PG No. 4 $C_{\rm s}$ m (b-axis setting) [monoclinic] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,0)}[q](A')$

** symmetry

$$y$$

** expression

$$- \frac{\sqrt{2} Q_{x} z}{2} + \frac{\sqrt{2} Q_{z} x}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](A'',1)$

** symmetry

$$x$$

** expression

$$\frac{\sqrt{2} Q_{y} z}{2} - \frac{\sqrt{2} Q_{z} y}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](A'',2)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2} Q_{x} y}{2} - \frac{\sqrt{2} Q_{y} x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,0)}[q](A',1)$

** symmetry

$$\sqrt{3} y z$$

** expression

$$\frac{\sqrt{2} Q_{x} \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{2} Q_{y} x y}{2} + \frac{\sqrt{2} Q_{z} x z}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A',2)$

** symmetry

$$\sqrt{3} x y$$

** expression

$$- \frac{\sqrt{2} Q_{x} x z}{2} + \frac{\sqrt{2} Q_{y} y z}{2} + \frac{\sqrt{2} Q_{z} \left(x - y\right) \left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A'',1)$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{6} Q_{x} y z}{2} - \frac{\sqrt{6} Q_{y} x z}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A'',2)$

** symmetry

$$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{2} Q_{x} y z}{2} + \frac{\sqrt{2} Q_{y} x z}{2} - \sqrt{2} Q_{z} x y$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A'',3)$

** symmetry

$$\sqrt{3} x z$$

** expression

$$\frac{\sqrt{2}Q_{x}xy}{2} - \frac{\sqrt{2}Q_{y}\left(x - z\right)\left(x + z\right)}{2} - \frac{\sqrt{2}Q_{z}yz}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A',1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_{x}x\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{5}Q_{y}y\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{5}Q_{z}z\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A',2)$

** symmetry

$$- \frac{y\left(3x^{2} - 2y^{2} + 3z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{x}z\left(x^{2} - 4y^{2} + z^{2}\right)}{4} - \frac{\sqrt{3}Q_{z}x\left(x^{2} - 4y^{2} + z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A',3)$

** symmetry

$$- \frac{\sqrt{15}y\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_{x}z\left(x^{2} + 2y^{2} - z^{2}\right)}{4} - \sqrt{5}Q_{y}xyz - \frac{\sqrt{5}Q_{z}x\left(x^{2} - 2y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A'',1)$

** symmetry

$$\frac{x\left(2x^{2} - 3y^{2} - 3z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_{y}z\left(4x^{2} - y^{2} - z^{2}\right)}{4} - \frac{\sqrt{3}Q_{z}y\left(4x^{2} - y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A'',2)$

** symmetry

$$- \frac{z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{3}Q_{x}y\left(x^{2} + y^{2} - 4z^{2}\right)}{4} + \frac{\sqrt{3}Q_{y}x\left(x^{2} + y^{2} - 4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A'',3)$

** symmetry

$$\frac{\sqrt{15}x\left(y - z\right)\left(y + z\right)}{2}$$

** expression

$$- \sqrt{5}Q_{x}xyz + \frac{\sqrt{5}Q_{y}z\left(2x^{2} + y^{2} - z^{2}\right)}{4} + \frac{\sqrt{5}Q_{z}y\left(2x^{2} - y^{2} + z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A'',4)$

** symmetry

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_{x}y\left(x^{2} - y^{2} + 2z^{2}\right)}{4} - \frac{\sqrt{5}Q_{y}x\left(x^{2} - y^{2} - 2z^{2}\right)}{4} - \sqrt{5}Q_{z}xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A', 1)$

** symmetry

$$\frac{\sqrt{35} y z \left(y - z\right) \left(y + z\right)}{2}$$

** expression

$$\frac{\sqrt{7} Q_{x} \left(y^{2} - 2 y z - z^{2}\right) \left(y^{2} + 2 y z - z^{2}\right)}{4} - \frac{\sqrt{7} Q_{y} x y \left(y^{2} - 3 z^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} x z \left(3 y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A', 2)$

** symmetry

$$\frac{\sqrt{35} x y \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$- \frac{\sqrt{7} Q_{x} x z \left(x^{2} - 3 y^{2}\right)}{4} + \frac{\sqrt{7} Q_{y} y z \left(3 x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} \left(x^{2} - 2 x y - y^{2}\right) \left(x^{2} + 2 x y - y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A', 3)$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x} \left(y - z\right) \left(y + z\right) \left(6 x^{2} - y^{2} - z^{2}\right)}{4} - \frac{Q_{y} x y \left(6 x^{2} - y^{2} - 15 z^{2}\right)}{4} + \frac{Q_{z} x z \left(6 x^{2} - 15 y^{2} - z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A', 4)$

** symmetry

$$- \frac{\sqrt{5} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x} x z \left(x^{2} + 15 y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{y} y z \left(15 x^{2} + y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{z} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A'', 1)$

** symmetry

$$\frac{\sqrt{21} \left(x^{4} - 3 x^{2} y^{2} - 3 x^{2} z^{2} + y^{4} - 3 y^{2} z^{2} + z^{4}\right)}{6}$$

** expression

$$- \frac{\sqrt{105} Q_{x} y z \left(y - z\right) \left(y + z\right)}{6} + \frac{\sqrt{105} Q_{y} x z \left(x - z\right) \left(x + z\right)}{6} - \frac{\sqrt{105} Q_{z} x y \left(x - y\right) \left(x + y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A'', 2)$

** symmetry

$$- \frac{\sqrt{15} \left(x^{4} - 12 x^{2} y^{2} + 6 x^{2} z^{2} + y^{4} + 6 y^{2} z^{2} - 2 z^{4}\right)}{12}$$

** expression

$$- \frac{\sqrt{3} Q_{x} y z \left(9 x^{2} + 2 y^{2} - 5 z^{2}\right)}{6} + \frac{\sqrt{3} Q_{y} x z \left(2 x^{2} + 9 y^{2} - 5 z^{2}\right)}{6} + \frac{7 \sqrt{3} Q_{z} x y \left(x - y\right) \left(x + y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A'', 3)$

** symmetry

$$\frac{\sqrt{5} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{4}$$

** expression

$$- \frac{Q_{x} y z \left(3 x^{2} - 4 y^{2} + 3 z^{2}\right)}{2} + \frac{Q_{y} x z \left(4 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2} - \frac{Q_{z} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A'', 4)$

** symmetry

$$- \frac{\sqrt{35} x z \left(x - z\right) \left(x + z\right)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_x xy\left(x^2-3z^2\right)}{4}+\frac{\sqrt{7}Q_y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{4}+\frac{\sqrt{7}Q_z yz\left(3x^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A'',5)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{Q_x xy\left(x^2-6y^2+15z^2\right)}{4}+\frac{Q_y\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{4}+\frac{Q_z yz\left(15x^2-6y^2+z^2\right)}{4}$$