PG No. 32 $O_{\rm h}$ $m\bar{3}m$ [cubic] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}^{(1,0)}_{1,1}[q](T_{1g})$, $\vec{\mathbb{G}}^{(1,0)}_{1,2}[q](T_{1g})$, $\vec{\mathbb{G}}^{(1,0)}_{1,3}[q](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$\frac{\sqrt{2}Q_{y}z}{2} - \frac{\sqrt{2}Q_{z}y}{2}$$

$$-\frac{\sqrt{2}Q_{x}z}{2} + \frac{\sqrt{2}Q_{z}x}{2}$$

$$\frac{\sqrt{2}Q_{x}y}{2} - \frac{\sqrt{2}Q_{y}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}^{(1,0)}_{2,1}[q](E_{u})$, $\vec{\mathbb{G}}^{(1,0)}_{2,2}[q](E_{u})$

** symmetry

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{6}Q_{x}yz}{2} - \frac{\sqrt{6}Q_{y}xz}{2}$$

$$\frac{\sqrt{2}Q_{x}yz}{2} + \frac{\sqrt{2}Q_{y}xz}{2} - \sqrt{2}Q_{z}xy$$

$\vec{\mathbb{G}}^{(1,0)}_{2,1}[q](T_{2u})$, $\vec{\mathbb{G}}^{(1,0)}_{2,2}[q](T_{2u})$, $\vec{\mathbb{G}}^{(1,0)}_{2,3}[q](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{x}\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}Q_{y}xy}{2} + \frac{\sqrt{2}Q_{z}xz}{2}$$

$$\frac{\sqrt{2}Q_{x}xy}{2} - \frac{\sqrt{2}Q_{y}\left(x - z\right)\left(x + z\right)}{2} - \frac{\sqrt{2}Q_{z}yz}{2}$$

$$-\frac{\sqrt{2}Q_{x}xz}{2} + \frac{\sqrt{2}Q_{y}yz}{2} + \frac{\sqrt{2}Q_{z}\left(x - y\right)\left(x + y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}^{(1,0)}_{3}[q](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_{x}x\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{5}Q_{y}y\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{5}Q_{z}z\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}^{(1,0)}_{3,1}[q](T_{1g})$, $\vec{\mathbb{G}}^{(1,0)}_{3,2}[q](T_{1g})$, $\vec{\mathbb{G}}^{(1,0)}_{3,3}[q](T_{1g})$

** symmetry

$$\frac{x \left(2 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2}$$

$$- \frac{y \left(3 x^{2} - 2 y^{2} + 3 z^{2}\right)}{2}$$

$$- \frac{z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{2}$$

** expression

$$\frac{\sqrt{3} Q_{y} z \left(4 x^{2} - y^{2} - z^{2}\right)}{4} - \frac{\sqrt{3} Q_{z} y \left(4 x^{2} - y^{2} - z^{2}\right)}{4}$$

$$\frac{\sqrt{3} Q_{x} z \left(x^{2} - 4 y^{2} + z^{2}\right)}{4} - \frac{\sqrt{3} Q_{z} x \left(x^{2} - 4 y^{2} + z^{2}\right)}{4}$$

$$- \frac{\sqrt{3} Q_{x} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{4} + \frac{\sqrt{3} Q_{y} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{4}$$

$\vec{\mathbb{G}}^{(1,0)}_{3,1}[q](T_{2g})$, $\vec{\mathbb{G}}^{(1,0)}_{3,2}[q](T_{2g})$, $\vec{\mathbb{G}}^{(1,0)}_{3,3}[q](T_{2g})$

** symmetry

$$\frac{\sqrt{15} x \left(y - z\right) \left(y + z\right)}{2}$$

$$- \frac{\sqrt{15} y \left(x - z\right) \left(x + z\right)}{2}$$

$$\frac{\sqrt{15} z \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$- \sqrt{5} Q_{x} x y z + \frac{\sqrt{5} Q_{y} z \left(2 x^{2} + y^{2} - z^{2}\right)}{4} + \frac{\sqrt{5} Q_{z} y \left(2 x^{2} - y^{2} + z^{2}\right)}{4}$$

$$\frac{\sqrt{5} Q_{x} z \left(x^{2} + 2 y^{2} - z^{2}\right)}{4} - \sqrt{5} Q_{y} x y z - \frac{\sqrt{5} Q_{z} x \left(x^{2} - 2 y^{2} - z^{2}\right)}{4}$$

$$\frac{\sqrt{5} Q_{x} y \left(x^{2} - y^{2} + 2 z^{2}\right)}{4} - \frac{\sqrt{5} Q_{y} x \left(x^{2} - y^{2} - 2 z^{2}\right)}{4} - \sqrt{5} Q_{z} x y z$$

* Harmonics for rank 4

$\vec{\mathbb{G}}^{(1,0)}_{4}[q](A_{1u})$

** symmetry

$$\frac{\sqrt{21} \left(x^{4} - 3 x^{2} y^{2} - 3 x^{2} z^{2} + y^{4} - 3 y^{2} z^{2} + z^{4}\right)}{6}$$

** expression

$$- \frac{\sqrt{105} Q_{x} y z \left(y - z\right) \left(y + z\right)}{6} + \frac{\sqrt{105} Q_{y} x z \left(x - z\right) \left(x + z\right)}{6} - \frac{\sqrt{105} Q_{z} x y \left(x - y\right) \left(x + y\right)}{6}$$

$\vec{\mathbb{G}}^{(1,0)}_{4,1}[q](E_{u})$, $\vec{\mathbb{G}}^{(1,0)}_{4,2}[q](E_{u})$

** symmetry

$$- \frac{\sqrt{15} \left(x^{4} - 12 x^{2} y^{2} + 6 x^{2} z^{2} + y^{4} + 6 y^{2} z^{2} - 2 z^{4}\right)}{12}$$

$$\frac{\sqrt{5} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{4}$$

** expression

$$- \frac{\sqrt{3} Q_{x} y z \left(9 x^{2} + 2 y^{2} - 5 z^{2}\right)}{6} + \frac{\sqrt{3} Q_{y} x z \left(2 x^{2} + 9 y^{2} - 5 z^{2}\right)}{6} + \frac{7 \sqrt{3} Q_{z} x y \left(x - y\right) \left(x + y\right)}{6}$$

$$- \frac{Q_{x} y z \left(3 x^{2} - 4 y^{2} + 3 z^{2}\right)}{2} + \frac{Q_{y} x z \left(4 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2} - \frac{Q_{z} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

$\vec{\mathbb{G}}^{(1,0)}_{4,1}[q](T_{1u})$, $\vec{\mathbb{G}}^{(1,0)}_{4,2}[q](T_{1u})$, $\vec{\mathbb{G}}^{(1,0)}_{4,3}[q](T_{1u})$

** symmetry

$$\frac{\sqrt{35} y z \left(y - z\right) \left(y + z\right)}{2}$$

$$- \frac{\sqrt{35} x z \left(x - z\right) \left(x + z\right)}{2}$$

$$\frac{\sqrt{35} x y \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{7} Q_{x} \left(y^{2} - 2 y z - z^{2}\right) \left(y^{2} + 2 y z - z^{2}\right)}{4} - \frac{\sqrt{7} Q_{y} x y \left(y^{2} - 3 z^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} x z \left(3 y^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7} Q_{x} x y \left(x^{2} - 3 z^{2}\right)}{4} + \frac{\sqrt{7} Q_{y} \left(x^{2} - 2 x z - z^{2}\right) \left(x^{2} + 2 x z - z^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} y z \left(3 x^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7} Q_{x} x z \left(x^{2} - 3 y^{2}\right)}{4} + \frac{\sqrt{7} Q_{y} y z \left(3 x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7} Q_{z} \left(x^{2} - 2 x y - y^{2}\right) \left(x^{2} + 2 x y - y^{2}\right)}{4}$$

$\vec{\mathbb{G}}^{(1,0)}_{4,1}[q](T_{2u})$, $\vec{\mathbb{G}}^{(1,0)}_{4,2}[q](T_{2u})$, $\vec{\mathbb{G}}^{(1,0)}_{4,3}[q](T_{2u})$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x z \left(x^{2} - 6 y^{2} + z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x} \left(y - z\right) \left(y + z\right) \left(6 x^{2} - y^{2} - z^{2}\right)}{4} - \frac{Q_{y} x y \left(6 x^{2} - y^{2} - 15 z^{2}\right)}{4} + \frac{Q_{z} x z \left(6 x^{2} - 15 y^{2} - z^{2}\right)}{4}$$

$$- \frac{Q_{x} x y \left(x^{2} - 6 y^{2} + 15 z^{2}\right)}{4} + \frac{Q_{y} \left(x - z\right) \left(x + z\right) \left(x^{2} - 6 y^{2} + z^{2}\right)}{4} + \frac{Q_{z} y z \left(15 x^{2} - 6 y^{2} + z^{2}\right)}{4}$$

$$\frac{Q_{x} x z \left(x^{2} + 15 y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{y} y z \left(15 x^{2} + y^{2} - 6 z^{2}\right)}{4} - \frac{Q_{z} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{4}$$