PG No. 32 $O_{\rm h}$ $m\bar{3}m$ [cubic] (axial, internal axial quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_{0}^{(2,2)}[g](A_{1u})$

** symmetry

$$1$$

** expression

$$- \frac{\sqrt{5} G_{u} \left(x^{2} + y^{2} - 2 z^{2}\right)}{10} + \frac{\sqrt{15} G_{v} \left(x - y\right) \left(x + y\right)}{10} + \frac{\sqrt{15} G_{xy} x y}{5} + \frac{\sqrt{15} G_{xz} x z}{5} + \frac{\sqrt{15} G_{yz} y z}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1,1}^{(2,0)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,2}^{(2,0)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,3}^{(2,0)}[g](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$- \frac{\sqrt{10} G_{u} x}{10} + \frac{\sqrt{30} G_{v} x}{10} + \frac{\sqrt{30} G_{xy} y}{10} + \frac{\sqrt{30} G_{xz} z}{10}$$

$$- \frac{\sqrt{10} G_{u} y}{10} - \frac{\sqrt{30} G_{v} y}{10} + \frac{\sqrt{30} G_{xy} x}{10} + \frac{\sqrt{30} G_{yz} z}{10}$$

$$\frac{\sqrt{10} G_{u} z}{5} + \frac{\sqrt{30} G_{xz} x}{10} + \frac{\sqrt{30} G_{yz} y}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(2,2)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,2}^{(2,2)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,3}^{(2,2)}[g](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$- \frac{3 \sqrt{35} G_{u} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{70} + \frac{\sqrt{105} G_{v} x \left(3 x^{2} - 7 y^{2} - 2 z^{2}\right)}{70} + \frac{\sqrt{105} G_{xy} y \left(4 x^{2} - y^{2} - z^{2}\right)}{35} + \frac{\sqrt{105} G_{xz} z \left(4 x^{2} - y^{2} - z^{2}\right)}{35} + \frac{\sqrt{105} G_{yz} x y z}{7}$$

$$- \frac{3 \sqrt{35} G_{u} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{70} + \frac{\sqrt{105} G_{v} y \left(7 x^{2} - 3 y^{2} + 2 z^{2}\right)}{70} - \frac{\sqrt{105} G_{xy} x \left(x^{2} - 4 y^{2} + z^{2}\right)}{35} + \frac{\sqrt{105} G_{xz} x y z}{7} - \frac{\sqrt{105} G_{yz} z \left(x^{2} - 4 y^{2} + z^{2}\right)}{35}$$

$$- \frac{3 \sqrt{35} G_{u} z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{70} + \frac{\sqrt{105} G_{v} z \left(x - y\right) \left(x + y\right)}{14} + \frac{\sqrt{105} G_{xy} x y z}{7} - \frac{\sqrt{105} G_{xz} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{35} - \frac{\sqrt{105} G_{yz} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{35}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E_{u})$, $\vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E_{u})$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$G_{u}$$

$$G_{v}$$

$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E_{u})$, $\vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E_{u})$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{14}G_{u}\left(x^{2}+y^{2}-2z^{2}\right)}{14}-\frac{\sqrt{42}G_{v}\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}G_{xy}xy}{7}+\frac{\sqrt{42}G_{xz}xz}{14}+\frac{\sqrt{42}G_{yz}yz}{14}$$

$$-\frac{\sqrt{42}G_{u}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{14}G_{v}\left(x^{2}+y^{2}-2z^{2}\right)}{14}+\frac{3\sqrt{14}G_{xz}xz}{14}-\frac{3\sqrt{14}G_{yz}yz}{14}$$

$\vec{\mathbb{G}}^{(2,2)}_{2,1}[g](E_u)$, $\vec{\mathbb{G}}^{(2,2)}_{2,2}[g](E_u)$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{14}G_{u}\left(3x^{4}+6x^{2}y^{2}-24x^{2}z^{2}+3y^{4}-24y^{2}z^{2}+8z^{4}\right)}{56}-\frac{5\sqrt{42}G_{v}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{168}-\frac{5\sqrt{42}G_{xy}xy\left(x^{2}+y^{2}-6z^{2}\right)}{84}-\frac{5\sqrt{42}G_{xz}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{84}-\frac{5\sqrt{42}G_{yz}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{84}$$

$$-\frac{5\sqrt{42}G_{u}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{168}+\frac{\sqrt{14}G_{v}\left(19x^{4}-102x^{2}y^{2}-12x^{2}z^{2}+19y^{4}-12y^{2}z^{2}+4z^{4}\right)}{168}+\frac{5\sqrt{14}G_{xy}xy\left(x-y\right)\left(x+y\right)}{12}+\frac{5\sqrt{14}G_{xz}xz\left(5x^{2}-9y^{2}-2z^{2}\right)}{84}+\frac{5\sqrt{14}G_{yz}yz\left(9x^{2}-5y^{2}+2z^{2}\right)}{84}$$

$\vec{\mathbb{G}}^{(2,-2)}_{2,1}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,-2)}_{2,2}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,-2)}_{2,3}[g](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$G_{yz}$$

$$G_{xz}$$

$$G_{xy}$$

$\vec{\mathbb{G}}^{(2,0)}_{2,1}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,0)}_{2,2}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,0)}_{2,3}[g](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{42}G_{u}yz}{14}-\frac{3\sqrt{14}G_{v}yz}{14}+\frac{3\sqrt{14}G_{xy}xz}{14}+\frac{3\sqrt{14}G_{xz}xy}{14}-\frac{\sqrt{14}G_{yz}\left(2x^{2}-y^{2}-z^{2}\right)}{14}$$

$$\frac{\sqrt{42}G_{u}xz}{14}+\frac{3\sqrt{14}G_{v}xz}{14}+\frac{3\sqrt{14}G_{xy}yz}{14}+\frac{\sqrt{14}G_{xz}\left(x^{2}-2y^{2}+z^{2}\right)}{14}+\frac{3\sqrt{14}G_{yz}xy}{14}$$

$$-\frac{\sqrt{42}G_{u}xy}{7}+\frac{\sqrt{14}G_{xy}\left(x^{2}+y^{2}-2z^{2}\right)}{14}+\frac{3\sqrt{14}G_{xz}yz}{14}+\frac{3\sqrt{14}G_{yz}xz}{14}$$

$\vec{\mathbb{G}}^{(2,2)}_{2,1}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,2)}_{2,2}[g](T_{2u})$, $\vec{\mathbb{G}}^{(2,2)}_{2,3}[g](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$- \frac{5\sqrt{42}G_{u}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{84}+\frac{5\sqrt{14}G_{v}yz\left(9x^{2}-5y^{2}+2z^{2}\right)}{84}-\frac{5\sqrt{14}G_{xy}xz\left(x^{2}-6y^{2}+z^{2}\right)}{42} - \frac{5\sqrt{14}G_{xz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{42}+\frac{\sqrt{14}G_{yz}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}-4y^{4}+27y^{2}z^{2}-4z^{4}\right)}{42}$$

$$- \frac{5\sqrt{42}G_{u}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{84}+\frac{5\sqrt{14}G_{v}xz\left(5x^{2}-9y^{2}-2z^{2}\right)}{84}+\frac{5\sqrt{14}G_{xy}yz\left(6x^{2}-y^{2}-z^{2}\right)}{42} - \frac{\sqrt{14}G_{xz}\left(4x^{4}+3x^{2}y^{2}-27x^{2}z^{2}-y^{4}+3y^{2}z^{2}+4z^{4}\right)}{42}-\frac{5\sqrt{14}G_{yz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{42}$$

$$- \frac{5\sqrt{42}G_{u}xy\left(x^{2}+y^{2}-6z^{2}\right)}{84}+\frac{5\sqrt{14}G_{v}xy\left(x-y\right)\left(x+y\right)}{12}-\frac{\sqrt{14}G_{xy}\left(4x^{4}-27x^{2}y^{2}+3x^{2}z^{2}+4y^{4}+3y^{2}z^{2}-z^{4}\right)}{42} + \frac{5\sqrt{14}G_{xz}yz\left(6x^{2}-y^{2}-z^{2}\right)}{42}-\frac{5\sqrt{14}G_{yz}xz\left(x^{2}-6y^{2}+z^{2}\right)}{42}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(2,-2)}[g](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}G_{xy}z}{3}+\frac{\sqrt{3}G_{xz}y}{3}+\frac{\sqrt{3}G_{yz}x}{3}$$

$\vec{\mathbb{G}}_{3}^{(2,0)}[g](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}G_{xy}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{6}+\frac{\sqrt{3}G_{xz}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{6}-\frac{\sqrt{3}G_{yz}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{6}$$

$\vec{\mathbb{G}}_{3}^{(2,2)}[g](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$- \frac{21\sqrt{22}G_{u}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{21\sqrt{66}G_{v}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}G_{xy}z\left(6x^{4}-51x^{2}y^{2}+5x^{2}z^{2}+6y^{4}+5y^{2}z^{2}-z^{4}\right)}{66} - \frac{\sqrt{66}G_{xz}y\left(6x^{4}+5x^{2}y^{2}-51x^{2}z^{2}-y^{4}+5y^{2}z^{2}+6z^{4}\right)}{66}+\frac{\sqrt{66}G_{yz}x\left(x^{4}-5x^{2}y^{2}-5x^{2}z^{2}-6y^{4}+51y^{2}z^{2}-6z^{4}\right)}{66}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(2,-2)}[g](T_{1g})$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$- \frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$- \frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{15}G_{u}x}{10}+\frac{3\sqrt{5}G_{v}x}{10}-\frac{\sqrt{5}G_{xy}y}{5}-\frac{\sqrt{5}G_{xz}z}{5}$$

$$- \frac{\sqrt{15}G_{u}y}{10}-\frac{3\sqrt{5}G_{v}y}{10}-\frac{\sqrt{5}G_{xy}x}{5}-\frac{\sqrt{5}G_{yz}z}{5}$$

$$\frac{\sqrt{15}G_u z}{5} - \frac{\sqrt{5}G_{xz}x}{5} - \frac{\sqrt{5}G_{yz}y}{5}$$

$\vec{\mathbb{G}}^{(2,0)}_{3,1}[g](T_{1g})$, $\vec{\mathbb{G}}^{(2,0)}_{3,2}[g](T_{1g})$, $\vec{\mathbb{G}}^{(2,0)}_{3,3}[g](T_{1g})$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u x\left(4x^2-21y^2+9z^2\right)}{60} + \frac{\sqrt{5}G_v x\left(4x^2-y^2-11z^2\right)}{20} + \frac{\sqrt{5}G_{xy}y\left(4x^2-y^2-z^2\right)}{20} + \frac{\sqrt{5}G_{xz}z\left(4x^2-y^2-z^2\right)}{20} - \sqrt{5}G_{yz}xyz$$

$$\frac{\sqrt{15}G_u y\left(21x^2-4y^2-9z^2\right)}{60} + \frac{\sqrt{5}G_v y\left(x^2-4y^2+11z^2\right)}{20} - \frac{\sqrt{5}G_{xy}x\left(x^2-4y^2+z^2\right)}{20} - \sqrt{5}G_{xz}xyz - \frac{\sqrt{5}G_{yz}z\left(x^2-4y^2+z^2\right)}{20}$$

$$-\frac{\sqrt{15}G_u z\left(3x^2+3y^2-2z^2\right)}{15} - \frac{\sqrt{5}G_v z\left(x-y\right)\left(x+y\right)}{2} - \sqrt{5}G_{xy}xyz - \frac{\sqrt{5}G_{xz}x\left(x^2+y^2-4z^2\right)}{20} - \frac{\sqrt{5}G_{yz}y\left(x^2+y^2-4z^2\right)}{20}$$

$\vec{\mathbb{G}}^{(2,2)}_{3,1}[g](T_{1g})$, $\vec{\mathbb{G}}^{(2,2)}_{3,2}[g](T_{1g})$, $\vec{\mathbb{G}}^{(2,2)}_{3,3}[g](T_{1g})$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{330}G_u x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264} + \frac{\sqrt{110}G_v x\left(4x^4-27x^2y^2-13x^2z^2+11y^4+15y^2z^2+4z^4\right)}{88} + \frac{\sqrt{110}G_{xy}y\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44} + \frac{\sqrt{110}G_{xz}z\left(8x^4-12x^2y^2-12x^2z^2+y^4+2y^2z^2+z^4\right)}{44} + \frac{7\sqrt{110}G_{yz}xyz\left(2x^2-y^2-z^2\right)}{44}$$

$$\frac{\sqrt{330}G_u y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264} - \frac{\sqrt{110}G_v y\left(11x^4-27x^2y^2+15x^2z^2+4y^4-13y^2z^2+4z^4\right)}{88} + \frac{\sqrt{110}G_{xy}x\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44} - \frac{7\sqrt{110}G_{xz}xyz\left(x^2-2y^2+z^2\right)}{44} + \frac{\sqrt{110}G_{yz}z\left(x^4-12x^2y^2+2x^2z^2+8y^4-12y^2z^2+z^4\right)}{44}$$

$$\frac{\sqrt{330}G_u z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{264} - \frac{7\sqrt{110}G_v z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88} - \frac{7\sqrt{110}G_{xy}xyz\left(x^2+y^2-2z^2\right)}{44} + \frac{\sqrt{110}G_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44} + \frac{\sqrt{110}G_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}^{(2,-2)}_{3,1}[g](T_{2g})$, $\vec{\mathbb{G}}^{(2,-2)}_{3,2}[g](T_{2g})$, $\vec{\mathbb{G}}^{(2,-2)}_{3,3}[g](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{G_u x}{2} - \frac{\sqrt{3}G_v x}{6} + \frac{\sqrt{3}G_{xy}y}{3} - \frac{\sqrt{3}G_{xz}z}{3}$$

$$\frac{G_u y}{2} - \frac{\sqrt{3}G_v y}{6} - \frac{\sqrt{3}G_{xy}x}{3} + \frac{\sqrt{3}G_{yz}z}{3}$$

$$\frac{\sqrt{3}G_{v}z}{3}+\frac{\sqrt{3}G_{xz}x}{3}-\frac{\sqrt{3}G_{yz}y}{3}$$

$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(2,0)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(2,0)}[g](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{G_{u}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{4}+\frac{\sqrt{3}G_{v}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{12}+\frac{\sqrt{3}G_{xy}y\left(6x^{2}+y^{2}-9z^{2}\right)}{12}-\frac{\sqrt{3}G_{xz}z\left(6x^{2}-9y^{2}+z^{2}\right)}{12}$$

$$\frac{G_{u}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{4}-\frac{\sqrt{3}G_{v}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{12}-\frac{\sqrt{3}G_{xy}x\left(x^{2}+6y^{2}-9z^{2}\right)}{12}-\frac{\sqrt{3}G_{yz}z\left(9x^{2}-6y^{2}-z^{2}\right)}{12}$$

$$\frac{\sqrt{3}G_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{6}+\frac{\sqrt{3}G_{xz}x\left(x^{2}-9y^{2}+6z^{2}\right)}{12}+\frac{\sqrt{3}G_{yz}y\left(9x^{2}-y^{2}-6z^{2}\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(2,2)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(2,2)}[g](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{22}G_{u}x\left(2x^{4}+11x^{2}y^{2}-31x^{2}z^{2}+9y^{4}-87y^{2}z^{2}+30z^{4}\right)}{88}-\frac{\sqrt{66}G_{v}x\left(2x^{4}-73x^{2}y^{2}+53x^{2}z^{2}+51y^{4}-87y^{2}z^{2}-12z^{4}\right)}{264}-\frac{\sqrt{66}G_{xy}y\left(12x^{4}-46x^{2}y^{2}+66x^{2}z^{2}+5y^{4}-4y^{2}z^{2}-9z^{4}\right)}{132}+\frac{\sqrt{66}G_{xz}z\left(12x^{4}+66x^{2}y^{2}-46x^{2}z^{2}-9y^{4}-4y^{2}z^{2}+5z^{4}\right)}{132}+\frac{21\sqrt{66}G_{yz}xyz\left(y-z\right)\left(y+z\right)}{44}$$

$$\frac{\sqrt{22}G_{u}y\left(9x^{4}+11x^{2}y^{2}-87x^{2}z^{2}+2y^{4}-31y^{2}z^{2}+30z^{4}\right)}{88}-\frac{\sqrt{66}G_{v}y\left(51x^{4}-73x^{2}y^{2}-87x^{2}z^{2}+2y^{4}+53y^{2}z^{2}-12z^{4}\right)}{264}+\frac{\sqrt{66}G_{xy}x\left(5x^{4}-46x^{2}y^{2}-4x^{2}z^{2}+12y^{4}+66y^{2}z^{2}-9z^{4}\right)}{132}-\frac{21\sqrt{66}G_{xz}xyz\left(x-z\right)\left(x+z\right)}{44}+\frac{\sqrt{66}G_{yz}z\left(9x^{4}-66x^{2}y^{2}+4x^{2}z^{2}-12y^{4}+46y^{2}z^{2}-5z^{4}\right)}{132}$$

$$-\frac{21\sqrt{22}G_{u}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{66}G_{v}z\left(39x^{4}-174x^{2}y^{2}-20x^{2}z^{2}+39y^{4}-20y^{2}z^{2}+4z^{4}\right)}{264}+\frac{21\sqrt{66}G_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}G_{xz}x\left(5x^{4}-4x^{2}y^{2}-46x^{2}z^{2}-9y^{4}+66y^{2}z^{2}+12z^{4}\right)}{132}-\frac{\sqrt{66}G_{yz}y\left(9x^{4}+4x^{2}y^{2}-66x^{2}z^{2}-5y^{4}+46y^{2}z^{2}-12z^{4}\right)}{132}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(2,-2)}[g](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{30}G_{u}\left(x^{2}+y^{2}-2z^{2}\right)}{20}+\frac{3\sqrt{10}G_{v}\left(x-y\right)\left(x+y\right)}{20}-\frac{\sqrt{10}G_{xy}xy}{5}-\frac{\sqrt{10}G_{xz}xz}{5}-\frac{\sqrt{10}G_{yz}yz}{5}$$

$\vec{\mathbb{G}}_{4}^{(2,0)}[g](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{165}G_{u}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{66}+\frac{\sqrt{55}G_{v}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{22}+\frac{\sqrt{55}G_{xy}xy\left(x^{2}+y^{2}-6z^{2}\right)}{22}+\frac{\sqrt{55}G_{xz}xz\left(x^{2}-6y^{2}+z^{2}\right)}{22}-\frac{\sqrt{55}G_{yz}yz\left(6x^{2}-y^{2}-z^{2}\right)}{22}$$

$\vec{\mathbb{G}}_{4}^{(2,2)}[g](A_{1u})$
** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$-\frac{\sqrt{15015}G_{u}\left(x^{6}-15x^{4}z^{2}+15x^{2}z^{4}+y^{6}-15y^{4}z^{2}+15y^{2}z^{4}-2z^{6}\right)}{572}+\frac{3\sqrt{5005}G_{v}\left(x-y\right)\left(x+y\right)\left(x^{4}-9x^{2}y^{2}-5x^{2}z^{2}+y^{4}-5y^{2}z^{2}+5z^{4}\right)}{572}+\frac{\sqrt{5005}G_{xy}xy\left(7x^{4}-19x^{2}y^{2}-13x^{2}z^{2}+7y^{4}-13y^{2}z^{2}+13z^{4}\right)}{286}+\frac{\sqrt{5005}G_{xz}xz\left(7x^{4}-13x^{2}y^{2}-19x^{2}z^{2}+13y^{4}-13y^{2}z^{2}+7z^{4}\right)}{286}+\frac{\sqrt{5005}G_{yz}yz\left(13x^{4}-13x^{2}y^{2}-13x^{2}z^{2}+7y^{4}-19y^{2}z^{2}+7z^{4}\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E_{u})$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E_{u})$
** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{42}G_{u}\left(x^{2}+y^{2}-2z^{2}\right)}{28}-\frac{3\sqrt{14}G_{v}\left(x-y\right)\left(x+y\right)}{28}+\frac{2\sqrt{14}G_{xy}xy}{7}-\frac{\sqrt{14}G_{xz}xz}{7}-\frac{\sqrt{14}G_{yz}yz}{7}$$

$$-\frac{3\sqrt{14}G_{u}\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{42}G_{v}\left(x^{2}+y^{2}-2z^{2}\right)}{28}-\frac{\sqrt{42}G_{xz}xz}{7}+\frac{\sqrt{42}G_{yz}yz}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E_{u})$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E_{u})$
** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{231}G_{u}\left(8x^{4}-33x^{2}y^{2}-15x^{2}z^{2}+8y^{4}-15y^{2}z^{2}+5z^{4}\right)}{231}+\frac{\sqrt{77}G_{v}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{77}+\frac{\sqrt{77}G_{xy}xy\left(x^{2}+y^{2}-6z^{2}\right)}{14}-\frac{\sqrt{77}G_{xz}xz\left(16x^{2}-33y^{2}-5z^{2}\right)}{154}+\frac{\sqrt{77}G_{yz}yz\left(33x^{2}-16y^{2}+5z^{2}\right)}{154}$$

$$\frac{\sqrt{77}G_{u}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{77}+\frac{\sqrt{231}G_{v}\left(2x^{4}-3x^{2}y^{2}-9x^{2}z^{2}+2y^{4}-9y^{2}z^{2}+3z^{4}\right)}{77}+\frac{\sqrt{231}G_{xy}xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{231}G_{xz}xz\left(2x^{2}-33y^{2}+9z^{2}\right)}{154}-\frac{\sqrt{231}G_{yz}yz\left(33x^{2}-2y^{2}-9z^{2}\right)}{154}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E_{u})$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E_{u})$
** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{429}G_u\left(x^6+45x^4y^2-60x^4z^2+45x^2y^4-540x^2y^2z^2+150x^2z^4+y^6-60y^4z^2+150y^2z^4-20z^6\right)}{1144}$$
$$-\frac{21\sqrt{143}G_v\left(x-y\right)\left(x+y\right)\left(x^4-20x^2y^2+6x^2z^2+y^4+6y^2z^2-6z^4\right)}{1144}-\frac{7\sqrt{143}G_{xy}xy\left(x^4-4x^2y^2+2x^2z^2+y^4+2y^2z^2-2z^4\right)}{52}$$
$$+\frac{7\sqrt{143}G_{xz}xz\left(x^4+56x^2y^2-22x^2z^2-11y^4-34y^2z^2+10z^4\right)}{572}-\frac{7\sqrt{143}G_{yz}yz\left(11x^4-56x^2y^2+34x^2z^2-y^4+22y^2z^2-10z^4\right)}{572}$$

$$-\frac{21\sqrt{143}G_u\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{\sqrt{429}G_v\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$
$$+\frac{21\sqrt{429}G_{xy}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{7\sqrt{429}G_{xz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{572}$$
$$+\frac{7\sqrt{429}G_{yz}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(2,-2)}[g](T_{1u})$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{2}G_uyz}{4}-\frac{\sqrt{6}G_vyz}{4}+\frac{\sqrt{6}G_{yz}\left(y-z\right)\left(y+z\right)}{4}$$

$$\frac{3\sqrt{2}G_uxz}{4}-\frac{\sqrt{6}G_vxz}{4}-\frac{\sqrt{6}G_{xz}\left(x-z\right)\left(x+z\right)}{4}$$

$$\frac{\sqrt{6}G_vxy}{2}+\frac{\sqrt{6}G_{xy}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(2,0)}[g](T_{1u})$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{11}G_uyz\left(18x^2+11y^2-17z^2\right)}{44}+\frac{\sqrt{33}G_vyz\left(6x^2-15y^2+13z^2\right)}{44}+\frac{7\sqrt{33}G_{xy}xz\left(3y^2-z^2\right)}{44}$$
$$+\frac{7\sqrt{33}G_{xz}xy\left(y^2-3z^2\right)}{44}-\frac{\sqrt{33}G_{yz}\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{44}$$

$$-\frac{\sqrt{11}G_uxz\left(11x^2+18y^2-17z^2\right)}{44}-\frac{\sqrt{33}G_vxz\left(15x^2-6y^2-13z^2\right)}{44}-\frac{7\sqrt{33}G_{xy}yz\left(3x^2-z^2\right)}{44}$$
$$-\frac{\sqrt{33}G_{xz}\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{44}-\frac{7\sqrt{33}G_{yz}xy\left(x^2-3z^2\right)}{44}$$

$$-\frac{7\sqrt{11}G_uxy\left(x-y\right)\left(x+y\right)}{11}+\frac{\sqrt{33}G_vxy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}G_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}G_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}G_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(2,2)}[g](T_{1u})$

** symmetry

$$\frac{\sqrt{35} yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35} xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35} xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{1001} G_{u} yz\left(2x^{4}+13x^{2}y^{2}-17x^{2}z^{2}+11y^{4}-41y^{2}z^{2}+14z^{4}\right)}{572}-\frac{\sqrt{3003} G_{v} yz\left(2x^{4}-47x^{2}y^{2}+43x^{2}z^{2}+17y^{4}-41y^{2}z^{2}+8z^{4}\right)}{572}-\frac{3\sqrt{3003} G_{xy} xz\left(3x^{2}y^{2}-x^{2}z^{2}-8y^{4}+13y^{2}z^{2}-z^{4}\right)}{286}-\frac{3\sqrt{3003} G_{xz} xy\left(x^{2}y^{2}-3x^{2}z^{2}+y^{4}-13y^{2}z^{2}+8z^{4}\right)}{286}+\frac{\sqrt{3003} G_{yz}\left(y-z\right)\left(y+z\right)\left(x^{4}-x^{2}y^{2}-x^{2}z^{2}-2y^{4}+29y^{2}z^{2}-2z^{4}\right)}{286}$$

$$\frac{3\sqrt{1001} G_{u} xz\left(11x^{4}+13x^{2}y^{2}-41x^{2}z^{2}+2y^{4}-17y^{2}z^{2}+14z^{4}\right)}{572}-\frac{\sqrt{3003} G_{v} xz\left(17x^{4}-47x^{2}y^{2}-41x^{2}z^{2}+2y^{4}+43y^{2}z^{2}+8z^{4}\right)}{572}-\frac{3\sqrt{3003} G_{xy} yz\left(8x^{4}-3x^{2}y^{2}-13x^{2}z^{2}+y^{2}z^{2}+z^{4}\right)}{286}+\frac{\sqrt{3003} G_{xz}\left(x-z\right)\left(x+z\right)\left(2x^{4}+x^{2}y^{2}-29x^{2}z^{2}-y^{4}+y^{2}z^{2}+2z^{4}\right)}{286}+\frac{3\sqrt{3003} G_{yz} xy\left(x^{4}+x^{2}y^{2}-13x^{2}z^{2}-3y^{2}z^{2}+8z^{4}\right)}{286}$$

$$-\frac{9\sqrt{1001} G_{u} xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}+\frac{\sqrt{3003} G_{v} xy\left(25x^{4}-82x^{2}y^{2}-4x^{2}z^{2}+25y^{4}-4y^{2}z^{2}+4z^{4}\right)}{572}-\frac{\sqrt{3003} G_{xy}\left(x-y\right)\left(x+y\right)\left(2x^{4}-29x^{2}y^{2}+x^{2}z^{2}+2y^{4}+y^{2}z^{2}-z^{4}\right)}{286}+\frac{3\sqrt{3003} G_{xz} yz\left(8x^{4}-13x^{2}y^{2}-3x^{2}z^{2}+y^{4}+y^{2}z^{2}\right)}{286}-\frac{3\sqrt{3003} G_{yz} xz\left(x^{4}-13x^{2}y^{2}+x^{2}z^{2}+8y^{4}-3y^{2}z^{2}\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(2,-2)}[g](T_{2u})$

** symmetry

$$\frac{\sqrt{5} yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5} xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5} xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{3\sqrt{14} G_{u} yz}{28}+\frac{3\sqrt{42} G_{v} yz}{28}+\frac{\sqrt{42} G_{xy} xz}{7}+\frac{\sqrt{42} G_{xz} xy}{7}+\frac{\sqrt{42} G_{yz}\left(2x^{2}-y^{2}-z^{2}\right)}{28}$$

$$-\frac{3\sqrt{14} G_{u} xz}{28}-\frac{3\sqrt{42} G_{v} xz}{28}+\frac{\sqrt{42} G_{xy} yz}{7}-\frac{\sqrt{42} G_{xz}\left(x^{2}-2y^{2}+z^{2}\right)}{28}+\frac{\sqrt{42} G_{yz} xy}{7}$$

$$\frac{3\sqrt{14} G_{u} xy}{14}-\frac{\sqrt{42} G_{xy}\left(x^{2}+y^{2}-2z^{2}\right)}{28}+\frac{\sqrt{42} G_{xz} yz}{7}+\frac{\sqrt{42} G_{yz} xz}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(2,0)}[g](T_{2u})$

** symmetry

$$\frac{\sqrt{5} yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5} xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5} xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{77} G_{u} yz\left(24x^{2}-25y^{2}+17z^{2}\right)}{308}+\frac{\sqrt{231} G_{v} yz\left(24x^{2}+3y^{2}-11z^{2}\right)}{308}+\frac{\sqrt{231} G_{xy} xz\left(18x^{2}+39y^{2}-31z^{2}\right)}{308}+\frac{\sqrt{231} G_{xz} xy\left(18x^{2}-31y^{2}+39z^{2}\right)}{308}-\frac{\sqrt{231} G_{yz}\left(12x^{4}-36x^{2}y^{2}-36x^{2}z^{2}+y^{4}+30y^{2}z^{2}+z^{4}\right)}{308}$$

$$\frac{\sqrt{77}G_u xz\left(25x^2-24y^2-17z^2\right)}{308}-\frac{\sqrt{231}G_v xz\left(3x^2+24y^2-11z^2\right)}{308}+\frac{\sqrt{231}G_{xy}yz\left(39x^2+18y^2-31z^2\right)}{308}-\frac{\sqrt{231}G_{xz}\left(x^4-36x^2y^2+30x^2z^2+12y^4-36y^2z^2+z^4\right)}{308}-\frac{\sqrt{231}G_{yz}xy\left(31x^2-18y^2-39z^2\right)}{308}$$

$$-\frac{2\sqrt{77}G_u xy\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}G_v xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{231}G_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}+\frac{\sqrt{231}G_{xz}yz\left(39x^2-31y^2+18z^2\right)}{308}-\frac{\sqrt{231}G_{yz}xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(2,2)}[g](T_{2u})$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{21\sqrt{143}G_u yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{572}+\frac{21\sqrt{429}G_v yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{572}-\frac{7\sqrt{429}G_{xy}xz\left(2x^4-23x^2y^2+x^2z^2+8y^4+7y^2z^2-z^4\right)}{286}-\frac{7\sqrt{429}G_{xz}xy\left(2x^4+x^2y^2-23x^2z^2-y^4+7y^2z^2+8z^4\right)}{286}+\frac{\sqrt{429}G_{yz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$

$$\frac{21\sqrt{143}G_u xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}-\frac{21\sqrt{429}G_v xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{572}-\frac{7\sqrt{429}G_{xy}yz\left(8x^4-23x^2y^2+7x^2z^2+2y^4+y^2z^2-z^4\right)}{286}+\frac{\sqrt{429}G_{xz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}+\frac{7\sqrt{429}G_{yz}xy\left(x^4-x^2y^2-7x^2z^2-2y^4+23y^2z^2-8z^4\right)}{286}$$

$$\frac{21\sqrt{143}G_u xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{21\sqrt{429}G_v xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{429}G_{xy}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}-\frac{7\sqrt{429}G_{xz}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{286}+\frac{7\sqrt{429}G_{yz}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{286}$$