PG No. 31 T_{d} $\bar{4}3m$ [cubic] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}^{(1,0)}_{1,1}[g](T_2)$, $\vec{\mathbb{Q}}^{(1,0)}_{1,2}[g](T_2)$, $\vec{\mathbb{Q}}^{(1,0)}_{1,3}[g](T_2)$

** symmetry

x

y

z

** expression

$$\frac{\sqrt{2}G_y z}{2} - \frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

$$\frac{\sqrt{2}G_x y}{2} - \frac{\sqrt{2}G_y x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}^{(1,0)}_{2,1}[g](E)$, $\vec{\mathbb{Q}}^{(1,0)}_{2,2}[g](E)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}G_x yz}{2} - \frac{\sqrt{6}G_y xz}{2}$$

$$\frac{\sqrt{2}G_x yz}{2} + \frac{\sqrt{2}G_y xz}{2} - \sqrt{2}G_z xy$$

$\vec{\mathbb{Q}}^{(1,0)}_{2,1}[g](T_2)$, $\vec{\mathbb{Q}}^{(1,0)}_{2,2}[g](T_2)$, $\vec{\mathbb{Q}}^{(1,0)}_{2,3}[g](T_2)$

** symmetry

$\sqrt{3}yz$

$\sqrt{3}xz$

$\sqrt{3}xy$

** expression

$$\frac{\sqrt{2}G_x\,(y-z)\,(y+z)}{2} - \frac{\sqrt{2}G_y xy}{2} + \frac{\sqrt{2}G_z xz}{2}$$

$$\frac{\sqrt{2}G_x xy}{2} - \frac{\sqrt{2}G_y\,(x-z)\,(x+z)}{2} - \frac{\sqrt{2}G_z yz}{2}$$

$$-\frac{\sqrt{2}G_x xz}{2} + \frac{\sqrt{2}G_y yz}{2} + \frac{\sqrt{2}G_z\,(x-y)\,(x+y)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}^{(1,0)}_{3}[g](A_1)$

** symmetry

$\sqrt{15}xyz$

** expression

$$\frac{\sqrt{5}G_x x\,(y-z)\,(y+z)}{2} - \frac{\sqrt{5}G_y y\,(x-z)\,(x+z)}{2} + \frac{\sqrt{5}G_z z\,(x-y)\,(x+y)}{2}$$

$\vec{\mathbb{Q}}^{(1,0)}_{3,1}[g](T_1)$, $\vec{\mathbb{Q}}^{(1,0)}_{3,2}[g](T_1)$, $\vec{\mathbb{Q}}^{(1,0)}_{3,3}[g](T_1)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\sqrt{5}G_{x}xyz+\frac{\sqrt{5}G_{y}z\left(2x^{2}+y^{2}-z^{2}\right)}{4}+\frac{\sqrt{5}G_{z}y\left(2x^{2}-y^{2}+z^{2}\right)}{4}$$

$$\frac{\sqrt{5}G_{x}z\left(x^{2}+2y^{2}-z^{2}\right)}{4}-\sqrt{5}G_{y}xyz-\frac{\sqrt{5}G_{z}x\left(x^{2}-2y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{5}G_{x}y\left(x^{2}-y^{2}+2z^{2}\right)}{4}-\frac{\sqrt{5}G_{y}x\left(x^{2}-y^{2}-2z^{2}\right)}{4}-\sqrt{5}G_{z}xyz$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](T_2)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](T_2)$, $\vec{\mathbb{Q}}_{3,3}^{(1,0)}[g](T_2)$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_{y}z\left(4x^{2}-y^{2}-z^{2}\right)}{4}-\frac{\sqrt{3}G_{z}y\left(4x^{2}-y^{2}-z^{2}\right)}{4}$$

$$\frac{\sqrt{3}G_{x}z\left(x^{2}-4y^{2}+z^{2}\right)}{4}-\frac{\sqrt{3}G_{z}x\left(x^{2}-4y^{2}+z^{2}\right)}{4}$$

$$-\frac{\sqrt{3}G_{x}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{3}G_{y}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_{4}^{(1,0)}[g](A_1)$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{105}G_{x}yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}G_{y}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}G_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{3}G_{x}yz\left(9x^{2}+2y^{2}-5z^{2}\right)}{6}+\frac{\sqrt{3}G_{y}xz\left(2x^{2}+9y^{2}-5z^{2}\right)}{6}+\frac{7\sqrt{3}G_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$$-\frac{G_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}+\frac{G_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}-\frac{G_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](T_1)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](T_1)$, $\vec{\mathbb{Q}}_{4,3}^{(1,0)}[g](T_1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_{x}\left(y^{2}-2yz-z^{2}\right)\left(y^{2}+2yz-z^{2}\right)}{4}-\frac{\sqrt{7}G_{y}xy\left(y^{2}-3z^{2}\right)}{4}+\frac{\sqrt{7}G_{z}xz\left(3y^{2}-z^{2}\right)}{4}$$

$$-\frac{\sqrt{7}G_{x}xy\left(x^{2}-3z^{2}\right)}{4}+\frac{\sqrt{7}G_{y}\left(x^{2}-2xz-z^{2}\right)\left(x^{2}+2xz-z^{2}\right)}{4}+\frac{\sqrt{7}G_{z}yz\left(3x^{2}-z^{2}\right)}{4}$$

$$-\frac{\sqrt{7}G_{x}xz\left(x^{2}-3y^{2}\right)}{4}+\frac{\sqrt{7}G_{y}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}G_{z}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](T_2)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](T_2)$, $\vec{\mathbb{Q}}_{4,3}^{(1,0)}[g](T_2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{G_{x}\left(y-z\right)\left(y+z\right)\left(6x^{2}-y^{2}-z^{2}\right)}{4}-\frac{G_{y}xy\left(6x^{2}-y^{2}-15z^{2}\right)}{4}+\frac{G_{z}xz\left(6x^{2}-15y^{2}-z^{2}\right)}{4}$$

$$-\frac{G_{x}xy\left(x^{2}-6y^{2}+15z^{2}\right)}{4}+\frac{G_{y}\left(x-z\right)\left(x+z\right)\left(x^{2}-6y^{2}+z^{2}\right)}{4}+\frac{G_{z}yz\left(15x^{2}-6y^{2}+z^{2}\right)}{4}$$

$$\frac{G_{x}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{4}-\frac{G_{y}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{4}-\frac{G_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$