PG No. 32 O_{h} $m\bar{3}m$ [cubic]

Table 1 Harmonics for rank 0.

No.	multipole	expression
1	$\mathbb{G}_0(A_{1u})$	1

Table 2 Harmonics for rank 1.

No.	multipole	expression
2	$\mathbb{G}_{1,1}(T_{1g})$	x
3	$\mathbb{G}_{1,2}(T_{1g})$	y
4	$\mathbb{G}_{1,3}(T_{1g})$	z

Table 3 Harmonics for rank 2.

No.	multipole	expression
5	$\mathbb{G}_{2,1}(E_u)$	$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$
6	$\mathbb{G}_{2,2}(E_u)$	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
7	$\mathbb{G}_{2,1}(T_{2u})$	$\sqrt{3}yz$
8	$\mathbb{G}_{2,2}(T_{2u})$	$\sqrt{3}xz$
9	$\mathbb{G}_{2,3}(T_{2u})$	$\sqrt{3}xy$

Table 4 Harmonics for rank 3.

No.	multipole	expression
10	$\mathbb{G}_3(A_{2g})$	$\sqrt{15}xyz$
11	$\mathbb{G}_{3,1}(T_{1g})$	$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$
12	$\mathbb{G}_{3,2}(T_{1g})$	$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$
13	$\mathbb{G}_{3,3}(T_{1g})$	$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$
14	$\mathbb{G}_{3,1}(T_{2g})$	$\frac{\sqrt{15}x(y-z)(y+z)}{2}$
15	$\mathbb{G}_{3,2}(T_{2g})$	$-\frac{\sqrt{15}y(x-z)(x+z)}{2}$
16	$\mathbb{G}_{3,3}(T_{2g})$	$\frac{\sqrt{15}z(x-y)(x+y)}{2}$

Table 5 Harmonics for rank 4.

No.	multipole	expression
17	$\mathbb{G}_4(A_{1u})$	$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$
18	$\mathbb{G}_{4,1}(E_u)$	$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$
19	$\mathbb{G}_{4,2}(E_u)$	$\frac{\sqrt{5}(x-y)(x+y)\left(x^2+y^2-6z^2\right)}{4}$
20	$\mathbb{G}_{4,1}(T_{1u})$	$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$
21	$\mathbb{G}_{4,2}(T_{1u})$	$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$
22	$\mathbb{G}_{4,3}(T_{1u})$	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$
23	$\mathbb{G}_{4,1}(T_{2u})$	$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$
24	$\mathbb{G}_{4,2}(T_{2u})$	$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$
25	$\mathbb{G}_{4,3}(T_{2u})$	$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$

Table 6 Harmonics for rank 5.

No.	multipole	expression
26	$\mathbb{G}_{5,1}(E_g)$	$\frac{3\sqrt{35}xyz(x-y)(x+y)}{2}$
27	$\mathbb{G}_{5,2}(E_g)$	$\frac{\sqrt{105}xyz\left(x^2+y^2-2z^2\right)}{2}$
28	$\mathbb{G}_{5,1}(T_{1g},1)$	$\frac{x\left(8x^4-40x^2y^2-40x^2z^2+15y^4+30y^2z^2+15z^4\right)}{8}$
29	$\mathbb{G}_{5,2}(T_{1g},1)$	$\frac{y\left(15x^4-40x^2y^2+30x^2z^2+8y^4-40y^2z^2+15z^4\right)}{8}$
30	$\mathbb{G}_{5,3}(T_{1g},1)$	$\frac{z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{8}$
31	$\mathbb{G}_{5,1}(T_{1g},2)$	$\frac{3\sqrt{35}x\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{8}$
32	$\mathbb{G}_{5,2}(T_{1g},2)$	$\frac{3\sqrt{35}y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{8}$
33	$\mathbb{G}_{5,3}(T_{1g},2)$	$\frac{3\sqrt{35}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$
34	$\mathbb{G}_{5,1}(T_{2g})$	$\frac{\sqrt{105}x(y-z)(y+z)\left(2x^2-y^2-z^2\right)}{4}$
35	$\mathbb{G}_{5,2}(T_{2g})$	$\frac{\sqrt{105}y(x-z)(x+z)\left(x^2-2y^2+z^2\right)}{4}$
36	$\mathbb{G}_{5,3}(T_{2g})$	$-\frac{\sqrt{105}z(x-y)(x+y)\left(x^2+y^2-2z^2\right)}{4}$

Table 7 Harmonics for rank 6.

No.	multipole	expression
37	$\mathbb{G}_6(A_{1u})$	$\frac{\sqrt{2}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{8}$
38	$\mathbb{G}_6(A_{2u})$	$-\frac{\sqrt{2310}(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{8}$
39	$\mathbb{G}_{6,1}(E_u)$	$-\frac{\sqrt{14}\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{8}$
40	$\mathbb{G}_{6,2}(E_u)$	$\frac{\sqrt{42}(x-y)(x+y)\left(x^4-9x^2y^2-5x^2z^2+y^4-5y^2z^2+5z^4\right)}{8}$
41	$\mathbb{G}_{6,1}(T_{1u})$	$\frac{3\sqrt{7}yz(y-z)(y+z)\left(10x^2-y^2-z^2\right)}{4}$
42	$\mathbb{G}_{6,2}(T_{1u})$	$\frac{3\sqrt{7}xz(x-z)(x+z)\left(x^2-10y^2+z^2\right)}{4}$
43	$\mathbb{G}_{6,3}(T_{1u})$	$-\frac{3\sqrt{7}xy(x-y)(x+y)\left(x^2+y^2-10z^2\right)}{4}$
44	$\mathbb{G}_{6,1}(T_{2u},1)$	$\frac{\sqrt{462}yz\left(y^2-3z^2\right)\left(3y^2-z^2\right)}{16}$
45	$\mathbb{G}_{6,2}(T_{2u},1)$	$\frac{\sqrt{462}xz\left(x^2-3z^2\right)\left(3x^2-z^2\right)}{16}$
46	$\mathbb{G}_{6,3}(T_{2u},1)$	$\frac{\sqrt{462}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{16}$
47	$\mathbb{G}_{6,1}(T_{2u},2)$	$\frac{\sqrt{210}yz\left(16x^4-16x^2y^2-16x^2z^2+y^4+2y^2z^2+z^4\right)}{16}$
48	$\mathbb{G}_{6,2}(T_{2u},2)$	$\frac{\sqrt{210}xz\left(x^4-16x^2y^2+2x^2z^2+16y^4-16y^2z^2+z^4\right)}{16}$
49	$\mathbb{G}_{6,3}(T_{2u},2)$	$\frac{\sqrt{210}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{16}$

Table 8 Harmonics for rank 7.

No.	multipole	expression
50	$\mathbb{G}_7(A_{2g})$	$\frac{\sqrt{91}xyz\left(3x^4-5x^2y^2-5x^2z^2+3y^4-5y^2z^2+3z^4\right)}{2}$
51	$\mathbb{G}_{7,1}(E_g)$	$-\frac{\sqrt{231}xyz(x-y)(x+y)\left(3x^2+3y^2-10z^2\right)}{4}$
52	$\mathbb{G}_{7,2}(E_g)$	$-\frac{\sqrt{77}xyz\left(3x^4-20x^2y^2+10x^2z^2+3y^4+10y^2z^2-6z^4\right)}{4}$
53	$\mathbb{G}_{7,1}(T_{1g},1)$	$\frac{x\left(16x^6-168x^4y^2-168x^4z^2+210x^2y^4+420x^2y^2z^2+210x^2z^4-35y^6-105y^4z^2-105y^2z^4-35z^6\right)}{16}$
54	$\mathbb{G}_{7,2}(T_{1g},1)$	$-\frac{y\left(35x^6-210x^4y^2+105x^4z^2+168x^2y^4-420x^2y^2z^2+105x^2z^4-16y^6+168y^4z^2-210y^2z^4+35z^6\right)}{16}$
55	$\mathbb{G}_{7,3}(T_{1g},1)$	$-\frac{z\left(35x^6+105x^4y^2-210x^4z^2+105x^2y^4-420x^2y^2z^2+168x^2z^4+35y^6-210y^4z^2+168y^2z^4-16z^6\right)}{16}$
56	$\mathbb{G}_{7,1}(T_{1g},2)$	$\frac{\sqrt{231}x\left(10x^2-3y^2-3z^2\right)\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{16}$
57	$\mathbb{G}_{7,2}(T_{1g},2)$	$-\frac{\sqrt{231}y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)\left(3x^2-10y^2+3z^2\right)}{16}$
58	$\mathbb{G}_{7,3}(T_{1g},2)$	$-\frac{\sqrt{231}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(3x^2+3y^2-10z^2\right)}{16}$
59	$\mathbb{G}_{7,1}(T_{2g},1)$	$\frac{\sqrt{6006}x(y-z)(y+z)\left(y^2-4yz+z^2\right)\left(y^2+4yz+z^2\right)}{32}$
60	$\mathbb{G}_{7,2}(T_{2g},1)$	$-\frac{\sqrt{6006}y(x-z)(x+z)\left(x^2-4xz+z^2\right)\left(x^2+4xz+z^2\right)}{32}$
61	$\mathbb{G}_{7,3}(T_{2g},1)$	$\frac{\sqrt{6006}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)}{32}$
62	$\mathbb{G}_{7,1}(T_{2g},2)$	$\frac{\sqrt{42}x(y-z)(y+z)\left(48x^4-80x^2y^2-80x^2z^2+15y^4+30y^2z^2+15z^4\right)}{32}$
63	$\mathbb{G}_{7,2}(T_{2g},2)$	$-\frac{\sqrt{42}y(x-z)(x+z)\left(15x^4-80x^2y^2+30x^2z^2+48y^4-80y^2z^2+15z^4\right)}{32}$
64	$\mathbb{G}_{7,3}(T_{2g},2)$	$\frac{\sqrt{42}z(x-y)(x+y)\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{32}$

Table 9 Harmonics for rank 8.

No.	multipole	expression
65	$\mathbb{G}_8(A_{1u})$	$\frac{\sqrt{33}\left(x^8-14x^6y^2-14x^6z^2+35x^4y^4+35x^4z^4-14x^2y^6-14x^2z^6+y^8-14y^6z^2+35y^4z^4-14y^2z^6+z^8\right)}{8}$
66	$\mathbb{G}_{8,1}(E_u,1)$	$\frac{\sqrt{286}\left(x^8-14x^6y^2-14x^6z^2+210x^4y^2z^2-14x^2y^6+210x^2y^4z^2-420x^2y^2z^4+28x^2z^6+y^8-14y^6z^2+28y^2z^6-2z^8\right)}{64}$
67	$\mathbb{G}_{8,2}(E_u,1)$	$-\frac{\sqrt{858}(x-y)(x+y)\left(x^2+y^2-14z^2\right)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)}{64}$
68	$\mathbb{G}_{8,1}(E_u,2)$	$\frac{\sqrt{210}\left(x^8-62x^6y^2+34x^6z^2+160x^4y^4-30x^4y^2z^2-80x^4z^4-62x^2y^6-30x^2y^4z^2+60x^2y^2z^4+28x^2z^6+y^8+34y^6z^2-80y^4z^4+28y^2z^6-2z^8\right)}{64}$
69	$\mathbb{G}_{8,2}(E_u,2)$	$-\frac{3\sqrt{70}(x-y)(x+y)\left(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6\right)}{64}$
70	$\mathbb{G}_{8,1}(T_{1u},1)$	$\frac{3\sqrt{715}yz(y-z)(y+z)\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{16}$
71	$\mathbb{G}_{8,2}(T_{1u},1)$	$-\frac{3\sqrt{715}xz(x-z)(x+z)\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{16}$
72	$\mathbb{G}_{8,3}(T_{1u},1)$	$\frac{3\sqrt{715}xy(x-y)(x+y)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{16}$
73	$\mathbb{G}_{8,1}(T_{1u},2)$	$\frac{3\sqrt{77}yz(y-z)(y+z)\left(40x^4-24x^2y^2-24x^2z^2+y^4+2y^2z^2+z^4\right)}{16}$
74	$\mathbb{G}_{8,2}(T_{1u},2)$	$-\frac{3\sqrt{77}xz(x-z)(x+z)\left(x^4-24x^2y^2+2x^2z^2+40y^4-24y^2z^2+z^4\right)}{16}$
75	$\mathbb{G}_{8,3}(T_{1u},2)$	$\frac{3\sqrt{77}xy(x-y)(x+y)\left(x^4+2x^2y^2-24x^2z^2+y^4-24y^2z^2+40z^4\right)}{16}$
76	$\mathbb{G}_{8,1}(T_{2u},1)$	$\frac{\sqrt{858}yz\left(y^2-3z^2\right)\left(3y^2-z^2\right)\left(14x^2-y^2-z^2\right)}{32}$
77	$\mathbb{G}_{8,2}(T_{2u},1)$	$-\frac{\sqrt{858}xz\left(x^2-3z^2\right)\left(3x^2-z^2\right)\left(x^2-14y^2+z^2\right)}{32}$
78	$\mathbb{G}_{8,3}(T_{2u},1)$	$-\frac{\sqrt{858}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(x^2+y^2-14z^2\right)}{32}$
79	$\mathbb{G}_{8,1}(T_{2u},2)$	$\frac{3\sqrt{70}yz\left(32x^6-80x^4y^2-80x^4z^2+30x^2y^4+60x^2y^2z^2+30x^2z^4-y^6-3y^4z^2-3y^2z^4-z^6\right)}{32}$
80	$\mathbb{G}_{8,2}(T_{2u},2)$	$-\frac{3\sqrt{70}xz\left(x^6-30x^4y^2+3x^4z^2+80x^2y^4-60x^2y^2z^2+3x^2z^4-32y^6+80y^4z^2-30y^2z^4+z^6\right)}{32}$
81	$\mathbb{G}_{8,3}(T_{2u},2)$	$-\frac{3\sqrt{70}xy\left(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6\right)}{32}$

Table 10 Harmonics for rank 9.

No.	multipole	expression
82	$\mathbb{G}_9(A_{1g})$	$-\frac{\sqrt{510510}xyz(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)}{8}$
83	$\mathbb{G}_9(A_{2g})$	$\frac{\sqrt{330}xyz\left(6x^6-21x^4y^2-21x^4z^2-21x^2y^4+140x^2y^2z^2-21x^2z^4+6y^6-21y^4z^2-21y^2z^4+6z^6\right)}{8}$
84	$\mathbb{G}_{9,1}(E_g)$	$\frac{\sqrt{4290}xyz(x-y)(x+y)\left(3x^4-11x^2y^2-7x^2z^2+3y^4-7y^2z^2+7z^4\right)}{8}$
85	$\mathbb{G}_{9,2}(E_g)$	$\frac{3\sqrt{1430}xyz\left(x^6-7x^4z^2+7x^2z^4+y^6-7y^4z^2+7y^2z^4-2z^6\right)}{8}$
86	$\mathbb{G}_{9,1}(T_{1g},1)$	$\frac{x\left(128x^8-2304x^6y^2-2304x^6z^2+6048x^4y^4+12096x^4y^2z^2+6048x^4z^4-3360x^2y^6-10080x^2y^4z^2-10080x^2y^2z^4-3360x^2z^6+315y^8+1260y^6z^2+1890y^4z^4+1260y^2z^6+315z^8\right)}{128}$
87	$\mathbb{G}_{9,2}(T_{1g},1)$	$\frac{y\left(315x^8-3360x^6y^2+1260x^6z^2+6048x^4y^4-10080x^4y^2z^2+1890x^4z^4-2304x^2y^6+12096x^2y^4z^2-10080x^2y^2z^4+1260x^2z^6+128y^8-2304y^6z^2+6048y^4z^4-3360y^2z^6+315z^8\right)}{128}$
88	$\mathbb{G}_{9,3}(T_{1g},1)$	$\frac{z\left(315x^8+1260x^6y^2-3360x^6z^2+1890x^4y^4-10080x^4y^2z^2+6048x^4z^4+1260x^2y^6-10080x^2y^4z^2+12096x^2y^2z^4-2304x^2z^6+315y^8-3360y^6z^2+6048y^4z^4-2304y^2z^6+128z^8\right)}{128}$
89	$\mathbb{G}_{9,1}(T_{1g},2)$	$\frac{3\sqrt{12155}x\left(y^4-4y^3z-6y^2z^2+4yz^3+z^4\right)\left(y^4+4y^3z-6y^2z^2-4yz^3+z^4\right)}{128}$
90	$\mathbb{G}_{9,2}(T_{1g},2)$	$\frac{3\sqrt{12155}y\left(x^4-4x^3z-6x^2z^2+4xz^3+z^4\right)\left(x^4+4x^3z-6x^2z^2-4xz^3+z^4\right)}{128}$
91	$\mathbb{G}_{9,3}(T_{1g},2)$	$\frac{3\sqrt{12155}z\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{128}$
92	$\mathbb{G}_{9,1}(T_{1g},3)$	$\frac{3\sqrt{5005}x\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)\left(8x^4-8x^2y^2-8x^2z^2+y^4+2y^2z^2+z^4\right)}{64}$
93	$\mathbb{G}_{9,2}(T_{1g},3)$	$\frac{3\sqrt{5005}y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)\left(x^4-8x^2y^2+2x^2z^2+8y^4-8y^2z^2+z^4\right)}{64}$
94	$\mathbb{G}_{9,3}(T_{1g},3)$	$\frac{3\sqrt{5005}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(x^4+2x^2y^2-8x^2z^2+y^4-8y^2z^2+8z^4\right)}{64}$
95	$\mathbb{G}_{9,1}(T_{2g},1)$	$\frac{\sqrt{4290}x(y-z)(y+z)\left(14x^2-3y^2-3z^2\right)\left(y^2-4yz+z^2\right)\left(y^2+4yz+z^2\right)}{64}$
96	$\mathbb{G}_{9,2}(T_{2g},1)$	$\frac{\sqrt{4290}y(x-z)(x+z)\left(x^2-4xz+z^2\right)\left(x^2+4xz+z^2\right)\left(3x^2-14y^2+3z^2\right)}{64}$
97	$\mathbb{G}_{9,3}(T_{2g},1)$	$-\frac{\sqrt{4290}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)\left(3x^2+3y^2-14z^2\right)}{64}$
98	$\mathbb{G}_{9,1}(T_{2g},2)$	$\frac{3\sqrt{110}x(y-z)(y+z)\left(32x^6-112x^4y^2-112x^4z^2+70x^2y^4+140x^2y^2z^2+70x^2z^4-7y^6-21y^4z^2-21y^2z^4-7z^6\right)}{64}$
99	$\mathbb{G}_{9,2}(T_{2g},2)$	$\frac{3\sqrt{110}y(x-z)(x+z)\left(7x^6-70x^4y^2+21x^4z^2+112x^2y^4-140x^2y^2z^2+21x^2z^4-32y^6+112y^4z^2-70y^2z^4+7z^6\right)}{64}$
100	$\mathbb{G}_{9,3}(T_{2g},2)$	$-\frac{3\sqrt{110}z(x-y)(x+y)\left(7x^6+21x^4y^2-70x^4z^2+21x^2y^4-140x^2y^2z^2+112x^2z^4+7y^6-70y^4z^2+112y^2z^4-32z^6\right)}{64}$

Table 11 Harmonics for rank 10.

No.	multipole	expression
101	$\mathbb{G}_{10}(A_{1u})$	$\frac{\sqrt{390}\left(2x^{10}-45x^8y^2-45x^8z^2+42x^6y^4+1008x^6y^2z^2+42x^6z^4+42x^4y^6-1260x^4y^4z^2-1260x^4y^2z^4+42x^4z^6-45x^2y^8+1008x^2y^6z^2-1260x^2y^4z^4+1008x^2y^2z^6-45x^2z^8+2y^{10}-45y^8z^2+42y^6z^4+42y^4z^6-45y^2z^8+2z^{10}\right)}{96}$
102	$\mathbb{G}_{10}(A_{2u})$	$-\frac{\sqrt{27170}(x-y)(x+y)(x-z)(x+z)(y-z)(y+z)\left(3x^4-11x^2y^2-11x^2z^2+3y^4-11y^2z^2+3z^4\right)}{32}$
103	$\mathbb{G}_{10,1}(E_u,1)$	$-\frac{\sqrt{420189}\left(22x^{10}+45x^8y^2-1035x^8z^2-105x^6y^4-630x^6y^2z^2+4935x^6z^4-105x^4y^6+3150x^4y^4z^2-1575x^4y^2z^4-4830x^4z^6+45x^2y^8-630x^2y^6z^2-1575x^2y^4z^4+1260x^2y^2z^6+990x^2z^8+22y^{10}-1035y^8z^2+4935y^6z^4-4830y^4z^6+990y^2z^8-44z^{10}\right)}{35952}$
104	$\mathbb{G}_{10,2}(E_u,1)$	$\frac{\sqrt{140063}(x-y)(x+y)\left(22x^8-653x^6y^2-315x^6z^2+2602x^4y^4-945x^4y^2z^2+1575x^4z^4-653x^2y^6-945x^2y^4z^2+3150x^2y^2z^4-1680x^2z^6+22y^8-315y^6z^2+1575y^4z^4-1680y^2z^6+360z^8\right)}{11984}$
105	$\mathbb{G}_{10,1}(E_u,2)$	$-\frac{\sqrt{3213210}\left(3x^{10}-96x^8y^2-39x^8z^2+224x^6y^4+1344x^6y^2z^2-42x^6z^4+224x^4y^6-6720x^4y^4z^2+3360x^4y^2z^4-182x^4z^6-96x^2y^8+1344x^2y^6z^2+3360x^2y^4z^4-2688x^2y^2z^6+135x^2z^8+3y^{10}-39y^8z^2-42y^6z^4-182y^4z^6+135y^2z^8-6z^{10}\right)}{23968}$
106	$\mathbb{G}_{10,2}(E_u,2)$	$\frac{\sqrt{1071070}(x-y)(x+y)\left(9x^8-165x^6y^2-231x^6z^2-25x^4y^4+3801x^4y^2z^2+406x^4z^4-165x^2y^6+3801x^2y^4z^2-9674x^2y^2z^4+266x^2z^6+9y^8-231y^6z^2+406y^4z^4+266y^2z^6-57z^8\right)}{23968}$
107	$\mathbb{G}_{10,1}(T_{1u},1)$	$\frac{\sqrt{12155}yz(y-z)(y+z)\left(18x^2-y^2-z^2\right)\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{32}$
108	$\mathbb{G}_{10,2}(T_{1u},1)$	$\frac{\sqrt{12155}xz(x-z)(x+z)\left(x^2-18y^2+z^2\right)\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{32}$
109	$\mathbb{G}_{10,3}(T_{1u},1)$	$-\frac{\sqrt{12155}xy(x-y)(x+y)\left(x^2+y^2-18z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{32}$
110	$\mathbb{G}_{10,1}(T_{1u},2)$	$\frac{\sqrt{2145}yz(y-z)(y+z)\left(112x^6-168x^4y^2-168x^4z^2+42x^2y^4+84x^2y^2z^2+42x^2z^4-y^6-3y^4z^2-3y^2z^4-z^6\right)}{32}$
111	$\mathbb{G}_{10,2}(T_{1u},2)$	$\frac{\sqrt{2145}xz(x-z)(x+z)\left(x^6-42x^4y^2+3x^4z^2+168x^2y^4-84x^2y^2z^2+3x^2z^4-112y^6+168y^4z^2-42y^2z^4+z^6\right)}{32}$
112	$\mathbb{G}_{10,3}(T_{1u},2)$	$-\frac{\sqrt{2145}xy(x-y)(x+y)\left(x^6+3x^4y^2-42x^4z^2+3x^2y^4-84x^2y^2z^2+168x^2z^4+y^6-42y^4z^2+168y^2z^4-112z^6\right)}{32}$
113	$\mathbb{G}_{10,1}(T_{2u},1)$	$\frac{\sqrt{92378}yz\left(y^4-10y^2z^2+5z^4\right)\left(5y^4-10y^2z^2+z^4\right)}{256}$
114	$\mathbb{G}_{10,2}(T_{2u},1)$	$\frac{\sqrt{92378}xz\left(x^4-10x^2z^2+5z^4\right)\left(5x^4-10x^2z^2+z^4\right)}{256}$
115	$\mathbb{G}_{10,3}(T_{2u},1)$	$\frac{\sqrt{92378}xy\left(x^4-10x^2y^2+5y^4\right)\left(5x^4-10x^2y^2+y^4\right)}{256}$
116	$\mathbb{G}_{10,1}(T_{2u},2)$	$\frac{\sqrt{4290}yz\left(y^2-3z^2\right)\left(3y^2-z^2\right)\left(224x^4-96x^2y^2-96x^2z^2+3y^4+6y^2z^2+3z^4\right)}{256}$
117	$\mathbb{G}_{10,2}(T_{2u},2)$	$\frac{\sqrt{4290}xz\left(x^2-3z^2\right)\left(3x^2-z^2\right)\left(3x^4-96x^2y^2+6x^2z^2+224y^4-96y^2z^2+3z^4\right)}{256}$
118	$\mathbb{G}_{10,3}(T_{2u},2)$	$\frac{\sqrt{4290}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-96x^2z^2+3y^4-96y^2z^2+224z^4\right)}{256}$
119	$\mathbb{G}_{10,1}(T_{2u},3)$	$\frac{\sqrt{165}yz\left(384x^8-1792x^6y^2-1792x^6z^2+1680x^4y^4+3360x^4y^2z^2+1680x^4z^4-336x^2y^6-1008x^2y^4z^2-1008x^2y^2z^4-336x^2z^6+7y^8+28y^6z^2+42y^4z^4+28y^2z^6+7z^8\right)}{128}$
120	$\mathbb{G}_{10,2}(T_{2u},3)$	$\frac{\sqrt{165}xz\left(7x^8-336x^6y^2+28x^6z^2+1680x^4y^4-1008x^4y^2z^2+42x^4z^4-1792x^2y^6+3360x^2y^4z^2-1008x^2y^2z^4+28x^2z^6+384y^8-1792y^6z^2+1680y^4z^4-336y^2z^6+7z^8\right)}{128}$
121	$\mathbb{G}_{10,3}(T_{2u},3)$	$\frac{\sqrt{165}xy\left(7x^8+28x^6y^2-336x^6z^2+42x^4y^4-1008x^4y^2z^2+1680x^4z^4+28x^2y^6-1008x^2y^4z^2+3360x^2y^2z^4-1792x^2z^6+7y^8-336y^6z^2+1680y^4z^4-1792y^2z^6+384z^8\right)}{128}$

Table 12 Harmonics for rank 11.

No.	multipole	expression
122	$\mathbb{G}_{11}(A_{2g})$	$\frac{\sqrt{2431}xyz\left(5x^8-30x^6y^2-30x^6z^2+63x^4y^4+63x^4z^4-30x^2y^6-30x^2z^6+5y^8-30y^6z^2+63y^4z^4-30y^2z^6+5z^8\right)}{8}$
123	$\mathbb{G}_{11,1}(E_g,1)$	$-\frac{\sqrt{692835}xyz(x-y)(x+y)\left(x^2+y^2-6z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{32}$
124	$\mathbb{G}_{11,2}(E_g,1)$	$-\frac{\sqrt{230945}xyz\left(x^8-6x^6y^2-6x^6z^2+42x^4y^2z^2-6x^2y^6+42x^2y^4z^2-84x^2y^2z^4+12x^2z^6+y^8-6y^6z^2+12y^2z^6-2z^8\right)}{32}$
125	$\mathbb{G}_{11,1}(E_g,2)$	$-\frac{3\sqrt{1001}xyz(x-y)(x+y)\left(5x^6+15x^4y^2-70x^4z^2+15x^2y^4-140x^2y^2z^2+168x^2z^4+5y^6-70y^4z^2+168y^2z^4-80z^6\right)}{32}$
126	$\mathbb{G}_{11,2}(E_g,2)$	$-\frac{\sqrt{3003}xyz\left(5x^8-150x^6y^2+90x^6z^2+336x^4y^4-70x^4y^2z^2-168x^4z^4-150x^2y^6-70x^2y^4z^2+140x^2y^2z^4+60x^2z^6+5y^8+90y^6z^2-168y^4z^4+60y^2z^6-10z^8\right)}{32}$
127	$\mathbb{G}_{11,1}(T_{1g},1)$	$\frac{x\left(256x^{10}-7040x^8y^2-7040x^8z^2+31680x^6y^4+63360x^6y^2z^2+31680x^6z^4-36960x^4y^6-110880x^4y^4z^2-110880x^4y^2z^4-36960x^4z^6+11550x^2y^8+46200x^2y^6z^2+69300x^2y^4z^4+46200x^2y^2z^6+11550x^2z^8-693y^{10}-3465y^8z^2-6930y^6z^4-6930y^4z^6-3465y^2z^8-693z^{10}\right)}{256}$
128	$\mathbb{G}_{11,2}(T_{1g},1)$	$-\frac{y\left(693x^{10}-11550x^8y^2+3465x^8z^2+36960x^6y^4-46200x^6y^2z^2+6930x^6z^4-31680x^4y^6+110880x^4y^4z^2-69300x^4y^2z^4+6930x^4z^6+7040x^2y^8-63360x^2y^6z^2+110880x^2y^4z^4-46200x^2y^2z^6+3465x^2z^8-256y^{10}+7040y^8z^2-31680y^6z^4+36960y^4z^6-11550y^2z^8+693z^{10}\right)}{256}$
129	$\mathbb{G}_{11,3}(T_{1g},1)$	$-\frac{z\left(693x^{10}+3465x^8y^2-11550x^8z^2+6930x^6y^4-46200x^6y^2z^2+36960x^6z^4+6930x^4y^6-69300x^4y^4z^2+110880x^4y^2z^4-31680x^4z^6+3465x^2y^8-46200x^2y^6z^2+110880x^2y^4z^4-63360x^2y^2z^6+7040x^2z^8+693y^{10}-11550y^8z^2+36960y^6z^4-31680y^4z^6+7040y^2z^8-256z^{10}\right)}{256}$
130	$\mathbb{G}_{11,1}(T_{1g},2)$	$\frac{\sqrt{692835}x\left(6x^2-y^2-z^2\right)\left(y^4-4y^3z-6y^2z^2+4yz^3+z^4\right)\left(y^4+4y^3z-6y^2z^2-4yz^3+z^4\right)}{256}$
131	$\mathbb{G}_{11,2}(T_{1g},2)$	$-\frac{\sqrt{692835}y\left(x^2-6y^2+z^2\right)\left(x^4-4x^3z-6x^2z^2+4xz^3+z^4\right)\left(x^4+4x^3z-6x^2z^2-4xz^3+z^4\right)}{256}$
132	$\mathbb{G}_{11,3}(T_{1g},2)$	$-\frac{\sqrt{692835}z\left(x^2+y^2-6z^2\right)\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{256}$
133	$\mathbb{G}_{11,1}(T_{1g},3)$	$\frac{3\sqrt{1001}x\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)\left(80x^6-168x^4y^2-168x^4z^2+70x^2y^4+140x^2y^2z^2+70x^2z^4-5y^6-15y^4z^2-15y^2z^4-5z^6\right)}{128}$
134	$\mathbb{G}_{11,2}(T_{1g},3)$	$-\frac{3\sqrt{1001}y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)\left(5x^6-70x^4y^2+15x^4z^2+168x^2y^4-140x^2y^2z^2+15x^2z^4-80y^6+168y^4z^2-70y^2z^4+5z^6\right)}{128}$
135	$\mathbb{G}_{11,3}(T_{1g},3)$	$-\frac{3\sqrt{1001}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(5x^6+15x^4y^2-70x^4z^2+15x^2y^4-140x^2y^2z^2+168x^2z^4+5y^6-70y^4z^2+168y^2z^4-80z^6\right)}{128}$
136	$\mathbb{G}_{11,1}(T_{2g},1)$	$\frac{\sqrt{1939938}x(y-z)(y+z)\left(y^4-4y^3z-14y^2z^2-4yz^3+z^4\right)\left(y^4+4y^3z-14y^2z^2+4yz^3+z^4\right)}{512}$
137	$\mathbb{G}_{11,2}(T_{2g},1)$	$-\frac{\sqrt{1939938}y(x-z)(x+z)\left(x^4-4x^3z-14x^2z^2-4xz^3+z^4\right)\left(x^4+4x^3z-14x^2z^2+4xz^3+z^4\right)}{512}$
138	$\mathbb{G}_{11,3}(T_{2g},1)$	$\frac{\sqrt{1939938}z(x-y)(x+y)\left(x^4-4x^3y-14x^2y^2-4xy^3+y^4\right)\left(x^4+4x^3y-14x^2y^2+4xy^3+y^4\right)}{512}$
139	$\mathbb{G}_{11,1}(T_{2g},2)$	$\frac{\sqrt{14586}x(y-z)(y+z)\left(y^2-4yz+z^2\right)\left(y^2+4yz+z^2\right)\left(224x^4-160x^2y^2-160x^2z^2+15y^4+30y^2z^2+15z^4\right)}{512}$
140	$\mathbb{G}_{11,2}(T_{2g},2)$	$-\frac{\sqrt{14586}y(x-z)(x+z)\left(x^2-4xz+z^2\right)\left(x^2+4xz+z^2\right)\left(15x^4-160x^2y^2+30x^2z^2+224y^4-160y^2z^2+15z^4\right)}{512}$
141	$\mathbb{G}_{11,3}(T_{2g},2)$	$\frac{\sqrt{14586}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)\left(15x^4+30x^2y^2-160x^2z^2+15y^4-160y^2z^2+224z^4\right)}{512}$
142	$\mathbb{G}_{11,1}(T_{2g},3)$	$\frac{\sqrt{2145}x(y-z)(y+z)\left(128x^8-768x^6y^2-768x^6z^2+1008x^4y^4+2016x^4y^2z^2+1008x^4z^4-336x^2y^6-1008x^2y^4z^2-1008x^2y^2z^4-336x^2z^6+21y^8+84y^6z^2+126y^4z^4+84y^2z^6+21z^8\right)}{256}$
143	$\mathbb{G}_{11,2}(T_{2g},3)$	$-\frac{\sqrt{2145}y(x-z)(x+z)\left(21x^8-336x^6y^2+84x^6z^2+1008x^4y^4-1008x^4y^2z^2+126x^4z^4-768x^2y^6+2016x^2y^4z^2-1008x^2y^2z^4+84x^2z^6+128y^8-768y^6z^2+1008y^4z^4-336y^2z^6+21z^8\right)}{256}$
144	$\mathbb{G}_{11,3}(T_{2g},3)$	$\frac{\sqrt{2145}z(x-y)(x+y)\left(21x^8+84x^6y^2-336x^6z^2+126x^4y^4-1008x^4y^2z^2+1008x^4z^4+84x^2y^6-1008x^2y^4z^2+2016x^2y^2z^4-768x^2z^6+21y^8-336y^6z^2+1008y^4z^4-768y^2z^6+128z^8\right)}{256}$