PG No. 13 C_{4v} $4mm$ [tetragonal] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}_{1}^{(1,0)}[g](A_1)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}G_{x}y}{2} - \frac{\sqrt{2}G_{y}x}{2}$$

$\vec{\mathbb{Q}}_{1,1}^{(1,0)}[g](E)$, $\vec{\mathbb{Q}}_{1,2}^{(1,0)}[g](E)$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}G_{y}z}{2} - \frac{\sqrt{2}G_{z}y}{2}$$

$$-\frac{\sqrt{2}G_{x}z}{2} + \frac{\sqrt{2}G_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_{2}^{(1,0)}[g](A_1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}G_{x}yz}{2} - \frac{\sqrt{6}G_{y}xz}{2}$$

$\vec{\mathbb{Q}}_{2}^{(1,0)}[g](B_1)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_{x}yz}{2} + \frac{\sqrt{2}G_{y}xz}{2} - \sqrt{2}G_{z}xy$$

$\vec{\mathbb{Q}}_{2}^{(1,0)}[g](B_2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{2}G_{x}xz}{2} + \frac{\sqrt{2}G_{y}yz}{2} + \frac{\sqrt{2}G_{z}\,(x-y)\,(x+y)}{2}$$

$\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E)$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E)$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}G_{x}xy}{2} - \frac{\sqrt{2}G_{y}\,(x-z)\,(x+z)}{2} - \frac{\sqrt{2}G_{z}yz}{2}$$

$$\frac{\sqrt{2}G_{x}\,(y-z)\,(y+z)}{2} - \frac{\sqrt{2}G_{y}xy}{2} + \frac{\sqrt{2}G_{z}xz}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}_{3}^{(1,0)}[g](A_1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}G_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}G_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_1)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}G_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}G_z xyz$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](B_2)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}G_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}G_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}G_z z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E,1)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_y z\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{3}G_z y\left(4x^2-y^2-z^2\right)}{4}$$

$$\frac{\sqrt{3}G_x z\left(x^2-4y^2+z^2\right)}{4}-\frac{\sqrt{3}G_z x\left(x^2-4y^2+z^2\right)}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E,2)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E,2)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\sqrt{5}G_x xyz+\frac{\sqrt{5}G_y z\left(2x^2+y^2-z^2\right)}{4}+\frac{\sqrt{5}G_z y\left(2x^2-y^2+z^2\right)}{4}$$

$$-\frac{\sqrt{5}G_x z\left(x^2+2y^2-z^2\right)}{4}+\sqrt{5}G_y xyz+\frac{\sqrt{5}G_z x\left(x^2-2y^2-z^2\right)}{4}$$

* Harmonics for rank 4
$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_1,1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}G_x yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}G_y xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}G_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_1,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{3}G_xyz\left(9x^2+2y^2-5z^2\right)}{6}+\frac{\sqrt{3}G_yxz\left(2x^2+9y^2-5z^2\right)}{6}+\frac{7\sqrt{3}G_zxy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_2)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{7}G_xxz\left(x^2-3y^2\right)}{4}+\frac{\sqrt{7}G_yyz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}G_z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_1)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{G_xyz\left(3x^2-4y^2+3z^2\right)}{2}+\frac{G_yxz\left(4x^2-3y^2-3z^2\right)}{2}-\frac{G_zxy\left(x^2+y^2-6z^2\right)}{2}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](B_2)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{G_xxz\left(x^2+15y^2-6z^2\right)}{4}-\frac{G_yyz\left(15x^2+y^2-6z^2\right)}{4}-\frac{G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E,1)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E,1)$
** symmetry

$$\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_xxy\left(x^2-3z^2\right)}{4}-\frac{\sqrt{7}G_y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{4}-\frac{\sqrt{7}G_zyz\left(3x^2-z^2\right)}{4}$$

$$\frac{\sqrt{7}G_x\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{4}-\frac{\sqrt{7}G_yxy\left(y^2-3z^2\right)}{4}+\frac{\sqrt{7}G_zxz\left(3y^2-z^2\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E,2)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{G_xxy\left(x^2-6y^2+15z^2\right)}{4}+\frac{G_y\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{4}+\frac{G_zyz\left(15x^2-6y^2+z^2\right)}{4}$$

$$\frac{G_x\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{4}-\frac{G_yxy\left(6x^2-y^2-15z^2\right)}{4}+\frac{G_zxz\left(6x^2-15y^2-z^2\right)}{4}$$