PG No. 5 $C_{2\mathrm{h}}$ $2/m$ (b-axis setting) [monoclinic] (polar, internal polar quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{Q}}_0^{(2,2)}[q](A_g)$

** symmetry

1

** expression

$$-\frac{\sqrt{5}Q_u\left(x^2+y^2-2z^2\right)}{10}+\frac{\sqrt{15}Q_v\left(x-y\right)\left(x+y\right)}{10}+\frac{\sqrt{15}Q_{xy}xy}{5}+\frac{\sqrt{15}Q_{xz}xz}{5}+\frac{\sqrt{15}Q_{yz}yz}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(2,0)}[q](A_u)$

** symmetry

y

** expression

$$-\frac{\sqrt{10}Q_u y}{10}-\frac{\sqrt{30}Q_v y}{10}+\frac{\sqrt{30}Q_{xy}x}{10}+\frac{\sqrt{30}Q_{yz}z}{10}$$

$\vec{\mathbb{Q}}_1^{(2,2)}[q](A_u)$

** symmetry

y

** expression

$$-\frac{3\sqrt{35}Q_u y\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}Q_v y\left(7x^2-3y^2+2z^2\right)}{70}-\frac{\sqrt{105}Q_{xy}x\left(x^2-4y^2+z^2\right)}{35}+\frac{\sqrt{105}Q_{xz}xyz}{7}-\frac{\sqrt{105}Q_{yz}z\left(x^2-4y^2+z^2\right)}{35}$$

$\vec{\mathbb{Q}}_1^{(2,0)}[q](B_u,1)$

** symmetry

x

** expression

$$-\frac{\sqrt{10}Q_u x}{10}+\frac{\sqrt{30}Q_v x}{10}+\frac{\sqrt{30}Q_{xy}y}{10}+\frac{\sqrt{30}Q_{xz}z}{10}$$

$\vec{\mathbb{Q}}_1^{(2,0)}[q](B_u,2)$

** symmetry

z

** expression

$$\frac{\sqrt{10}Q_u z}{5}+\frac{\sqrt{30}Q_{xz}x}{10}+\frac{\sqrt{30}Q_{yz}y}{10}$$

$\vec{\mathbb{Q}}_1^{(2,2)}[q](B_u,1)$

** symmetry

x

** expression

$$-\frac{3\sqrt{35}Q_u x\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}Q_v x\left(3x^2-7y^2-2z^2\right)}{70}+\frac{\sqrt{105}Q_{xy}y\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}Q_{xz}z\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}Q_{yz}xyz}{7}$$

$\vec{\mathbb{Q}}_1^{(2,2)}[q](B_u,2)$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}Q_u z\left(3x^2+3y^2-2z^2\right)}{70}+\frac{\sqrt{105}Q_{vz}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}Q_{xy}xyz}{7}-\frac{\sqrt{105}Q_{xz}x\left(x^2+y^2-4z^2\right)}{35}-\frac{\sqrt{105}Q_{yz}y\left(x^2+y^2-4z^2\right)}{35}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(2,-2)}[q](A_g,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$Q_u$$

$\vec{\mathbb{Q}}_2^{(2,-2)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$Q_v$$

$\vec{\mathbb{Q}}_2^{(2,-2)}[q](A_g, 3)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$Q_{xz}$$

$\vec{\mathbb{Q}}_2^{(2,0)}[q](A_g, 1)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{14}Q_u\left(x^2+y^2-2z^2\right)}{14} - \frac{\sqrt{42}Q_v\,(x-y)\,(x+y)}{14} - \frac{\sqrt{42}Q_{xy}xy}{7} + \frac{\sqrt{42}Q_{xz}xz}{14} + \frac{\sqrt{42}Q_{yz}yz}{14}$$

$\vec{\mathbb{Q}}_2^{(2,0)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{42}Q_u\,(x-y)\,(x+y)}{14} + \frac{\sqrt{14}Q_v\left(x^2+y^2-2z^2\right)}{14} + \frac{3\sqrt{14}Q_{xz}xz}{14} - \frac{3\sqrt{14}Q_{yz}yz}{14}$$

$\vec{\mathbb{Q}}_2^{(2,0)}[q](A_g, 3)$
** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}Q_uxz}{14} + \frac{3\sqrt{14}Q_vxz}{14} + \frac{3\sqrt{14}Q_{xy}yz}{14} + \frac{\sqrt{14}Q_{xz}\left(x^2-2y^2+z^2\right)}{14} + \frac{3\sqrt{14}Q_{yz}xy}{14}$$

$\vec{\mathbb{Q}}_2^{(2,2)}[q](A_g, 1)$
** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}Q_u\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{56} - \frac{5\sqrt{42}Q_v\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{168}$$
$$- \frac{5\sqrt{42}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{84} - \frac{5\sqrt{42}Q_{xz}xz\left(3x^2+3y^2-4z^2\right)}{84} - \frac{5\sqrt{42}Q_{yz}yz\left(3x^2+3y^2-4z^2\right)}{84}$$

$\vec{\mathbb{Q}}_2^{(2,2)}[q](A_g, 2)$
** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{5\sqrt{42}Q_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{168} + \frac{\sqrt{14}Q_v\left(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4\right)}{168}$$
$$+ \frac{5\sqrt{14}Q_{xy}xy\,(x-y)\,(x+y)}{12} + \frac{5\sqrt{14}Q_{xz}xz\left(5x^2-9y^2-2z^2\right)}{84} + \frac{5\sqrt{14}Q_{yz}yz\left(9x^2-5y^2+2z^2\right)}{84}$$

$\vec{\mathbb{Q}}_2^{(2,2)}[q](A_g, 3)$
** symmetry

$\sqrt{3}xz$

** expression

$$-\frac{5\sqrt{42}Q_u xz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}Q_v xz\left(5x^2-9y^2-2z^2\right)}{84}+\frac{5\sqrt{14}Q_{xy}yz\left(6x^2-y^2-z^2\right)}{42}$$
$$-\frac{\sqrt{14}Q_{xz}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{42}-\frac{5\sqrt{14}Q_{yz}xy\left(x^2+y^2-6z^2\right)}{42}$$

$\vec{\mathbb{Q}}_2^{(2,-2)}[q](B_g, 1)$
** symmetry

$\sqrt{3}yz$

** expression

Q_{yz}

$\vec{\mathbb{Q}}_2^{(2,-2)}[q](B_g, 2)$
** symmetry

$\sqrt{3}xy$

** expression

Q_{xy}

$\vec{\mathbb{Q}}_2^{(2,0)}[q](B_g, 1)$
** symmetry

$\sqrt{3}yz$

** expression

$$\frac{\sqrt{42}Q_u yz}{14}-\frac{3\sqrt{14}Q_v yz}{14}+\frac{3\sqrt{14}Q_{xy}xz}{14}+\frac{3\sqrt{14}Q_{xz}xy}{14}-\frac{\sqrt{14}Q_{yz}\left(2x^2-y^2-z^2\right)}{14}$$

$\vec{\mathbb{Q}}_2^{(2,0)}[q](B_g, 2)$
** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{\sqrt{42}Q_u xy}{7}+\frac{\sqrt{14}Q_{xy}\left(x^2+y^2-2z^2\right)}{14}+\frac{3\sqrt{14}Q_{xz}yz}{14}+\frac{3\sqrt{14}Q_{yz}xz}{14}$$

$\vec{\mathbb{Q}}_2^{(2,2)}[q](B_g, 1)$
** symmetry

$\sqrt{3}yz$

** expression

$$-\frac{5\sqrt{42}Q_u yz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}Q_v yz\left(9x^2-5y^2+2z^2\right)}{84}-\frac{5\sqrt{14}Q_{xy}xz\left(x^2-6y^2+z^2\right)}{42}$$
$$-\frac{5\sqrt{14}Q_{xz}xy\left(x^2+y^2-6z^2\right)}{42}+\frac{\sqrt{14}Q_{yz}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{42}$$

$\vec{\mathbb{Q}}_2^{(2,2)}[q](B_g, 2)$
** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{5\sqrt{42}Q_u xy\left(x^2+y^2-6z^2\right)}{84}+\frac{5\sqrt{14}Q_v xy\left(x-y\right)\left(x+y\right)}{12}-\frac{\sqrt{14}Q_{xy}\left(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4\right)}{42}$$
$$+\frac{5\sqrt{14}Q_{xz}yz\left(6x^2-y^2-z^2\right)}{42}-\frac{5\sqrt{14}Q_{yz}xz\left(x^2-6y^2+z^2\right)}{42}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_3^{(2,-2)}[q](A_u, 1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}Q_{xy}z}{3}+\frac{\sqrt{3}Q_{xz}y}{3}+\frac{\sqrt{3}Q_{yz}x}{3}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](A_u,2)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_uy}{10}-\frac{3\sqrt{5}Q_vy}{10}-\frac{\sqrt{5}Q_{xy}x}{5}-\frac{\sqrt{5}Q_{yz}z}{5}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](A_u,3)$
** symmetry

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{Q_uy}{2}-\frac{\sqrt{3}Q_vy}{6}-\frac{\sqrt{3}Q_{xy}x}{3}+\frac{\sqrt{3}Q_{yz}z}{3}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](A_u,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}Q_{xy}z\left(3x^2+3y^2-2z^2\right)}{6}+\frac{\sqrt{3}Q_{xz}y\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}Q_{yz}x\left(2x^2-3y^2-3z^2\right)}{6}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](A_u,2)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_uy\left(21x^2-4y^2-9z^2\right)}{60}+\frac{\sqrt{5}Q_vy\left(x^2-4y^2+11z^2\right)}{20}-\frac{\sqrt{5}Q_{xy}x\left(x^2-4y^2+z^2\right)}{20}-\sqrt{5}Q_{xz}xyz-\frac{\sqrt{5}Q_{yz}z\left(x^2-4y^2+z^2\right)}{20}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](A_u,3)$
** symmetry

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{Q_uy\left(3x^2-2y^2+3z^2\right)}{4}-\frac{\sqrt{3}Q_vy\left(3x^2-2y^2+3z^2\right)}{12}-\frac{\sqrt{3}Q_{xy}x\left(x^2+6y^2-9z^2\right)}{12}-\frac{\sqrt{3}Q_{yz}z\left(9x^2-6y^2-z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](A_u,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{21\sqrt{22}Q_uxyz\left(x^2+y^2-2z^2\right)}{44}+\frac{21\sqrt{66}Q_vxyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xy}z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{66}$$
$$-\frac{\sqrt{66}Q_{xz}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{66}+\frac{\sqrt{66}Q_{yz}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{66}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](A_u,2)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{330}Q_u y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264}-\frac{\sqrt{110}Q_v y\left(11x^4-27x^2y^2+15x^2z^2+4y^4-13y^2z^2+4z^4\right)}{88}$$
$$+\frac{\sqrt{110}Q_{xy}x\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}-\frac{7\sqrt{110}Q_{xz}xyz\left(x^2-2y^2+z^2\right)}{44}+\frac{\sqrt{110}Q_{yz}z\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](A_u,3)$

** symmetry

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{22}Q_u y\left(9x^4+11x^2y^2-87x^2z^2+2y^4-31y^2z^2+30z^4\right)}{88}-\frac{\sqrt{66}Q_v y\left(51x^4-73x^2y^2-87x^2z^2+2y^4+53y^2z^2-12z^4\right)}{264}$$
$$+\frac{\sqrt{66}Q_{xy}x\left(5x^4-46x^2y^2-4x^2z^2+12y^4+66y^2z^2-9z^4\right)}{132}-\frac{21\sqrt{66}Q_{xz}xyz\left(x-z\right)\left(x+z\right)}{44}$$
$$+\frac{\sqrt{66}Q_{yz}z\left(9x^4-66x^2y^2+4x^2z^2-12y^4+46y^2z^2-5z^4\right)}{132}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](B_u,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_u x}{10}+\frac{3\sqrt{5}Q_v x}{10}-\frac{\sqrt{5}Q_{xy}y}{5}-\frac{\sqrt{5}Q_{xz}z}{5}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](B_u,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_u z}{5}-\frac{\sqrt{5}Q_{xz}x}{5}-\frac{\sqrt{5}Q_{yz}y}{5}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](B_u,3)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$-\frac{Q_u x}{2}-\frac{\sqrt{3}Q_v x}{6}+\frac{\sqrt{3}Q_{xy}y}{3}-\frac{\sqrt{3}Q_{xz}z}{3}$$

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](B_u,4)$

** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_v z}{3}+\frac{\sqrt{3}Q_{xz}x}{3}-\frac{\sqrt{3}Q_{yz}y}{3}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](B_u,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_u x\left(4x^2-21y^2+9z^2\right)}{60}+\frac{\sqrt{5}Q_v x\left(4x^2-y^2-11z^2\right)}{20}+\frac{\sqrt{5}Q_{xy}y\left(4x^2-y^2-z^2\right)}{20}+\frac{\sqrt{5}Q_{xz}z\left(4x^2-y^2-z^2\right)}{20}-\sqrt{5}Q_{yz}xyz$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](B_u,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_u z\left(3x^2+3y^2-2z^2\right)}{15}-\frac{\sqrt{5}Q_v z\,(x-y)\,(x+y)}{2}-\sqrt{5}Q_{xy}xyz-\frac{\sqrt{5}Q_{xz}x\left(x^2+y^2-4z^2\right)}{20}-\frac{\sqrt{5}Q_{yz}y\left(x^2+y^2-4z^2\right)}{20}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](B_u,3)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{Q_u x\left(2x^2-3y^2-3z^2\right)}{4}+\frac{\sqrt{3}Q_v x\left(2x^2-3y^2-3z^2\right)}{12}+\frac{\sqrt{3}Q_{xy}y\left(6x^2+y^2-9z^2\right)}{12}-\frac{\sqrt{3}Q_{xz}z\left(6x^2-9y^2+z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](B_u,4)$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}Q_v z\left(3x^2+3y^2-2z^2\right)}{6}+\frac{\sqrt{3}Q_{xz}x\left(x^2-9y^2+6z^2\right)}{12}+\frac{\sqrt{3}Q_{yz}y\left(9x^2-y^2-6z^2\right)}{12}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](B_u,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{330}Q_u x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264}+\frac{\sqrt{110}Q_v x\left(4x^4-27x^2y^2-13x^2z^2+11y^4+15y^2z^2+4z^4\right)}{88}$$
$$+\frac{\sqrt{110}Q_{xy}y\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}+\frac{\sqrt{110}Q_{xz}z\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44}+\frac{7\sqrt{110}Q_{yz}xyz\left(2x^2-y^2-z^2\right)}{44}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](B_u,2)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{330}Q_u z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{264}-\frac{7\sqrt{110}Q_v z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}-\frac{7\sqrt{110}Q_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{\sqrt{110}Q_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{110}Q_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](B_u,3)$
** symmetry

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{\sqrt{22}Q_u x\left(2x^4+11x^2y^2-31x^2z^2+9y^4-87y^2z^2+30z^4\right)}{88}-\frac{\sqrt{66}Q_v x\left(2x^4-73x^2y^2+53x^2z^2+51y^4-87y^2z^2-12z^4\right)}{264}$$
$$-\frac{\sqrt{66}Q_{xy}y\left(12x^4-46x^2y^2+66x^2z^2+5y^4-4y^2z^2-9z^4\right)}{132}$$
$$+\frac{\sqrt{66}Q_{xz}z\left(12x^4+66x^2y^2-46x^2z^2-9y^4-4y^2z^2+5z^4\right)}{132}+\frac{21\sqrt{66}Q_{yz}xyz\,(y-z)\,(y+z)}{44}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](B_u,4)$
** symmetry

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{21\sqrt{22}Q_u z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{66}Q_v z\left(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4\right)}{264}+\frac{21\sqrt{66}Q_{xy}xyz\,(x-y)\,(x+y)}{44}$$
$$-\frac{\sqrt{66}Q_{xz}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{132}-\frac{\sqrt{66}Q_{yz}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{132}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](A_g,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{30}Q_u\left(x^2+y^2-2z^2\right)}{20}+\frac{3\sqrt{10}Q_v\left(x-y\right)\left(x+y\right)}{20}-\frac{\sqrt{10}Q_{xy}xy}{5}-\frac{\sqrt{10}Q_{xz}xz}{5}-\frac{\sqrt{10}Q_{yz}yz}{5}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](A_g,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{42}Q_u\left(x^2+y^2-2z^2\right)}{28}-\frac{3\sqrt{14}Q_v\left(x-y\right)\left(x+y\right)}{28}+\frac{2\sqrt{14}Q_{xy}xy}{7}-\frac{\sqrt{14}Q_{xz}xz}{7}-\frac{\sqrt{14}Q_{yz}yz}{7}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](A_g,3)$

** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{14}Q_u\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{42}Q_v\left(x^2+y^2-2z^2\right)}{28}-\frac{\sqrt{42}Q_{xz}xz}{7}+\frac{\sqrt{42}Q_{yz}yz}{7}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](A_g,4)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{3\sqrt{2}Q_uxz}{4}-\frac{\sqrt{6}Q_vxz}{4}-\frac{\sqrt{6}Q_{xz}\left(x-z\right)\left(x+z\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](A_g,5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{14}Q_uxz}{28}-\frac{3\sqrt{42}Q_vxz}{28}+\frac{\sqrt{42}Q_{xy}yz}{7}-\frac{\sqrt{42}Q_{xz}\left(x^2-2y^2+z^2\right)}{28}+\frac{\sqrt{42}Q_{yz}xy}{7}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{165}Q_u\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{66}+\frac{\sqrt{55}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{55}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{22}+\frac{\sqrt{55}Q_{xz}xz\left(x^2-6y^2+z^2\right)}{22}-\frac{\sqrt{55}Q_{yz}yz\left(6x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$\frac{\sqrt{231}Q_u\left(8x^4-33x^2y^2-15x^2z^2+8y^4-15y^2z^2+5z^4\right)}{231}+\frac{\sqrt{77}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{77}$$
$$+\frac{\sqrt{77}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{14}-\frac{\sqrt{77}Q_{xz}xz\left(16x^2-33y^2-5z^2\right)}{154}+\frac{\sqrt{77}Q_{yz}yz\left(33x^2-16y^2+5z^2\right)}{154}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g,3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{77}Q_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{231}Q_v\left(2x^4-3x^2y^2-9x^2z^2+2y^4-9y^2z^2+3z^4\right)}{77}$$
$$+\frac{\sqrt{231}Q_{xy}xy\,(x-y)\,(x+y)}{22}-\frac{\sqrt{231}Q_{xz}xz\left(2x^2-33y^2+9z^2\right)}{154}-\frac{\sqrt{231}Q_{yz}yz\left(33x^2-2y^2-9z^2\right)}{154}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g,4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{11}Q_uxz\left(11x^2+18y^2-17z^2\right)}{44}-\frac{\sqrt{33}Q_vxz\left(15x^2-6y^2-13z^2\right)}{44}-\frac{7\sqrt{33}Q_{xy}yz\left(3x^2-z^2\right)}{44}$$
$$-\frac{\sqrt{33}Q_{xz}\,(x-z)\,(x+z)\left(x^2-6y^2+z^2\right)}{44}-\frac{7\sqrt{33}Q_{yz}xy\left(x^2-3z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](A_g,5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$\frac{\sqrt{77}Q_uxz\left(25x^2-24y^2-17z^2\right)}{308}-\frac{\sqrt{231}Q_vxz\left(3x^2+24y^2-11z^2\right)}{308}+\frac{\sqrt{231}Q_{xy}yz\left(39x^2+18y^2-31z^2\right)}{308}$$
$$-\frac{\sqrt{231}Q_{xz}\left(x^4-36x^2y^2+30x^2z^2+12y^4-36y^2z^2+z^4\right)}{308}-\frac{\sqrt{231}Q_{yz}xy\left(31x^2-18y^2-39z^2\right)}{308}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{15015}Q_u\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{572}$$
$$+\frac{3\sqrt{5005}Q_v\,(x-y)\,(x+y)\left(x^4-9x^2y^2-5x^2z^2+y^4-5y^2z^2+5z^4\right)}{572}+\frac{\sqrt{5005}Q_{xy}xy\left(7x^4-19x^2y^2-13x^2z^2+7y^4-13y^2z^2+13z^4\right)}{286}$$
$$+\frac{\sqrt{5005}Q_{xz}xz\left(7x^4-13x^2y^2-19x^2z^2+13y^4-13y^2z^2+7z^4\right)}{286}+\frac{\sqrt{5005}Q_{yz}yz\left(13x^4-13x^2y^2-13x^2z^2+7y^4-19y^2z^2+7z^4\right)}{286}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{429}Q_u\left(x^6+45x^4y^2-60x^4z^2+45x^2y^4-540x^2y^2z^2+150x^2z^4+y^6-60y^4z^2+150y^2z^4-20z^6\right)}{1144}$$
$$-\frac{21\sqrt{143}Q_v\,(x-y)\,(x+y)\left(x^4-20x^2y^2+6x^2z^2+y^4+6y^2z^2-6z^4\right)}{1144}-\frac{7\sqrt{143}Q_{xy}xy\left(x^4-4x^2y^2+2x^2z^2+y^4+2y^2z^2-2z^4\right)}{52}$$
$$+\frac{7\sqrt{143}Q_{xz}xz\left(x^4+56x^2y^2-22x^2z^2-11y^4-34y^2z^2+10z^4\right)}{572}-\frac{7\sqrt{143}Q_{yz}yz\left(11x^4-56x^2y^2+34x^2z^2-y^4+22y^2z^2-10z^4\right)}{572}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g,3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{143}Q_u\,(x-y)\,(x+y)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{\sqrt{429}Q_v\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$
$$+\frac{21\sqrt{429}Q_{xy}xy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}+\frac{7\sqrt{429}Q_{xz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{572}$$
$$+\frac{7\sqrt{429}Q_{yz}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{572}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g,4)$
** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$\frac{3\sqrt{1001}Q_uxz\left(11x^4+13x^2y^2-41x^2z^2+2y^4-17y^2z^2+14z^4\right)}{572}$$
$$-\frac{\sqrt{3003}Q_vxz\left(17x^4-47x^2y^2-41x^2z^2+2y^4+43y^2z^2+8z^4\right)}{572}-\frac{3\sqrt{3003}Q_{xy}yz\left(8x^4-3x^2y^2-13x^2z^2+y^2z^2+z^4\right)}{286}$$
$$+\frac{\sqrt{3003}Q_{xz}\,(x-z)\,(x+z)\left(2x^4+x^2y^2-29x^2z^2-y^4+y^2z^2+2z^4\right)}{286}+\frac{3\sqrt{3003}Q_{yz}xy\left(x^4+x^2y^2-13x^2z^2-3y^2z^2+8z^4\right)}{286}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](A_g,5)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{143}Q_uxz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}-\frac{21\sqrt{429}Q_vxz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{572}$$
$$-\frac{7\sqrt{429}Q_{xy}yz\left(8x^4-23x^2y^2+7x^2z^2+2y^4+y^2z^2-z^4\right)}{286}$$
$$+\frac{\sqrt{429}Q_{xz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$
$$+\frac{7\sqrt{429}Q_{yz}xy\left(x^4-x^2y^2-7x^2z^2-2y^4+23y^2z^2-8z^4\right)}{286}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](B_g,1)$
** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{3\sqrt{2}Q_uyz}{4}-\frac{\sqrt{6}Q_vyz}{4}+\frac{\sqrt{6}Q_{yz}\,(y-z)\,(y+z)}{4}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](B_g,2)$
** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}Q_vxy}{2}+\frac{\sqrt{6}Q_{xy}\,(x-y)\,(x+y)}{4}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](B_g,3)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{14}Q_uyz}{28}+\frac{3\sqrt{42}Q_vyz}{28}+\frac{\sqrt{42}Q_{xy}xz}{7}+\frac{\sqrt{42}Q_{xz}xy}{7}+\frac{\sqrt{42}Q_{yz}\left(2x^2-y^2-z^2\right)}{28}$$

$\vec{\mathbb{Q}}_4^{(2,-2)}[q](B_g,4)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{14}Q_u xy}{14}-\frac{\sqrt{42}Q_{xy}\left(x^2+y^2-2z^2\right)}{28}+\frac{\sqrt{42}Q_{xz}yz}{7}+\frac{\sqrt{42}Q_{yz}xz}{7}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](B_g,1)$
** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{11}Q_u yz\left(18x^2+11y^2-17z^2\right)}{44}+\frac{\sqrt{33}Q_v yz\left(6x^2-15y^2+13z^2\right)}{44}+\frac{7\sqrt{33}Q_{xy}xz\left(3y^2-z^2\right)}{44}$$
$$+\frac{7\sqrt{33}Q_{xz}xy\left(y^2-3z^2\right)}{44}-\frac{\sqrt{33}Q_{yz}\,(y-z)\,(y+z)\left(6x^2-y^2-z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](B_g,2)$
** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{7\sqrt{11}Q_u xy\,(x-y)\,(x+y)}{11}+\frac{\sqrt{33}Q_v xy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}Q_{xy}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}Q_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}Q_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](B_g,3)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_u yz\left(24x^2-25y^2+17z^2\right)}{308}+\frac{\sqrt{231}Q_v yz\left(24x^2+3y^2-11z^2\right)}{308}+\frac{\sqrt{231}Q_{xy}xz\left(18x^2+39y^2-31z^2\right)}{308}$$
$$+\frac{\sqrt{231}Q_{xz}xy\left(18x^2-31y^2+39z^2\right)}{308}-\frac{\sqrt{231}Q_{yz}\left(12x^4-36x^2y^2-36x^2z^2+y^4+30y^2z^2+z^4\right)}{308}$$

$\vec{\mathbb{Q}}_4^{(2,0)}[q](B_g,4)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{2\sqrt{77}Q_u xy\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}Q_v xy\,(x-y)\,(x+y)}{22}-\frac{\sqrt{231}Q_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}$$
$$+\frac{\sqrt{231}Q_{xz}yz\left(39x^2-31y^2+18z^2\right)}{308}-\frac{\sqrt{231}Q_{yz}xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](B_g,1)$
** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{3\sqrt{1001}Q_u yz\left(2x^4+13x^2y^2-17x^2z^2+11y^4-41y^2z^2+14z^4\right)}{572}-\frac{\sqrt{3003}Q_v yz\left(2x^4-47x^2y^2+43x^2z^2+17y^4-41y^2z^2+8z^4\right)}{572}$$
$$-\frac{3\sqrt{3003}Q_{xy}xz\left(3x^2y^2-x^2z^2-8y^4+13y^2z^2-z^4\right)}{286}-\frac{3\sqrt{3003}Q_{xz}xy\left(x^2y^2-3x^2z^2+y^4-13y^2z^2+8z^4\right)}{286}$$
$$+\frac{\sqrt{3003}Q_{yz}\,(y-z)\,(y+z)\left(x^4-x^2y^2-x^2z^2-2y^4+29y^2z^2-2z^4\right)}{286}$$

$\vec{\mathbb{Q}}_4^{(2,2)}[q](B_g,2)$

** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{9\sqrt{1001}Q_{u}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}+\frac{\sqrt{3003}Q_{v}xy\left(25x^{4}-82x^{2}y^{2}-4x^{2}z^{2}+25y^{4}-4y^{2}z^{2}+4z^{4}\right)}{572}$$
$$-\frac{\sqrt{3003}Q_{xy}\left(x-y\right)\left(x+y\right)\left(2x^{4}-29x^{2}y^{2}+x^{2}z^{2}+2y^{4}+y^{2}z^{2}-z^{4}\right)}{286}$$
$$+\frac{3\sqrt{3003}Q_{xz}yz\left(8x^{4}-13x^{2}y^{2}-3x^{2}z^{2}+y^{4}+y^{2}z^{2}\right)}{286}-\frac{3\sqrt{3003}Q_{yz}xz\left(x^{4}-13x^{2}y^{2}+x^{2}z^{2}+8y^{4}-3y^{2}z^{2}\right)}{286}$$

$\vec{\mathbb{Q}}_{4}^{(2,2)}[q](B_{g},3)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

** expression

$$-\frac{21\sqrt{143}Q_{u}yz\left(8x^{4}+7x^{2}y^{2}-23x^{2}z^{2}-y^{4}+y^{2}z^{2}+2z^{4}\right)}{572}+\frac{21\sqrt{429}Q_{v}yz\left(8x^{4}-13x^{2}y^{2}-3x^{2}z^{2}+y^{4}+y^{2}z^{2}\right)}{572}$$
$$-\frac{7\sqrt{429}Q_{xy}xz\left(2x^{4}-23x^{2}y^{2}+x^{2}z^{2}+8y^{4}+7y^{2}z^{2}-z^{4}\right)}{286}-\frac{7\sqrt{429}Q_{xz}xy\left(2x^{4}+x^{2}y^{2}-23x^{2}z^{2}-y^{4}+7y^{2}z^{2}+8z^{4}\right)}{286}$$
$$+\frac{\sqrt{429}Q_{yz}\left(2x^{6}-15x^{4}y^{2}-15x^{4}z^{2}-15x^{2}y^{4}+180x^{2}y^{2}z^{2}-15x^{2}z^{4}+2y^{6}-15y^{4}z^{2}-15y^{2}z^{4}+2z^{6}\right)}{286}$$

$\vec{\mathbb{Q}}_{4}^{(2,2)}[q](B_{g},4)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{21\sqrt{143}Q_{u}xy\left(x^{4}+2x^{2}y^{2}-16x^{2}z^{2}+y^{4}-16y^{2}z^{2}+16z^{4}\right)}{572}-\frac{21\sqrt{429}Q_{v}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}$$
$$+\frac{\sqrt{429}Q_{xy}\left(2x^{6}-15x^{4}y^{2}-15x^{4}z^{2}-15x^{2}y^{4}+180x^{2}y^{2}z^{2}-15x^{2}z^{4}+2y^{6}-15y^{4}z^{2}-15y^{2}z^{4}+2z^{6}\right)}{286}$$
$$-\frac{7\sqrt{429}Q_{xz}yz\left(8x^{4}+7x^{2}y^{2}-23x^{2}z^{2}-y^{4}+y^{2}z^{2}+2z^{4}\right)}{286}+\frac{7\sqrt{429}Q_{yz}xz\left(x^{4}-7x^{2}y^{2}-x^{2}z^{2}-8y^{4}+23y^{2}z^{2}-2z^{4}\right)}{286}$$