-172z^4\right)}{16016}$$

$$-\frac{\sqrt{6006}G_{3y}x\left(53x^4-20x^2y^2-510x^2z^2-73y^4+498y^2z^2+172z^4\right)}{16016}+\frac{3\sqrt{1001}G_{az}xyz\left(x^2+y^2-2z^2\right)}{286}-\frac{3\sqrt{15015}G_{bz}xyz\,(x-y)\,(x+y)}{143}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E_{2u},1),\ \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E_{2u},1)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{1430}G_1y\left(1499x^6-7515x^4y^2+60x^4z^2+4497x^2y^4+120x^2y^2z^2-120x^2z^4-217y^6+60y^4z^2-120y^2z^4+32z^6\right)}{22880}$$

$$+\frac{\sqrt{1430}G_2x\left(217x^6-4497x^4y^2-60x^4z^2+7515x^2y^4-120x^2y^2z^2+120x^2z^4-1499y^6-60y^4z^2+120y^2z^4-32z^6\right)}{22880}$$

$$+\frac{3\sqrt{2145}G_3xyz\left(207x^4-730x^2y^2+40x^2z^2+207y^4+40y^2z^2-24z^4\right)}{5720}$$

$$-\frac{3\sqrt{858}G_{3x}x\left(7x^6-51x^4y^2-96x^4z^2-35x^2y^4+720x^2y^2z^2+40x^2z^4+23y^6-240y^4z^2-120y^2z^4\right)}{4576}$$

$$-\frac{3\sqrt{858}G_{3y}y\left(23x^6-35x^4y^2-240x^4z^2-51x^2y^4+720x^2y^2z^2-120x^2z^4+7y^6-96y^4z^2+40y^2z^4\right)}{4576}$$

$$-\frac{3\sqrt{143}G_{az}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(3x^2+3y^2-10z^2\right)}{208}$$

$$+\frac{3\sqrt{2145}G_{bz}z\,(x-y)\,(x+y)\left(79x^4-986x^2y^2-40x^2z^2+79y^4-40y^2z^2+24z^4\right)}{11440}$$

$$-\frac{\sqrt{1430}G_1 x \left(53x^6 - 1128x^4y^2 + 15x^4z^2 + 1875x^2y^4 + 30x^2y^2z^2 - 30x^2z^4 - 376y^6 + 15y^4z^2 - 30y^2z^4 + 8z^6\right)}{5720}$$

$$+\frac{\sqrt{1430}G_2 y \left(376x^6 - 1875x^4y^2 - 15x^4z^2 + 1128x^2y^4 - 30x^2y^2z^2 + 30x^2z^4 - 53y^6 - 15y^4z^2 + 30y^2z^4 - 8z^6\right)}{5720}$$

$$-\frac{3\sqrt{2145}G_3 z \left(x - y\right)\left(x + y\right)\left(8x^4 - 127x^2y^2 + 5x^2z^2 + 8y^4 + 5y^2z^2 - 3z^4\right)}{1430}$$

$$-\frac{3\sqrt{858}G_{3x} y \left(8x^6 - 5x^4y^2 - 105x^4z^2 - 12x^2y^4 + 150x^2y^2z^2 + 30x^2z^4 + y^6 - 9y^4z^2 - 10y^2z^4\right)}{1144}$$

$$+\frac{3\sqrt{858}G_{3y} x \left(x^6 - 12x^4y^2 - 9x^4z^2 - 5x^2y^4 + 150x^2y^2z^2 - 10x^2z^4 + 8y^6 - 105y^4z^2 + 30y^2z^4\right)}{1144}$$

$$-\frac{3\sqrt{143}G_{az} xyz \left(x - y\right)\left(x + y\right)\left(3x^2 + 3y^2 - 10z^2\right)}{52} + \frac{3\sqrt{2145}G_{bz} xyz \left(111x^4 - 350x^2y^2 - 20x^2z^2 + 111y^4 - 20y^2z^2 + 12z^4\right)}{2860}$$

$\vec{\mathbb{G}}_{4,1}^{(3,3)}[g](E_{2u}, 2), \vec{\mathbb{G}}_{4,2}^{(3,3)}[g](E_{2u}, 2)$

** symmetry

$$-\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy \left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{10010}G_1 y \left(85x^6 - 75x^4y^2 - 1050x^4z^2 - 141x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 19y^6 - 258y^4z^2 + 120y^2z^4 - 32z^6\right)}{11440}$$

$$-\frac{\sqrt{10010}G_2 x \left(19x^6 - 141x^4y^2 - 258x^4z^2 - 75x^2y^4 + 1860x^2y^2z^2 + 120x^2z^4 + 85y^6 - 1050y^4z^2 + 120y^2z^4 - 32z^6\right)}{11440}$$

$$-\frac{3\sqrt{15015}G_3 xyz \left(x - y\right)\left(x + y\right)\left(3x^2 + 3y^2 - 10z^2\right)}{260}$$

$$+\frac{\sqrt{6006}G_{3x} x \left(7x^6 + 3x^4y^2 - 150x^4z^2 - 15x^2y^4 + 60x^2y^2z^2 + 240x^2z^4 - 11y^6 + 210y^4z^2 - 240y^2z^4 - 32z^6\right)}{2288}$$

$$+\frac{\sqrt{6006}G_{3y} y \left(11x^6 + 15x^4y^2 - 210x^4z^2 - 3x^2y^4 - 60x^2y^2z^2 + 240x^2z^4 - 7y^6 + 150y^4z^2 - 240y^2z^4 + 32z^6\right)}{2288}$$

$$+\frac{3\sqrt{1001}G_{az} z \left(x - y\right)\left(x + y\right)\left(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4\right)}{1144}$$

$$-\frac{\sqrt{15015}G_{bz} z \left(67x^6 - 195x^4y^2 - 270x^4z^2 - 195x^2y^4 + 780x^2y^2z^2 + 84x^2z^4 + 67y^6 - 270y^4z^2 + 84y^2z^4 - 8z^6\right)}{5720}$$

$$-\frac{\sqrt{10010}G_1 x \left(7x^6 - 78x^4y^2 - 69x^4z^2 - 45x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 40y^6 - 465y^4z^2 - 60y^2z^4 + 16z^6\right)}{5720}$$

$$+\frac{\sqrt{10010}G_2 y \left(40x^6 - 45x^4y^2 - 465x^4z^2 - 78x^2y^4 + 1050x^2y^2z^2 - 60x^2z^4 + 7y^6 - 69y^4z^2 - 60y^2z^4 + 16z^6\right)}{5720}$$

$$-\frac{\sqrt{15015}G_3 z \left(8x^6 - 75x^4y^2 - 15x^4z^2 - 75x^2y^4 + 300x^2y^2z^2 - 21x^2z^4 + 8y^6 - 15y^4z^2 - 21y^2z^4 + 2z^6\right)}{1430}$$

$$-\frac{\sqrt{6006}G_{3x} y \left(8x^6 + 15x^4y^2 - 165x^4z^2 + 6x^2y^4 - 150x^2y^2z^2 + 240x^2z^4 - y^6 + 15y^4z^2 - 16z^6\right)}{1144}$$

$$+\frac{\sqrt{6006}G_{3y} x \left(x^6 - 6x^4y^2 - 15x^4z^2 - 15x^2y^4 + 150x^2y^2z^2 - 8y^6 + 165y^4z^2 - 240y^2z^4 + 16z^6\right)}{1144}$$

$$-\frac{3\sqrt{1001}G_{az} xyz \left(15x^4 + 30x^2y^2 - 80x^2z^2 + 15y^4 - 80y^2z^2 + 48z^4\right)}{572} + \frac{3\sqrt{15015}G_{bz} xyz \left(x - y\right)\left(x + y\right)\left(3x^2 + 3y^2 - 10z^2\right)}{260}$$