PG No. 35 $C_{3v}(1)$ $3m$ (31m setting) [trigonal] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(1,1)}[g](A_2)$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_2)$

** symmetry

z

** expression

G_z

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_2)$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x xz}{10} + \frac{3\sqrt{10}G_y yz}{10} - \frac{\sqrt{10}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](E)$

** symmetry

$-y$

x

** expression

$-G_y$

G_x

$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(1,1)}[g](E)$

** symmetry

$-y$

x

** expression

$$-\frac{3\sqrt{10}G_x xy}{10} + \frac{\sqrt{10}G_y\left(x^2-2y^2+z^2\right)}{10} - \frac{3\sqrt{10}G_z yz}{10}$$

$$\frac{\sqrt{10}G_x\left(2x^2-y^2-z^2\right)}{10} + \frac{3\sqrt{10}G_y xy}{10} + \frac{3\sqrt{10}G_z xz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_x x\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_y y\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_y z}{2}+\frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2}-\frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E,2)$
** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x y}{2}+\frac{\sqrt{2}G_y x}{2}$$

$$\frac{\sqrt{2}G_x x}{2}-\frac{\sqrt{2}G_y y}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7}-\frac{\sqrt{7}G_y z\left(x^2-4y^2+z^2\right)}{7}-\frac{\sqrt{7}G_z y\left(x^2+y^2-4z^2\right)}{7}$$

$$-\frac{\sqrt{7}G_x z\left(4x^2-y^2-z^2\right)}{7}-\frac{5\sqrt{7}G_y xyz}{7}+\frac{\sqrt{7}G_z x\left(x^2+y^2-4z^2\right)}{7}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E,2)$
** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}G_x y\left(4x^2-y^2-z^2\right)}{7}-\frac{\sqrt{7}G_y x\left(x^2-4y^2+z^2\right)}{7}+\frac{5\sqrt{7}G_z xyz}{7}$$

$$\frac{\sqrt{7}G_x x\left(3x^2-7y^2-2z^2\right)}{14}+\frac{\sqrt{7}G_y y\left(7x^2-3y^2+2z^2\right)}{14}+\frac{5\sqrt{7}G_z z\,(x-y)\,(x+y)}{14}$$

* Harmonics for rank 3
$\vec{\mathbb{G}}_{3}^{(1,-1)}[g](A_1)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{6}G_x xy}{2} + \frac{\sqrt{6}G_y\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_1)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{10}G_x xy\left(15x^2-13y^2-6z^2\right)}{24} - \frac{\sqrt{10}G_y\left(3x^4-21x^2y^2+3x^2z^2+4y^4-3y^2z^2\right)}{24} + \frac{7\sqrt{10}G_z yz\left(3x^2-y^2\right)}{24}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xz}{5} - \frac{\sqrt{15}G_y yz}{5} - \frac{\sqrt{15}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{6}G_x\left(x-y\right)\left(x+y\right)}{4} - \frac{\sqrt{6}G_y xy}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{5G_x xz\left(3x^2+3y^2-4z^2\right)}{12} - \frac{5G_y yz\left(3x^2+3y^2-4z^2\right)}{12} + \frac{G_z\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{10}G_x\left(4x^4-21x^2y^2-3x^2z^2+3y^4+3y^2z^2\right)}{24} + \frac{\sqrt{10}G_y xy\left(13x^2-15y^2+6z^2\right)}{24} + \frac{7\sqrt{10}G_z xz\left(x^2-3y^2\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E,1)$
** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{\sqrt{10}G_x xy}{10} + \frac{\sqrt{10}G_y\left(x^2+3y^2-4z^2\right)}{20} - \frac{2\sqrt{10}G_z yz}{5}$$

$$-\frac{\sqrt{10}G_x\left(3x^2+y^2-4z^2\right)}{20} - \frac{\sqrt{10}G_y xy}{10} + \frac{2\sqrt{10}G_z xz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$G_x yz + G_y xz + G_z xy$$

$$G_x xz - G_y yz + \frac{G_z\,(x-y)\,(x+y)}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E,1)$

** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{5\sqrt{6}G_x xy\left(x^2+y^2-6z^2\right)}{24} - \frac{\sqrt{6}G_y\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{24} + \frac{5\sqrt{6}G_z yz\left(3x^2+3y^2-4z^2\right)}{24}$$

$$-\frac{\sqrt{6}G_x\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{24} - \frac{5\sqrt{6}G_y xy\left(x^2+y^2-6z^2\right)}{24} - \frac{5\sqrt{6}G_z xz\left(3x^2+3y^2-4z^2\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{15}G_x yz\left(6x^2-y^2-z^2\right)}{6} - \frac{\sqrt{15}G_y xz\left(x^2-6y^2+z^2\right)}{6} - \frac{\sqrt{15}G_z xy\left(x^2+y^2-6z^2\right)}{6}$$

$$\frac{\sqrt{15}G_x xz\left(5x^2-9y^2-2z^2\right)}{12} + \frac{\sqrt{15}G_y yz\left(9x^2-5y^2+2z^2\right)}{12} - \frac{\sqrt{15}G_z\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_1)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{10}G_x xyz}{4} + \frac{3\sqrt{10}G_y z\,(x-y)\,(x+y)}{8} + \frac{\sqrt{10}G_z y\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_1)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{154}G_x xyz\left(7x^2-5y^2-2z^2\right)}{44} - \frac{3\sqrt{154}G_y z\left(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2\right)}{44} - \frac{\sqrt{154}G_z y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_2,1)$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{3\sqrt{7}G_x x\left(x^2+y^2-4z^2\right)}{28} + \frac{3\sqrt{7}G_y y\left(x^2+y^2-4z^2\right)}{28} - \frac{\sqrt{7}G_z z\left(3x^2+3y^2-2z^2\right)}{7}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{10}G_xz\left(x-y\right)\left(x+y\right)}{8}-\frac{3\sqrt{10}G_yxyz}{4}+\frac{\sqrt{10}G_zx\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{3\sqrt{55}G_xx\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}+\frac{3\sqrt{55}G_yy\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}$$
$$+\frac{\sqrt{55}G_zz\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{88}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{154}G_xz\left(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2\right)}{44}+\frac{3\sqrt{154}G_yxyz\left(5x^2-7y^2+2z^2\right)}{44}-\frac{\sqrt{154}G_zx\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{70}G_xxyz}{28}-\frac{\sqrt{70}G_yz\left(3x^2+9y^2-4z^2\right)}{56}-\frac{3\sqrt{70}G_zy\left(x^2+y^2-4z^2\right)}{56}$$

$$\frac{\sqrt{70}G_xz\left(9x^2+3y^2-4z^2\right)}{56}+\frac{3\sqrt{70}G_yxyz}{28}+\frac{3\sqrt{70}G_zx\left(x^2+y^2-4z^2\right)}{56}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

** expression

$$-\frac{\sqrt{5}G_xy\left(3x^2-y^2\right)}{4}-\frac{\sqrt{5}G_yx\left(x^2-3y^2\right)}{4}$$

$$\frac{\sqrt{5}G_xx\left(x^2-3y^2\right)}{4}-\frac{\sqrt{5}G_yy\left(3x^2-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E,3)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{35}G_x y\left(3x^2+y^2-6z^2\right)}{28}-\frac{\sqrt{35}G_y x\left(x^2+3y^2-6z^2\right)}{28}+\frac{3\sqrt{35}G_z xyz}{7}$$

$$-\frac{\sqrt{35}G_x x\left(x^2-3z^2\right)}{14}+\frac{\sqrt{35}G_y y\left(y^2-3z^2\right)}{14}+\frac{3\sqrt{35}G_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{22}G_x xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{22}G_y z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{44}+\frac{3\sqrt{22}G_z y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$$\frac{\sqrt{22}G_x z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{44}+\frac{21\sqrt{22}G_y xyz\left(x^2+y^2-2z^2\right)}{44}-\frac{3\sqrt{22}G_z x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

** expression

$$-\frac{\sqrt{77}G_x y\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}+\frac{\sqrt{77}G_y x\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}-\frac{9\sqrt{77}G_z xyz\left(x-y\right)\left(x+y\right)}{22}$$

$$\frac{\sqrt{77}G_x x\left(5x^4-46x^2y^2-4x^2z^2+21y^4+12y^2z^2\right)}{88}+\frac{\sqrt{77}G_y y\left(21x^4-46x^2y^2+12x^2z^2+5y^4-4y^2z^2\right)}{88}+\frac{9\sqrt{77}G_z z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E,3)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{11}G_x y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}G_y x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}G_z xyz\left(x^2+y^2-2z^2\right)}{22}$$

$$-\frac{\sqrt{11}G_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$-\frac{\sqrt{11}G_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}-\frac{21\sqrt{11}G_z z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$