PG No. 15 D_{4h} $4/mmm$ [tetragonal] (polar, internal axial quadrupole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(2,1)}[g](A_{2u})$

** symmetry

z

** expression

$$\frac{\sqrt{30}G_v xy}{5} - \frac{\sqrt{30}G_{xy}\left(x-y\right)\left(x+y\right)}{10} + \frac{\sqrt{30}G_{xz}yz}{10} - \frac{\sqrt{30}G_{yz}xz}{10}$$

$\vec{\mathbb{Q}}_{1,1}^{(2,1)}[g](E_u)$, $\vec{\mathbb{Q}}_{1,2}^{(2,1)}[g](E_u)$

** symmetry

x

y

** expression

$$-\frac{3\sqrt{10}G_u yz}{10} - \frac{\sqrt{30}G_v yz}{10} + \frac{\sqrt{30}G_{xy}xz}{10} - \frac{\sqrt{30}G_{xz}xy}{10} - \frac{\sqrt{30}G_{yz}\left(y-z\right)\left(y+z\right)}{10}$$

$$\frac{3\sqrt{10}G_u xz}{10} - \frac{\sqrt{30}G_v xz}{10} - \frac{\sqrt{30}G_{xy}yz}{10} + \frac{\sqrt{30}G_{xz}\left(x-z\right)\left(x+z\right)}{10} + \frac{\sqrt{30}G_{yz}xy}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(2,-1)}[g](A_{1g})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}G_{xz}y}{2} - \frac{\sqrt{2}G_{yz}x}{2}$$

$\vec{\mathbb{Q}}_2^{(2,1)}[g](A_{1g})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}G_v xyz}{14} - \frac{5\sqrt{42}G_{xy}z\left(x-y\right)\left(x+y\right)}{28} - \frac{\sqrt{42}G_{xz}y\left(x^2+y^2-4z^2\right)}{28} + \frac{\sqrt{42}G_{yz}x\left(x^2+y^2-4z^2\right)}{28}$$

$\vec{\mathbb{Q}}_2^{(2,-1)}[g](B_{1g})$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{6}G_{xy}z}{3} - \frac{\sqrt{6}G_{xz}y}{6} - \frac{\sqrt{6}G_{yz}x}{6}$$

$\vec{\mathbb{Q}}_2^{(2,1)}[g](B_{1g})$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{5\sqrt{42}G_u xyz}{14} + \frac{\sqrt{14}G_{xy}z\left(3x^2+3y^2-2z^2\right)}{28} - \frac{\sqrt{14}G_{xz}y\left(9x^2-y^2-6z^2\right)}{28} + \frac{\sqrt{14}G_{yz}x\left(x^2-9y^2+6z^2\right)}{28}$$

$\vec{\mathbb{Q}}_2^{(2,-1)}[g](B_{2g})$

** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{\sqrt{6}G_v z}{3}+\frac{\sqrt{6}G_{xz}x}{6}-\frac{\sqrt{6}G_{yz}y}{6}$$

$\vec{\mathbb{Q}}_2^{(2,1)}[g](B_{2g})$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{5\sqrt{42}G_u z\left(x-y\right)\left(x+y\right)}{28}-\frac{\sqrt{14}G_v z\left(3x^2+3y^2-2z^2\right)}{28}+\frac{\sqrt{14}G_{xz}x\left(2x^2-3y^2-3z^2\right)}{14}+\frac{\sqrt{14}G_{yz}y\left(3x^2-2y^2+3z^2\right)}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E_g)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E_g)$
** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_u x}{2}+\frac{\sqrt{6}G_v x}{6}+\frac{\sqrt{6}G_{xy}y}{6}-\frac{\sqrt{6}G_{xz}z}{6}$$

$$-\frac{\sqrt{2}G_u y}{2}+\frac{\sqrt{6}G_v y}{6}-\frac{\sqrt{6}G_{xy}x}{6}+\frac{\sqrt{6}G_{yz}z}{6}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E_g)$, $\vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E_g)$
** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{42}G_u x\left(x^2+y^2-4z^2\right)}{28}-\frac{\sqrt{14}G_v x\left(x^2-9y^2+6z^2\right)}{28}-\frac{\sqrt{14}G_{xy}y\left(3x^2-2y^2+3z^2\right)}{14}+\frac{\sqrt{14}G_{xz}z\left(3x^2+3y^2-2z^2\right)}{14}$$

$$\frac{\sqrt{42}G_u y\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{14}G_v y\left(9x^2-y^2-6z^2\right)}{28}-\frac{\sqrt{14}G_{xy}x\left(2x^2-3y^2-3z^2\right)}{14}-\frac{\sqrt{14}G_{yz}z\left(3x^2+3y^2-2z^2\right)}{14}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_3^{(2,-1)}[g](A_{2u})$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{30}G_v xy}{10}+\frac{\sqrt{30}G_{xy}\left(x-y\right)\left(x+y\right)}{20}+\frac{\sqrt{30}G_{xz}yz}{5}-\frac{\sqrt{30}G_{yz}xz}{5}$$

$\vec{\mathbb{Q}}_3^{(2,1)}[g](A_{2u})$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_v xy\left(x^2+y^2-6z^2\right)}{6}+\frac{\sqrt{15}G_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{12}-\frac{\sqrt{15}G_{xz}yz\left(3x^2+3y^2-4z^2\right)}{12}+\frac{\sqrt{15}G_{yz}xz\left(3x^2+3y^2-4z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(2,-1)}[g](B_{1u})$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}G_u\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{2}G_v\left(x^2+y^2-2z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(2,1)}[g](B_{1u})$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}G_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{6}-\frac{G_v\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{6}$$
$$-\frac{7G_{xy}xy\,(x-y)\,(x+y)}{6}+\frac{7G_{xz}xz\,(x-z)\,(x+z)}{6}-\frac{7G_{yz}yz\,(y-z)\,(y+z)}{6}$$

$\vec{\mathbb{Q}}_3^{(2,-1)}[g](B_{2u})$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{6}G_uxy}{2}-\frac{\sqrt{2}G_{xy}\left(x^2+y^2-2z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(2,1)}[g](B_{2u})$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_uxy\left(x^2+y^2-6z^2\right)}{3}+\frac{7G_vxy\,(x-y)\,(x+y)}{6}-\frac{G_{xy}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{12}$$
$$-\frac{7G_{xz}yz\left(3x^2+y^2-2z^2\right)}{12}-\frac{7G_{yz}xz\left(x^2+3y^2-2z^2\right)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E_u,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{10}G_uyz}{20}+\frac{\sqrt{30}G_vyz}{20}+\frac{\sqrt{30}G_{xy}xz}{5}-\frac{\sqrt{30}G_{xz}xy}{5}+\frac{\sqrt{30}G_{yz}\,(y-z)\,(y+z)}{20}$$

$$-\frac{3\sqrt{10}G_uxz}{20}+\frac{\sqrt{30}G_vxz}{20}-\frac{\sqrt{30}G_{xy}yz}{5}-\frac{\sqrt{30}G_{xz}\,(x-z)\,(x+z)}{20}+\frac{\sqrt{30}G_{yz}xy}{5}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E_u,2)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

$$\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{\sqrt{6}G_uyz}{4}-\frac{3\sqrt{2}G_vyz}{4}+\frac{\sqrt{2}G_{yz}\left(2x^2-y^2-z^2\right)}{4}$$

$$-\frac{\sqrt{6}G_uxz}{4}-\frac{3\sqrt{2}G_vxz}{4}+\frac{\sqrt{2}G_{xz}\left(x^2-2y^2+z^2\right)}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](E_u,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{5}G_u yz\left(6x^2-y^2-z^2\right)}{4}-\frac{\sqrt{15}G_v yz\left(6x^2-y^2-z^2\right)}{12}+\frac{\sqrt{15}G_{xy} xz\left(4x^2-3y^2-3z^2\right)}{12}$$
$$-\frac{\sqrt{15}G_{xz} xy\left(4x^2-3y^2-3z^2\right)}{12}-\frac{\sqrt{15}G_{yz}\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{12}$$

$$-\frac{\sqrt{5}G_u xz\left(x^2-6y^2+z^2\right)}{4}+\frac{\sqrt{15}G_v xz\left(x^2-6y^2+z^2\right)}{12}+\frac{\sqrt{15}G_{xy} yz\left(3x^2-4y^2+3z^2\right)}{12}$$
$$-\frac{\sqrt{15}G_{xz}\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{12}-\frac{\sqrt{15}G_{yz} xy\left(3x^2-4y^2+3z^2\right)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](E_u,2)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_u yz\left(12x^2-9y^2+5z^2\right)}{12}-\frac{G_v yz\left(36x^2+y^2-13z^2\right)}{12}+\frac{7G_{xy} xz\left(2x^2-3y^2-z^2\right)}{12}$$
$$+\frac{7G_{xz} xy\left(2x^2-y^2-3z^2\right)}{12}-\frac{G_{yz}\left(4x^4-12x^2y^2-12x^2z^2+5y^4-18y^2z^2+5z^4\right)}{12}$$

$$\frac{\sqrt{3}G_u xz\left(9x^2-12y^2-5z^2\right)}{12}-\frac{G_v xz\left(x^2+36y^2-13z^2\right)}{12}+\frac{7G_{xy} yz\left(3x^2-2y^2+z^2\right)}{12}$$
$$+\frac{G_{xz}\left(5x^4-12x^2y^2-18x^2z^2+4y^4-12y^2z^2+5z^4\right)}{12}+\frac{7G_{yz} xy\left(x^2-2y^2+3z^2\right)}{12}$$

* Harmonics for rank 4
$\vec{\mathbb{Q}}_4^{(2,-1)}[g](A_{1g},1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{5}G_{xy} z\left(x-y\right)\left(x+y\right)}{2}-\frac{\sqrt{5}G_{xz} y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}G_{yz} x\left(y-z\right)\left(y+z\right)}{2}$$

$\vec{\mathbb{Q}}_4^{(2,-1)}[g](A_{1g},2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{6\sqrt{7}G_v xyz}{7}-\frac{\sqrt{7}G_{xy} z\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{7}G_{xz} y\left(4x^2-3y^2+5z^2\right)}{14}+\frac{\sqrt{7}G_{yz} x\left(3x^2-4y^2-5z^2\right)}{14}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_{1g},1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{3\sqrt{2310}G_u xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{3\sqrt{770}G_v xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{770}G_{xy} z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{22}$$
$$-\frac{\sqrt{770}G_{xz} y\left(x-z\right)\left(x+z\right)\left(x^2-2y^2+z^2\right)}{22}-\frac{\sqrt{770}G_{yz} x\left(y-z\right)\left(y+z\right)\left(2x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_{1g},2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$\frac{21\sqrt{66}G_u xyz\,(x-y)\,(x+y)}{44}-\frac{21\sqrt{22}G_v xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{7\sqrt{22}G_{xy}z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{\sqrt{22}G_{xz}y\left(17x^4-22x^2y^2-36x^2z^2+3y^4-8y^2z^2+10z^4\right)}{44}-\frac{\sqrt{22}G_{yz}x\left(3x^4-22x^2y^2-8x^2z^2+17y^4-36y^2z^2+10z^4\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,-1)}[g](A_{2g})$
** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{3}G_v z\,(x-y)\,(x+y)}{2}+\sqrt{3}G_{xy}xyz+\frac{\sqrt{3}G_{xz}x\left(x^2-3y^2\right)}{4}-\frac{\sqrt{3}G_{yz}y\left(3x^2-y^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_{2g})$
** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{9\sqrt{154}G_u z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}-\frac{\sqrt{462}G_v z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{462}G_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{\sqrt{462}G_{xz}x\left(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2\right)}{44}+\frac{\sqrt{462}G_{yz}y\left(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,-1)}[g](B_{1g})$
** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{6\sqrt{7}G_u xyz}{7}+\frac{\sqrt{21}G_{xy}z\left(3x^2+3y^2-2z^2\right)}{14}-\frac{\sqrt{21}G_{xz}y\left(2x^2-y^2+z^2\right)}{14}+\frac{\sqrt{21}G_{yz}x\left(x^2-2y^2-z^2\right)}{14}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](B_{1g})$
** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{22}G_u xyz\left(x^2+y^2-2z^2\right)}{44}-\frac{21\sqrt{66}G_v xyz\,(x-y)\,(x+y)}{44}+\frac{\sqrt{66}G_{xy}z\left(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4\right)}{44}$$
$$-\frac{\sqrt{66}G_{xz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}-\frac{\sqrt{66}G_{yz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,-1)}[g](B_{2g})$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{7}G_u z\,(x-y)\,(x+y)}{7}+\frac{\sqrt{21}G_v z\left(3x^2+3y^2-2z^2\right)}{14}-\frac{\sqrt{21}G_{xz}x\left(x^2-5y^2+2z^2\right)}{28}-\frac{\sqrt{21}G_{yz}y\left(5x^2-y^2-2z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](B_{2g})$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{21\sqrt{22}G_u z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}-\frac{\sqrt{66}G_v z\left(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4\right)}{88}-\frac{21\sqrt{66}G_{xy}xyz\,(x-y)\,(x+y)}{44}$$
$$-\frac{\sqrt{66}G_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{66}G_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E_g,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{3G_{u}x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}G_{v}x\left(y-z\right)\left(y+z\right)}{4}+\frac{\sqrt{3}G_{xy}y\left(y^{2}-3z^{2}\right)}{4}-\frac{\sqrt{3}G_{xz}z\left(3y^{2}-z^{2}\right)}{4}+\sqrt{3}G_{yz}xyz$$

$$-\frac{3G_{u}y\left(x-z\right)\left(x+z\right)}{4}+\frac{\sqrt{3}G_{v}y\left(x-z\right)\left(x+z\right)}{4}-\frac{\sqrt{3}G_{xy}x\left(x^{2}-3z^{2}\right)}{4}-\sqrt{3}G_{xz}xyz+\frac{\sqrt{3}G_{yz}z\left(3x^{2}-z^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

** expression

$$\frac{3\sqrt{7}G_{u}x\left(2x^{2}-5y^{2}-z^{2}\right)}{28}+\frac{\sqrt{21}G_{v}x\left(2x^{2}+3y^{2}-9z^{2}\right)}{28}-\frac{\sqrt{21}G_{xy}y\left(2x^{2}+y^{2}-5z^{2}\right)}{28}+\frac{\sqrt{21}G_{xz}z\left(2x^{2}-5y^{2}+z^{2}\right)}{28}$$

$$\frac{3\sqrt{7}G_{u}y\left(5x^{2}-2y^{2}+z^{2}\right)}{28}+\frac{\sqrt{21}G_{v}y\left(3x^{2}+2y^{2}-9z^{2}\right)}{28}+\frac{\sqrt{21}G_{xy}x\left(x^{2}+2y^{2}-5z^{2}\right)}{28}+\frac{\sqrt{21}G_{yz}z\left(5x^{2}-2y^{2}-z^{2}\right)}{28}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E_g,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\frac{3\sqrt{154}G_{u}x\left(x^{2}y^{2}-x^{2}z^{2}+y^{4}-9y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{462}G_{v}x\left(x^{2}y^{2}-x^{2}z^{2}-5y^{4}+27y^{2}z^{2}-4z^{4}\right)}{88}$$
$$-\frac{\sqrt{462}G_{xy}y\left(2x^{2}y^{2}-6x^{2}z^{2}-y^{4}+8y^{2}z^{2}-3z^{4}\right)}{44}+\frac{\sqrt{462}G_{xz}z\left(6x^{2}y^{2}-2x^{2}z^{2}+3y^{4}-8y^{2}z^{2}+z^{4}\right)}{44}-\frac{\sqrt{462}G_{yz}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{44}$$

$$\frac{3\sqrt{154}G_{u}y\left(x^{4}+x^{2}y^{2}-9x^{2}z^{2}-y^{2}z^{2}+2z^{4}\right)}{88}+\frac{\sqrt{462}G_{v}y\left(5x^{4}-x^{2}y^{2}-27x^{2}z^{2}+y^{2}z^{2}+4z^{4}\right)}{88}$$
$$-\frac{\sqrt{462}G_{xy}x\left(x^{4}-2x^{2}y^{2}-8x^{2}z^{2}+6y^{2}z^{2}+3z^{4}\right)}{44}-\frac{\sqrt{462}G_{xz}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{44}-\frac{\sqrt{462}G_{yz}z\left(3x^{4}+6x^{2}y^{2}-8x^{2}z^{2}-2y^{2}z^{2}+z^{4}\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

** expression

$$-\frac{3\sqrt{22}G_{u}x\left(2x^{4}-3x^{2}y^{2}-17x^{2}z^{2}-5y^{4}+39y^{2}z^{2}+2z^{4}\right)}{88}-\frac{\sqrt{66}G_{v}x\left(2x^{4}-31x^{2}y^{2}+11x^{2}z^{2}+9y^{4}+39y^{2}z^{2}-12z^{4}\right)}{88}$$
$$-\frac{\sqrt{66}G_{xy}y\left(8x^{4}-12x^{2}y^{2}-12x^{2}z^{2}+y^{4}+2y^{2}z^{2}+z^{4}\right)}{44}+\frac{\sqrt{66}G_{xz}z\left(8x^{4}-12x^{2}y^{2}-12x^{2}z^{2}+y^{4}+2y^{2}z^{2}+z^{4}\right)}{44}-\frac{21\sqrt{66}G_{yz}xyz\left(y-z\right)\left(y+z\right)}{44}$$

$$-\frac{3\sqrt{22}G_{u}y\left(5x^{4}+3x^{2}y^{2}-39x^{2}z^{2}-2y^{4}+17y^{2}z^{2}-2z^{4}\right)}{88}-\frac{\sqrt{66}G_{v}y\left(9x^{4}-31x^{2}y^{2}+39x^{2}z^{2}+2y^{4}+11y^{2}z^{2}-12z^{4}\right)}{88}$$
$$+\frac{\sqrt{66}G_{xy}x\left(x^{4}-12x^{2}y^{2}+2x^{2}z^{2}+8y^{4}-12y^{2}z^{2}+z^{4}\right)}{44}+\frac{21\sqrt{66}G_{xz}xyz\left(x-z\right)\left(x+z\right)}{44}-\frac{\sqrt{66}G_{yz}z\left(x^{4}-12x^{2}y^{2}+2x^{2}z^{2}+8y^{4}-12y^{2}z^{2}+z^{4}\right)}{44}$$