PG No. 9 C_4 4 [tetragonal] (axial, internal axial quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(2,2)}[g](A)$

** symmetry

1

** expression

$$-\frac{\sqrt{5}G_u\left(x^2+y^2-2z^2\right)}{10}+\frac{\sqrt{15}G_v\left(x-y\right)\left(x+y\right)}{10}+\frac{\sqrt{15}G_{xy}xy}{5}+\frac{\sqrt{15}G_{xz}xz}{5}+\frac{\sqrt{15}G_{yz}yz}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(2,0)}[g](A)$

** symmetry

z

** expression

$$\frac{\sqrt{10}G_uz}{5}+\frac{\sqrt{30}G_{xz}x}{10}+\frac{\sqrt{30}G_{yz}y}{10}$$

$\vec{\mathbb{G}}_1^{(2,2)}[g](A)$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}G_uz\left(3x^2+3y^2-2z^2\right)}{70}+\frac{\sqrt{105}G_vz\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}G_{xy}xyz}{7}-\frac{\sqrt{105}G_{xz}x\left(x^2+y^2-4z^2\right)}{35}-\frac{\sqrt{105}G_{yz}y\left(x^2+y^2-4z^2\right)}{35}$$

$\vec{\mathbb{G}}_{1,1}^{(2,0)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(2,0)}[g](E)$

** symmetry

x

y

** expression

$$-\frac{\sqrt{10}G_ux}{10}+\frac{\sqrt{30}G_vx}{10}+\frac{\sqrt{30}G_{xy}y}{10}+\frac{\sqrt{30}G_{xz}z}{10}$$

$$-\frac{\sqrt{10}G_uy}{10}-\frac{\sqrt{30}G_vy}{10}+\frac{\sqrt{30}G_{xy}x}{10}+\frac{\sqrt{30}G_{yz}z}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(2,2)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(2,2)}[g](E)$

** symmetry

x

y

** expression

$$-\frac{3\sqrt{35}G_ux\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}G_vx\left(3x^2-7y^2-2z^2\right)}{70}+\frac{\sqrt{105}G_{xy}y\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}G_{xz}z\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}G_{yz}xyz}{7}$$

$$-\frac{3\sqrt{35}G_uy\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}G_vy\left(7x^2-3y^2+2z^2\right)}{70}-\frac{\sqrt{105}G_{xy}x\left(x^2-4y^2+z^2\right)}{35}+\frac{\sqrt{105}G_{xz}xyz}{7}-\frac{\sqrt{105}G_{yz}z\left(x^2-4y^2+z^2\right)}{35}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(2,-2)}[g](A)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

G_u

$\vec{\mathbb{G}}_2^{(2,0)}[g](A)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{14}G_u\left(x^2+y^2-2z^2\right)}{14}-\frac{\sqrt{42}G_v\,(x-y)\,(x+y)}{14}-\frac{\sqrt{42}G_{xy}xy}{7}+\frac{\sqrt{42}G_{xz}xz}{14}+\frac{\sqrt{42}G_{yz}yz}{14}$$

$\vec{\mathbb{G}}_2^{(2,2)}[g](A)$
** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\frac{\sqrt{14}G_u\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{56}-\frac{5\sqrt{42}G_v\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{168}$$
$$-\frac{5\sqrt{42}G_{xy}xy\left(x^2+y^2-6z^2\right)}{84}-\frac{5\sqrt{42}G_{xz}xz\left(3x^2+3y^2-4z^2\right)}{84}-\frac{5\sqrt{42}G_{yz}yz\left(3x^2+3y^2-4z^2\right)}{84}$$

$\vec{\mathbb{G}}_2^{(2,-2)}[g](B,1)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$G_v$$

$\vec{\mathbb{G}}_2^{(2,-2)}[g](B,2)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$G_{xy}$$

$\vec{\mathbb{G}}_2^{(2,0)}[g](B,1)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{42}G_u\,(x-y)\,(x+y)}{14}+\frac{\sqrt{14}G_v\left(x^2+y^2-2z^2\right)}{14}+\frac{3\sqrt{14}G_{xz}xz}{14}-\frac{3\sqrt{14}G_{yz}yz}{14}$$

$\vec{\mathbb{G}}_2^{(2,0)}[g](B,2)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}G_uxy}{7}+\frac{\sqrt{14}G_{xy}\left(x^2+y^2-2z^2\right)}{14}+\frac{3\sqrt{14}G_{xz}yz}{14}+\frac{3\sqrt{14}G_{yz}xz}{14}$$

$\vec{\mathbb{G}}_2^{(2,2)}[g](B,1)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{5\sqrt{42}G_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{168}+\frac{\sqrt{14}G_v\left(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4\right)}{168}$$
$$+\frac{5\sqrt{14}G_{xy}xy\,(x-y)\,(x+y)}{12}+\frac{5\sqrt{14}G_{xz}xz\left(5x^2-9y^2-2z^2\right)}{84}+\frac{5\sqrt{14}G_{yz}yz\left(9x^2-5y^2+2z^2\right)}{84}$$

$\vec{\mathbb{G}}_2^{(2,2)}[g](B,2)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}G_uxy\left(x^2+y^2-6z^2\right)}{84}+\frac{5\sqrt{14}G_vxy\,(x-y)\,(x+y)}{12}-\frac{\sqrt{14}G_{xy}\left(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4\right)}{42}$$
$$+\frac{5\sqrt{14}G_{xz}yz\left(6x^2-y^2-z^2\right)}{42}-\frac{5\sqrt{14}G_{yz}xz\left(x^2-6y^2+z^2\right)}{42}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E)$, $\vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E)$
** symmetry

$\sqrt{3}xz$

$\sqrt{3}yz$

** expression

G_{xz}

G_{yz}

$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E)$, $\vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E)$
** symmetry

$\sqrt{3}xz$

$\sqrt{3}yz$

** expression

$$\frac{\sqrt{42}G_u xz}{14} + \frac{3\sqrt{14}G_v xz}{14} + \frac{3\sqrt{14}G_{xy} yz}{14} + \frac{\sqrt{14}G_{xz}\left(x^2 - 2y^2 + z^2\right)}{14} + \frac{3\sqrt{14}G_{yz} xy}{14}$$

$$\frac{\sqrt{42}G_u yz}{14} - \frac{3\sqrt{14}G_v yz}{14} + \frac{3\sqrt{14}G_{xy} xz}{14} + \frac{3\sqrt{14}G_{xz} xy}{14} - \frac{\sqrt{14}G_{yz}\left(2x^2 - y^2 - z^2\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E)$, $\vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E)$
** symmetry

$\sqrt{3}xz$

$\sqrt{3}yz$

** expression

$$-\frac{5\sqrt{42}G_u xz\left(3x^2 + 3y^2 - 4z^2\right)}{84} + \frac{5\sqrt{14}G_v xz\left(5x^2 - 9y^2 - 2z^2\right)}{84} + \frac{5\sqrt{14}G_{xy} yz\left(6x^2 - y^2 - z^2\right)}{42}$$
$$- \frac{\sqrt{14}G_{xz}\left(4x^4 + 3x^2y^2 - 27x^2z^2 - y^4 + 3y^2z^2 + 4z^4\right)}{42} - \frac{5\sqrt{14}G_{yz} xy\left(x^2 + y^2 - 6z^2\right)}{42}$$

$$-\frac{5\sqrt{42}G_u yz\left(3x^2 + 3y^2 - 4z^2\right)}{84} + \frac{5\sqrt{14}G_v yz\left(9x^2 - 5y^2 + 2z^2\right)}{84} - \frac{5\sqrt{14}G_{xy} xz\left(x^2 - 6y^2 + z^2\right)}{42}$$
$$- \frac{5\sqrt{14}G_{xz} xy\left(x^2 + y^2 - 6z^2\right)}{42} + \frac{\sqrt{14}G_{yz}\left(x^4 - 3x^2y^2 - 3x^2z^2 - 4y^4 + 27y^2z^2 - 4z^4\right)}{42}$$

* Harmonics for rank 3
$\vec{\mathbb{G}}_{3}^{(2,-2)}[g](A)$
** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_u z}{5} - \frac{\sqrt{5}G_{xz} x}{5} - \frac{\sqrt{5}G_{yz} y}{5}$$

$\vec{\mathbb{G}}_{3}^{(2,0)}[g](A)$
** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u z\left(3x^2 + 3y^2 - 2z^2\right)}{15} - \frac{\sqrt{5}G_v z\left(x - y\right)\left(x + y\right)}{2} - \sqrt{5}G_{xy} xyz - \frac{\sqrt{5}G_{xz} x\left(x^2 + y^2 - 4z^2\right)}{20} - \frac{\sqrt{5}G_{yz} y\left(x^2 + y^2 - 4z^2\right)}{20}$$

$\vec{\mathbb{G}}_{3}^{(2,2)}[g](A)$
** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{330}G_u z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{264}-\frac{7\sqrt{110}G_{vz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}-\frac{7\sqrt{110}G_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{\sqrt{110}G_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{110}G_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_3^{(2,-2)}[g](B,1)$
** symmetry

$\sqrt{15}xyz$

** expression

$$\frac{\sqrt{3}G_{xy}z}{3}+\frac{\sqrt{3}G_{xz}y}{3}+\frac{\sqrt{3}G_{yz}x}{3}$$

$\vec{\mathbb{G}}_3^{(2,-2)}[g](B,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_v z}{3}+\frac{\sqrt{3}G_{xz}x}{3}-\frac{\sqrt{3}G_{yz}y}{3}$$

$\vec{\mathbb{G}}_3^{(2,0)}[g](B,1)$
** symmetry

$\sqrt{15}xyz$

** expression

$$\frac{\sqrt{3}G_{xy}z\left(3x^2+3y^2-2z^2\right)}{6}+\frac{\sqrt{3}G_{xz}y\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}G_{yz}x\left(2x^2-3y^2-3z^2\right)}{6}$$

$\vec{\mathbb{G}}_3^{(2,0)}[g](B,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_v z\left(3x^2+3y^2-2z^2\right)}{6}+\frac{\sqrt{3}G_{xz}x\left(x^2-9y^2+6z^2\right)}{12}+\frac{\sqrt{3}G_{yz}y\left(9x^2-y^2-6z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(2,2)}[g](B,1)$
** symmetry

$\sqrt{15}xyz$

** expression

$$-\frac{21\sqrt{22}G_u xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{21\sqrt{66}G_v xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}G_{xy}z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{66}$$
$$-\frac{\sqrt{66}G_{xz}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{66}+\frac{\sqrt{66}G_{yz}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{66}$$

$\vec{\mathbb{G}}_3^{(2,2)}[g](B,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{21\sqrt{22}G_u z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{66}G_v z\left(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4\right)}{264}+\frac{21\sqrt{66}G_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}$$
$$-\frac{\sqrt{66}G_{xz}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{132}-\frac{\sqrt{66}G_{yz}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{132}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u x}{10}+\frac{3\sqrt{5}G_v x}{10}-\frac{\sqrt{5}G_{xy}y}{5}-\frac{\sqrt{5}G_{xz}z}{5}$$

$$-\frac{\sqrt{15}G_u y}{10}-\frac{3\sqrt{5}G_v y}{10}-\frac{\sqrt{5}G_{xy}x}{5}-\frac{\sqrt{5}G_{yz}z}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{G_u y}{2}-\frac{\sqrt{3}G_v y}{6}-\frac{\sqrt{3}G_{xy}x}{3}+\frac{\sqrt{3}G_{yz}z}{3}$$

$$-\frac{G_u x}{2}-\frac{\sqrt{3}G_v x}{6}+\frac{\sqrt{3}G_{xy}y}{3}-\frac{\sqrt{3}G_{xz}z}{3}$$

$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u x\left(4x^2-21y^2+9z^2\right)}{60}+\frac{\sqrt{5}G_v x\left(4x^2-y^2-11z^2\right)}{20}+\frac{\sqrt{5}G_{xy}y\left(4x^2-y^2-z^2\right)}{20}+\frac{\sqrt{5}G_{xz}z\left(4x^2-y^2-z^2\right)}{20}-\sqrt{5}G_{yz}xyz$$

$$\frac{\sqrt{15}G_u y\left(21x^2-4y^2-9z^2\right)}{60}+\frac{\sqrt{5}G_v y\left(x^2-4y^2+11z^2\right)}{20}-\frac{\sqrt{5}G_{xy}x\left(x^2-4y^2+z^2\right)}{20}-\sqrt{5}G_{xz}xyz-\frac{\sqrt{5}G_{yz}z\left(x^2-4y^2+z^2\right)}{20}$$

$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{G_u y\left(3x^2-2y^2+3z^2\right)}{4}-\frac{\sqrt{3}G_v y\left(3x^2-2y^2+3z^2\right)}{12}-\frac{\sqrt{3}G_{xy}x\left(x^2+6y^2-9z^2\right)}{12}-\frac{\sqrt{3}G_{yz}z\left(9x^2-6y^2-z^2\right)}{12}$$

$$\frac{G_u x\left(2x^2-3y^2-3z^2\right)}{4}+\frac{\sqrt{3}G_v x\left(2x^2-3y^2-3z^2\right)}{12}+\frac{\sqrt{3}G_{xy}y\left(6x^2+y^2-9z^2\right)}{12}-\frac{\sqrt{3}G_{xz}z\left(6x^2-9y^2+z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{330}G_u x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264}+\frac{\sqrt{110}G_v x\left(4x^4-27x^2y^2-13x^2z^2+11y^4+15y^2z^2+4z^4\right)}{88}$$
$$+\frac{\sqrt{110}G_{xy}y\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}+\frac{\sqrt{110}G_{xz}z\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}+\frac{7\sqrt{110}G_{yz}xyz\left(2x^2-y^2-z^2\right)}{44}$$

$$\frac{\sqrt{330}G_u y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264}-\frac{\sqrt{110}G_v y\left(11x^4-27x^2y^2+15x^2z^2+4y^4-13y^2z^2+4z^4\right)}{88}$$
$$+\frac{\sqrt{110}G_{xy}x\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}-\frac{7\sqrt{110}G_{xz}xyz\left(x^2-2y^2+z^2\right)}{44}+\frac{\sqrt{110}G_{yz}z\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{22}G_u y\left(9x^4+11x^2y^2-87x^2z^2+2y^4-31y^2z^2+30z^4\right)}{88}-\frac{\sqrt{66}G_v y\left(51x^4-73x^2y^2-87x^2z^2+2y^4+53y^2z^2-12z^4\right)}{264}$$
$$+\frac{\sqrt{66}G_{xy}x\left(5x^4-46x^2y^2-4x^2z^2+12y^4+66y^2z^2-9z^4\right)}{132}-\frac{21\sqrt{66}G_{xz}xyz\,(x-z)\,(x+z)}{44}$$
$$+\frac{\sqrt{66}G_{yz}z\left(9x^4-66x^2y^2+4x^2z^2-12y^4+46y^2z^2-5z^4\right)}{132}$$

$$-\frac{\sqrt{22}G_u x\left(2x^4+11x^2y^2-31x^2z^2+9y^4-87y^2z^2+30z^4\right)}{88}-\frac{\sqrt{66}G_v x\left(2x^4-73x^2y^2+53x^2z^2+51y^4-87y^2z^2-12z^4\right)}{264}$$
$$-\frac{\sqrt{66}G_{xy}y\left(12x^4-46x^2y^2+66x^2z^2+5y^4-4y^2z^2-9z^4\right)}{132}$$
$$+\frac{\sqrt{66}G_{xz}z\left(12x^4+66x^2y^2-46x^2z^2-9y^4-4y^2z^2+5z^4\right)}{132}+\frac{21\sqrt{66}G_{yz}xyz\,(y-z)\,(y+z)}{44}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{30}G_u\left(x^2+y^2-2z^2\right)}{20}+\frac{3\sqrt{10}G_v\,(x-y)\,(x+y)}{20}-\frac{\sqrt{10}G_{xy}xy}{5}-\frac{\sqrt{10}G_{xz}xz}{5}-\frac{\sqrt{10}G_{yz}yz}{5}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{42}G_u\left(x^2+y^2-2z^2\right)}{28}-\frac{3\sqrt{14}G_v\,(x-y)\,(x+y)}{28}+\frac{2\sqrt{14}G_{xy}xy}{7}-\frac{\sqrt{14}G_{xz}xz}{7}-\frac{\sqrt{14}G_{yz}yz}{7}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A,3)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}G_v xy}{2}+\frac{\sqrt{6}G_{xy}\,(x-y)\,(x+y)}{4}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{165}G_u\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{66}+\frac{\sqrt{55}G_v\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{55}G_{xy}xy\left(x^2+y^2-6z^2\right)}{22}+\frac{\sqrt{55}G_{xz}xz\left(x^2-6y^2+z^2\right)}{22}-\frac{\sqrt{55}G_{yz}yz\left(6x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$\frac{\sqrt{231}G_u\left(8x^4-33x^2y^2-15x^2z^2+8y^4-15y^2z^2+5z^4\right)}{231}+\frac{\sqrt{77}G_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{77}$$
$$+\frac{\sqrt{77}G_{xy}xy\left(x^2+y^2-6z^2\right)}{14}-\frac{\sqrt{77}G_{xz}xz\left(16x^2-33y^2-5z^2\right)}{154}+\frac{\sqrt{77}G_{yz}yz\left(33x^2-16y^2+5z^2\right)}{154}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A,3)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{7\sqrt{11}G_uxy\left(x-y\right)\left(x+y\right)}{11}+\frac{\sqrt{33}G_vxy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}G_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}G_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}G_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{15015}G_u\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{572}$$
$$+\frac{3\sqrt{5005}G_v\left(x-y\right)\left(x+y\right)\left(x^4-9x^2y^2-5x^2z^2+y^4-5y^2z^2+5z^4\right)}{572}+\frac{\sqrt{5005}G_{xy}xy\left(7x^4-19x^2y^2-13x^2z^2+7y^4-13y^2z^2+13z^4\right)}{286}$$
$$+\frac{\sqrt{5005}G_{xz}xz\left(7x^4-13x^2y^2-19x^2z^2+13y^4-13y^2z^2+7z^4\right)}{286}+\frac{\sqrt{5005}G_{yz}yz\left(13x^4-13x^2y^2-13x^2z^2+7y^4-19y^2z^2+7z^4\right)}{286}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{429}G_u\left(x^6+45x^4y^2-60x^4z^2+45x^2y^4-540x^2y^2z^2+150x^2z^4+y^6-60y^4z^2+150y^2z^4-20z^6\right)}{1144}$$
$$-\frac{21\sqrt{143}G_v\left(x-y\right)\left(x+y\right)\left(x^4-20x^2y^2+6x^2z^2+y^4+6y^2z^2-6z^4\right)}{1144}-\frac{7\sqrt{143}G_{xy}xy\left(x^4-4x^2y^2+2x^2z^2+y^4+2y^2z^2-2z^4\right)}{52}$$
$$+\frac{7\sqrt{143}G_{xz}xz\left(x^4+56x^2y^2-22x^2z^2-11y^4-34y^2z^2+10z^4\right)}{572}-\frac{7\sqrt{143}G_{yz}yz\left(11x^4-56x^2y^2+34x^2z^2-y^4+22y^2z^2-10z^4\right)}{572}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A,3)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{9\sqrt{1001}G_uxy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{3003}G_vxy\left(25x^4-82x^2y^2-4x^2z^2+25y^4-4y^2z^2+4z^4\right)}{572}$$
$$-\frac{\sqrt{3003}G_{xy}\left(x-y\right)\left(x+y\right)\left(2x^4-29x^2y^2+x^2z^2+2y^4+y^2z^2-z^4\right)}{286}$$
$$+\frac{3\sqrt{3003}G_{xz}yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{286}-\frac{3\sqrt{3003}G_{yz}xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{286}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](B,1)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{14}G_u\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{42}G_v\left(x^2+y^2-2z^2\right)}{28}-\frac{\sqrt{42}G_{xz}xz}{7}+\frac{\sqrt{42}G_{yz}yz}{7}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](B,2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{14}G_u xy}{14}-\frac{\sqrt{42}G_{xy}\left(x^2+y^2-2z^2\right)}{28}+\frac{\sqrt{42}G_{xz}yz}{7}+\frac{\sqrt{42}G_{yz}xz}{7}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](B,1)$
** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{77}G_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{231}G_v\left(2x^4-3x^2y^2-9x^2z^2+2y^4-9y^2z^2+3z^4\right)}{77}$$
$$+\frac{\sqrt{231}G_{xy}xy\,(x-y)\,(x+y)}{22}-\frac{\sqrt{231}G_{xz}xz\left(2x^2-33y^2+9z^2\right)}{154}-\frac{\sqrt{231}G_{yz}yz\left(33x^2-2y^2-9z^2\right)}{154}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](B,2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{2\sqrt{77}G_u xy\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}G_v xy\,(x-y)\,(x+y)}{22}-\frac{\sqrt{231}G_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}$$
$$+\frac{\sqrt{231}G_{xz}yz\left(39x^2-31y^2+18z^2\right)}{308}-\frac{\sqrt{231}G_{yz}xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](B,1)$
** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{143}G_u\,(x-y)\,(x+y)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{\sqrt{429}G_v\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$
$$+\frac{21\sqrt{429}G_{xy}xy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}+\frac{7\sqrt{429}G_{xz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{572}$$
$$+\frac{7\sqrt{429}G_{yz}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{572}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](B,2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{143}G_u xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{21\sqrt{429}G_v xy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}$$
$$+\frac{\sqrt{429}G_{xy}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$
$$-\frac{7\sqrt{429}G_{xz}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{286}+\frac{7\sqrt{429}G_{yz}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,1)$
** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{3\sqrt{2}G_u yz}{4}-\frac{\sqrt{6}G_v yz}{4}+\frac{\sqrt{6}G_{yz}\left(y-z\right)\left(y+z\right)}{4}$$

$$\frac{3\sqrt{2}G_u xz}{4}-\frac{\sqrt{6}G_v xz}{4}-\frac{\sqrt{6}G_{xz}\left(x-z\right)\left(x+z\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{14}G_u xz}{28}-\frac{3\sqrt{42}G_v xz}{28}+\frac{\sqrt{42}G_{xy}yz}{7}-\frac{\sqrt{42}G_{xz}\left(x^2-2y^2+z^2\right)}{28}+\frac{\sqrt{42}G_{yz}xy}{7}$$

$$-\frac{3\sqrt{14}G_u yz}{28}+\frac{3\sqrt{42}G_v yz}{28}+\frac{\sqrt{42}G_{xy}xz}{7}+\frac{\sqrt{42}G_{xz}xy}{7}+\frac{\sqrt{42}G_{yz}\left(2x^2-y^2-z^2\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{11}G_u yz\left(18x^2+11y^2-17z^2\right)}{44}+\frac{\sqrt{33}G_v yz\left(6x^2-15y^2+13z^2\right)}{44}+\frac{7\sqrt{33}G_{xy}xz\left(3y^2-z^2\right)}{44}$$
$$+\frac{7\sqrt{33}G_{xz}xy\left(y^2-3z^2\right)}{44}-\frac{\sqrt{33}G_{yz}\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{44}$$

$$-\frac{\sqrt{11}G_u xz\left(11x^2+18y^2-17z^2\right)}{44}-\frac{\sqrt{33}G_v xz\left(15x^2-6y^2-13z^2\right)}{44}-\frac{7\sqrt{33}G_{xy}yz\left(3x^2-z^2\right)}{44}$$
$$-\frac{\sqrt{33}G_{xz}\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{44}-\frac{7\sqrt{33}G_{yz}xy\left(x^2-3z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{\sqrt{77}G_u xz\left(25x^2-24y^2-17z^2\right)}{308}-\frac{\sqrt{231}G_v xz\left(3x^2+24y^2-11z^2\right)}{308}+\frac{\sqrt{231}G_{xy}yz\left(39x^2+18y^2-31z^2\right)}{308}$$
$$-\frac{\sqrt{231}G_{xz}\left(x^4-36x^2y^2+30x^2z^2+12y^4-36y^2z^2+z^4\right)}{308}-\frac{\sqrt{231}G_{yz}xy\left(31x^2-18y^2-39z^2\right)}{308}$$

$$-\frac{\sqrt{77}G_u yz\left(24x^2-25y^2+17z^2\right)}{308}+\frac{\sqrt{231}G_v yz\left(24x^2+3y^2-11z^2\right)}{308}+\frac{\sqrt{231}G_{xy}xz\left(18x^2+39y^2-31z^2\right)}{308}$$
$$+\frac{\sqrt{231}G_{xz}xy\left(18x^2-31y^2+39z^2\right)}{308}-\frac{\sqrt{231}G_{yz}\left(12x^4-36x^2y^2-36x^2z^2+y^4+30y^2z^2+z^4\right)}{308}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{3\sqrt{1001}G_u yz\left(2x^4+13x^2y^2-17x^2z^2+11y^4-41y^2z^2+14z^4\right)}{572}-\frac{\sqrt{3003}G_v yz\left(2x^4-47x^2y^2+43x^2z^2+17y^4-41y^2z^2+8z^4\right)}{572}$$
$$-\frac{3\sqrt{3003}G_{xy}xz\left(3x^2y^2-x^2z^2-8y^4+13y^2z^2-z^4\right)}{286}-\frac{3\sqrt{3003}G_{xz}xy\left(x^2y^2-3x^2z^2+y^4-13y^2z^2+8z^4\right)}{286}$$
$$+\frac{\sqrt{3003}G_{yz}\left(y-z\right)\left(y+z\right)\left(x^4-x^2y^2-x^2z^2-2y^4+29y^2z^2-2z^4\right)}{286}$$

$$\frac{3\sqrt{1001}G_u xz\left(11x^4+13x^2y^2-41x^2z^2+2y^4-17y^2z^2+14z^4\right)}{572}$$
$$-\frac{\sqrt{3003}G_v xz\left(17x^4-47x^2y^2-41x^2z^2+2y^4+43y^2z^2+8z^4\right)}{572}-\frac{3\sqrt{3003}G_{xy}yz\left(8x^4-3x^2y^2-13x^2z^2+y^2z^2+z^4\right)}{286}$$
$$+\frac{\sqrt{3003}G_{xz}\left(x-z\right)\left(x+z\right)\left(2x^4+x^2y^2-29x^2z^2-y^4+y^2z^2+2z^4\right)}{286}+\frac{3\sqrt{3003}G_{yz}xy\left(x^4+x^2y^2-13x^2z^2-3y^2z^2+8z^4\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{143}G_u xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}-\frac{21\sqrt{429}G_v xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{572}$$
$$-\frac{7\sqrt{429}G_{xy}yz\left(8x^4-23x^2y^2+7x^2z^2+2y^4+y^2z^2-z^4\right)}{286}$$
$$+\frac{\sqrt{429}G_{xz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$
$$+\frac{7\sqrt{429}G_{yz}xy\left(x^4-x^2y^2-7x^2z^2-2y^4+23y^2z^2-8z^4\right)}{286}$$

$$-\frac{21\sqrt{143}G_u yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{572}+\frac{21\sqrt{429}G_v yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{572}$$
$$-\frac{7\sqrt{429}G_{xy}xz\left(2x^4-23x^2y^2+x^2z^2+8y^4+7y^2z^2-z^4\right)}{286}-\frac{7\sqrt{429}G_{xz}xy\left(2x^4+x^2y^2-23x^2z^2-y^4+7y^2z^2+8z^4\right)}{286}$$
$$+\frac{\sqrt{429}G_{yz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$