# PG No. 32 $O_{\mathrm{h}}$ $m\bar{3}m$ [ cubic ] (axial, internal polar octupole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}^{(3,2)}_{1,1}[q](T_{1g})$, $\vec{\mathbb{G}}^{(3,2)}_{1,2}[q](T_{1g})$, $\vec{\mathbb{G}}^{(3,2)}_{1,3}[q](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$\frac{3\sqrt{70}Q_{1}z\left(x-y\right)\left(x+y\right)}{56}-\frac{3\sqrt{70}Q_{2}xyz}{28}-\frac{\sqrt{105}Q_{3}x\left(y-z\right)\left(y+z\right)}{14}-\frac{5\sqrt{42}Q_{3x}xyz}{28}-\frac{\sqrt{42}Q_{3y}z\left(x^{2}+11y^{2}-4z^{2}\right)}{56}+\frac{3\sqrt{7}Q_{az}y\left(x^{2}+y^{2}-4z^{2}\right)}{28}-\frac{\sqrt{105}Q_{bz}y\left(x^{2}-y^{2}+2z^{2}\right)}{28}$$

$$-\frac{3\sqrt{70}Q_{1}xyz}{28}-\frac{3\sqrt{70}Q_{2}z\left(x-y\right)\left(x+y\right)}{56}+\frac{\sqrt{105}Q_{3}y\left(x-z\right)\left(x+z\right)}{14}+\frac{\sqrt{42}Q_{3x}z\left(11x^{2}+y^{2}-4z^{2}\right)}{56}+\frac{5\sqrt{42}Q_{3y}xyz}{28}-\frac{3\sqrt{7}Q_{az}x\left(x^{2}+y^{2}-4z^{2}\right)}{28}+\frac{\sqrt{105}Q_{bz}x\left(x^{2}-y^{2}-2z^{2}\right)}{28}$$

$$-\frac{3\sqrt{70}Q_{1}x\left(x^{2}-3y^{2}\right)}{56}+\frac{3\sqrt{70}Q_{2}y\left(3x^{2}-y^{2}\right)}{56}-\frac{\sqrt{105}Q_{3}z\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}Q_{3x}y\left(x^{2}+y^{2}-4z^{2}\right)}{56}+\frac{\sqrt{42}Q_{3y}x\left(x^{2}+y^{2}-4z^{2}\right)}{56}+\frac{\sqrt{105}Q_{bz}xyz}{7}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}^{(3,0)}_{2,1}[q](E_{u})$, $\vec{\mathbb{G}}^{(3,0)}_{2,2}[q](E_{u})$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{210}Q_{3}\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{21}Q_{3x}yz}{7}-\frac{\sqrt{21}Q_{3y}xz}{7}+\frac{\sqrt{210}Q_{bz}xy}{14}$$

$$\frac{\sqrt{105}Q_{1}xz}{14}-\frac{\sqrt{105}Q_{2}yz}{14}-\frac{\sqrt{70}Q_{3}\left(x^{2}+y^{2}-2z^{2}\right)}{28}-\frac{3\sqrt{7}Q_{3x}yz}{14}-\frac{3\sqrt{7}Q_{3y}xz}{14}+\frac{\sqrt{42}Q_{az}xy}{14}$$

$\vec{\mathbb{G}}^{(3,2)}_{2,1}[q](E_{u})$, $\vec{\mathbb{G}}^{(3,2)}_{2,2}[q](E_{u})$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{70}Q_{1}xz\left(x^{2}-3y^{2}\right)}{8}+\frac{\sqrt{70}Q_{2}yz\left(3x^{2}-y^{2}\right)}{8}+\frac{\sqrt{105}Q_{3}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{42}-\frac{5\sqrt{42}Q_{3x}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{168}+\frac{5\sqrt{42}Q_{3y}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{168}-\frac{\sqrt{105}Q_{bz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{21}$$

$$\frac{\sqrt{210}Q_{1}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{168}-\frac{\sqrt{210}Q_{2}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{168}+\frac{\sqrt{35}Q_{3}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{42}-\frac{5\sqrt{14}Q_{3x}yz\left(27x^{2}-y^{2}-8z^{2}\right)}{168}+\frac{5\sqrt{14}Q_{3y}xz\left(x^{2}-27y^{2}+8z^{2}\right)}{168}+\frac{5\sqrt{21}Q_{az}xy\left(x^{2}+y^{2}-6z^{2}\right)}{42}-\frac{\sqrt{35}Q_{bz}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}^{(3,0)}_{2,1}[q](T_{2u})$, $\vec{\mathbb{G}}^{(3,0)}_{2,2}[q](T_{2u})$, $\vec{\mathbb{G}}^{(3,0)}_{2,3}[q](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{105}Q_1xy}{14} + \frac{\sqrt{105}Q_2\left(x-y\right)\left(x+y\right)}{28} + \frac{\sqrt{7}Q_{3x}\left(5x^2-y^2-4z^2\right)}{28} + \frac{3\sqrt{7}Q_{3y}xy}{14} + \frac{\sqrt{42}Q_{az}xz}{7}$$

$$-\frac{\sqrt{105}Q_1\left(x-y\right)\left(x+y\right)}{28} + \frac{\sqrt{105}Q_2xy}{14} - \frac{3\sqrt{7}Q_{3x}xy}{14} + \frac{\sqrt{7}Q_{3y}\left(x^2-5y^2+4z^2\right)}{28} - \frac{\sqrt{42}Q_{az}yz}{7}$$

$$-\frac{\sqrt{105}Q_1yz}{14} - \frac{\sqrt{105}Q_2xz}{14} + \frac{3\sqrt{7}Q_{3x}xz}{14} - \frac{3\sqrt{7}Q_{3y}yz}{14} - \frac{\sqrt{42}Q_{az}\left(x-y\right)\left(x+y\right)}{28} + \frac{\sqrt{70}Q_{bz}\left(x^2+y^2-2z^2\right)}{28}$$

$\vec{\mathbb{G}}^{(3,2)}_{2,1}[q](T_{2u})$, $\vec{\mathbb{G}}^{(3,2)}_{2,2}[q](T_{2u})$, $\vec{\mathbb{G}}^{(3,2)}_{2,3}[q](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{210}Q_1xy\left(11x^2-17y^2+18z^2\right)}{168} - \frac{\sqrt{210}Q_2\left(2x^4-21x^2y^2+9x^2z^2+5y^4-9y^2z^2\right)}{168} + \frac{\sqrt{35}Q_3yz\left(y-z\right)\left(y+z\right)}{6} - \frac{5\sqrt{14}Q_{3x}\left(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4\right)}{168} - \frac{5\sqrt{14}Q_{3y}xy\left(x^2+y^2-6z^2\right)}{168} - \frac{5\sqrt{21}Q_{az}xz\left(3x^2+3y^2-4z^2\right)}{84} + \frac{\sqrt{35}Q_{bz}xz\left(x^2+3y^2-2z^2\right)}{12}$$

$$-\frac{\sqrt{210}Q_1\left(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2\right)}{168} + \frac{\sqrt{210}Q_2xy\left(17x^2-11y^2-18z^2\right)}{168} - \frac{\sqrt{35}Q_3xz\left(x-z\right)\left(x+z\right)}{6} + \frac{5\sqrt{14}Q_{3x}xy\left(x^2+y^2-6z^2\right)}{168} + \frac{5\sqrt{14}Q_{3y}\left(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4\right)}{168} + \frac{5\sqrt{21}Q_{az}yz\left(3x^2+3y^2-4z^2\right)}{84} + \frac{\sqrt{35}Q_{bz}yz\left(3x^2+y^2-2z^2\right)}{12}$$

$$-\frac{\sqrt{210}Q_1yz\left(3x^2+3y^2-4z^2\right)}{168} - \frac{\sqrt{210}Q_2xz\left(3x^2+3y^2-4z^2\right)}{168} + \frac{\sqrt{35}Q_3xy\left(x-y\right)\left(x+y\right)}{6} + \frac{5\sqrt{14}Q_{3x}xz\left(13x^2-15y^2-8z^2\right)}{168} + \frac{5\sqrt{14}Q_{3y}yz\left(15x^2-13y^2+8z^2\right)}{168} - \frac{5\sqrt{21}Q_{az}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{84} + \frac{\sqrt{35}Q_{bz}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{84}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}^{(3,-2)}_{3}[q](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{2}Q_1y}{4} + \frac{\sqrt{2}Q_2x}{4} + \frac{\sqrt{30}Q_{3x}x}{12} - \frac{\sqrt{30}Q_{3y}y}{12} - \frac{\sqrt{3}Q_{bz}z}{3}$$

$\vec{\mathbb{G}}^{(3,0)}_{3}[q](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{Q_1y\left(x^2+y^2-4z^2\right)}{4} + \frac{Q_2x\left(x^2+y^2-4z^2\right)}{4} + \frac{\sqrt{15}Q_{3x}x\left(x^2-3y^2\right)}{12} + \frac{\sqrt{15}Q_{3y}y\left(3x^2-y^2\right)}{12} + \frac{\sqrt{10}Q_{az}z\left(x-y\right)\left(x+y\right)}{4} + \frac{\sqrt{6}Q_{bz}z\left(3x^2+3y^2-2z^2\right)}{12}$$

$\vec{\mathbb{G}}^{(3,2)}_{3}[q](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{22}Q_1y\left(25x^4-55x^2y^2+15x^2z^2+4y^4+15y^2z^2-10z^4\right)}{88} - \frac{\sqrt{22}Q_2x\left(4x^4-55x^2y^2+15x^2z^2+25y^4+15y^2z^2-10z^4\right)}{88} - \frac{\sqrt{330}Q_{3x}x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264} - \frac{\sqrt{330}Q_{3y}y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264} - \frac{7\sqrt{55}Q_{az}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44} + \frac{\sqrt{33}Q_{bz}z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{132}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(3,-2)}[q](T_{1g})$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{30}Q_1z}{8}-\frac{\sqrt{2}Q_{3y}z}{8}+\frac{\sqrt{3}Q_{az}y}{4}-\frac{\sqrt{5}Q_{bz}y}{4}$$

$$\frac{\sqrt{30}Q_2z}{8}+\frac{\sqrt{2}Q_{3x}z}{8}-\frac{\sqrt{3}Q_{az}x}{4}-\frac{\sqrt{5}Q_{bz}x}{4}$$

$$\frac{\sqrt{2}Q_{3x}y}{2}-\frac{\sqrt{2}Q_{3y}x}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,0)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(3,0)}[q](T_{1g})$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}Q_1z\left(3x^2-z^2\right)}{12}-\frac{\sqrt{15}Q_2xyz}{4}-\frac{\sqrt{10}Q_3x\left(y-z\right)\left(y+z\right)}{4}-\frac{5Q_{3x}xyz}{4}$$
$$-\frac{Q_{3y}z\left(x^2-4y^2+z^2\right)}{4}+\frac{\sqrt{6}Q_{az}y\left(3x^2-2y^2+3z^2\right)}{12}-\frac{\sqrt{10}Q_{bz}y\left(x-z\right)\left(x+z\right)}{4}$$

$$-\frac{\sqrt{15}Q_1xyz}{4}+\frac{\sqrt{15}Q_2z\left(3y^2-z^2\right)}{12}+\frac{\sqrt{10}Q_3y\left(x-z\right)\left(x+z\right)}{4}-\frac{Q_{3x}z\left(4x^2-y^2-z^2\right)}{4}$$
$$+\frac{5Q_{3y}xyz}{4}+\frac{\sqrt{6}Q_{az}x\left(2x^2-3y^2-3z^2\right)}{12}-\frac{\sqrt{10}Q_{bz}x\left(y-z\right)\left(y+z\right)}{4}$$

$$\frac{\sqrt{15}Q_1x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{15}Q_2y\left(3x^2-y^2\right)}{12}-\frac{\sqrt{10}Q_3z\left(x-y\right)\left(x+y\right)}{4}-\frac{Q_{3x}y\left(x^2+y^2-4z^2\right)}{4}+\frac{Q_{3y}x\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{10}Q_{bz}xyz}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(3,2)}[q](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(3,2)}[q](T_{1g})$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{330}Q_1z\left(30x^4-87x^2y^2-31x^2z^2+9y^4+11y^2z^2+2z^4\right)}{528}-\frac{7\sqrt{330}Q_2xyz\left(2x^2-y^2-z^2\right)}{88}-\frac{7\sqrt{55}Q_3x\left(y-z\right)\left(y+z\right)\left(2x^2-y^2-z^2\right)}{44}$$
$$-\frac{35\sqrt{22}Q_{3x}xyz\left(2x^2-y^2-z^2\right)}{88}-\frac{5\sqrt{22}Q_{3y}z\left(2x^4+39x^2y^2-17x^2z^2-5y^4-3y^2z^2+2z^4\right)}{176}$$
$$+\frac{5\sqrt{33}Q_{az}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{264}-\frac{\sqrt{55}Q_{bz}y\left(10x^4-29x^2y^2+27x^2z^2+3y^4-y^2z^2-4z^4\right)}{88}$$

$$\frac{7\sqrt{330}Q_1xyz\left(x^2-2y^2+z^2\right)}{88}+\frac{\sqrt{330}Q_2z\left(9x^4-87x^2y^2+11x^2z^2+30y^4-31y^2z^2+2z^4\right)}{528}-\frac{7\sqrt{55}Q_3y\left(x-z\right)\left(x+z\right)\left(x^2-2y^2+z^2\right)}{44}$$
$$-\frac{5\sqrt{22}Q_{3x}z\left(5x^4-39x^2y^2+3x^2z^2-2y^4+17y^2z^2-2z^4\right)}{176}-\frac{35\sqrt{22}Q_{3y}xyz\left(x^2-2y^2+z^2\right)}{88}$$
$$+\frac{5\sqrt{33}Q_{az}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{264}-\frac{\sqrt{55}Q_{bz}x\left(3x^4-29x^2y^2-x^2z^2+10y^4+27y^2z^2-4z^4\right)}{88}$$

$$\frac{7\sqrt{330}Q_1x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{528}-\frac{7\sqrt{330}Q_2y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{528}+\frac{7\sqrt{55}Q_3z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{5\sqrt{22}Q_{3x}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{5\sqrt{22}Q_{3y}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{7\sqrt{55}Q_{bz}xyz\left(x^2+y^2-2z^2\right)}{22}$$

$\vec{\mathbb{G}}_{3,1}^{(3,-2)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,-2)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(3,-2)}[q](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{2}Q_1z}{8}+\frac{\sqrt{3}Q_3x}{3}-\frac{\sqrt{30}Q_{3y}z}{24}+\frac{\sqrt{5}Q_{az}y}{4}-\frac{\sqrt{3}Q_{bz}y}{12}$$

$$\frac{3\sqrt{2}Q_2z}{8}+\frac{\sqrt{3}Q_3y}{3}-\frac{\sqrt{30}Q_{3x}z}{24}+\frac{\sqrt{5}Q_{az}x}{4}+\frac{\sqrt{3}Q_{bz}x}{12}$$

$$-\frac{\sqrt{2}Q_1x}{4}+\frac{\sqrt{2}Q_2y}{4}+\frac{\sqrt{3}Q_3z}{3}-\frac{\sqrt{30}Q_{3x}y}{12}-\frac{\sqrt{30}Q_{3y}x}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(3,0)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,0)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(3,0)}[q](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{Q_1z\left(x^2-4y^2+z^2\right)}{4}-\frac{5Q_2xyz}{4}+\frac{\sqrt{6}Q_3x\left(2x^2-3y^2-3z^2\right)}{12}+\frac{\sqrt{15}Q_{3x}xyz}{4}$$
$$+\frac{\sqrt{15}Q_{3y}z\left(3x^2-z^2\right)}{12}-\frac{\sqrt{10}Q_{az}y\left(x-z\right)\left(x+z\right)}{4}-\frac{\sqrt{6}Q_{bz}y\left(3x^2-2y^2+3z^2\right)}{12}$$

$$\frac{5Q_1xyz}{4}+\frac{Q_2z\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{6}Q_3y\left(3x^2-2y^2+3z^2\right)}{12}+\frac{\sqrt{15}Q_{3x}z\left(3y^2-z^2\right)}{12}$$
$$+\frac{\sqrt{15}Q_{3y}xyz}{4}-\frac{\sqrt{10}Q_{az}x\left(y-z\right)\left(y+z\right)}{4}-\frac{\sqrt{6}Q_{bz}x\left(2x^2-3y^2-3z^2\right)}{12}$$

$$-\frac{Q_1x\left(x^2+y^2-4z^2\right)}{4}+\frac{Q_2y\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{6}Q_3z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{\sqrt{15}Q_{3x}y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{15}Q_{3y}x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{10}Q_{az}xyz}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(3,2)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(3,2)}[q](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(3,2)}[q](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{22}Q_{1}z\left(40x^{4}-165x^{2}y^{2}-25x^{2}z^{2}+5y^{4}+45y^{2}z^{2}-2z^{4}\right)}{176}-\frac{35\sqrt{22}Q_{2}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{88}-\frac{\sqrt{33}Q_{3}x\left(8x^{4}-40x^{2}y^{2}-40x^{2}z^{2}+15y^{4}+30y^{2}z^{2}+15z^{4}\right)}{132}+\frac{7\sqrt{330}Q_{3x}xyz\left(2x^{2}-y^{2}-z^{2}\right)}{88}-\frac{\sqrt{330}Q_{3y}z\left(24x^{4}-57x^{2}y^{2}-29x^{2}z^{2}+45y^{4}-71y^{2}z^{2}+10z^{4}\right)}{528}-\frac{\sqrt{55}Q_{az}y\left(4x^{4}+x^{2}y^{2}-27x^{2}z^{2}-3y^{4}+29y^{2}z^{2}-10z^{4}\right)}{88}+\frac{\sqrt{33}Q_{bz}y\left(60x^{4}-55x^{2}y^{2}-195x^{2}z^{2}+11y^{4}-55y^{2}z^{2}+60z^{4}\right)}{264}$$

$$-\frac{35\sqrt{22}Q_{1}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{88}-\frac{\sqrt{22}Q_{2}z\left(5x^{4}-165x^{2}y^{2}+45x^{2}z^{2}+40y^{4}-25y^{2}z^{2}-2z^{4}\right)}{176}-\frac{\sqrt{33}Q_{3}y\left(15x^{4}-40x^{2}y^{2}+30x^{2}z^{2}+8y^{4}-40y^{2}z^{2}+15z^{4}\right)}{132}-\frac{\sqrt{330}Q_{3x}z\left(45x^{4}-57x^{2}y^{2}-71x^{2}z^{2}+24y^{4}-29y^{2}z^{2}+10z^{4}\right)}{528}-\frac{7\sqrt{330}Q_{3y}xyz\left(x^{2}-2y^{2}+z^{2}\right)}{88}+\frac{\sqrt{55}Q_{az}x\left(3x^{4}-x^{2}y^{2}-29x^{2}z^{2}-4y^{4}+27y^{2}z^{2}+10z^{4}\right)}{88}-\frac{\sqrt{33}Q_{bz}x\left(11x^{4}-55x^{2}y^{2}-55x^{2}z^{2}+60y^{4}-195y^{2}z^{2}+60z^{4}\right)}{264}$$

$$-\frac{\sqrt{22}Q_{1}x\left(13x^{4}-100x^{2}y^{2}-30x^{2}z^{2}+55y^{4}-30y^{2}z^{2}+20z^{4}\right)}{176}+\frac{\sqrt{22}Q_{2}y\left(55x^{4}-100x^{2}y^{2}-30x^{2}z^{2}+13y^{4}-30y^{2}z^{2}+20z^{4}\right)}{176}-\frac{\sqrt{33}Q_{3}z\left(15x^{4}+30x^{2}y^{2}-40x^{2}z^{2}+15y^{4}-40y^{2}z^{2}+8z^{4}\right)}{132}+\frac{\sqrt{330}Q_{3x}y\left(15x^{4}+16x^{2}y^{2}-138x^{2}z^{2}+y^{4}-26y^{2}z^{2}+36z^{4}\right)}{528}+\frac{\sqrt{330}Q_{3y}x\left(x^{4}+16x^{2}y^{2}-26x^{2}z^{2}+15y^{4}-138y^{2}z^{2}+36z^{4}\right)}{528}+\frac{7\sqrt{55}Q_{az}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{22}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(3,-2)}[q](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$\frac{\sqrt{6}Q_{1}xz}{4}+\frac{\sqrt{6}Q_{2}yz}{4}+\frac{\sqrt{10}Q_{3x}yz}{4}-\frac{\sqrt{10}Q_{3y}xz}{4}-Q_{bz}xy$$

$\vec{\mathbb{G}}_{4}^{(3,0)}[q](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$\frac{5\sqrt{33}Q_{1}xz\left(5x^{2}-9y^{2}-2z^{2}\right)}{132}-\frac{5\sqrt{33}Q_{2}yz\left(9x^{2}-5y^{2}+2z^{2}\right)}{132}-\frac{\sqrt{55}Q_{3x}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{44}+\frac{\sqrt{55}Q_{3y}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{44}+\frac{7\sqrt{330}Q_{az}xy\left(x-y\right)\left(x+y\right)}{132}-\frac{5\sqrt{22}Q_{bz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{44}$$

$\vec{\mathbb{G}}_{4}^{(3,2)}[q](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+z^{4}\right)}{6}$$

** expression

$$\frac{\sqrt{4290}Q_{1}xz\left(17x^{4}-29x^{2}y^{2}-47x^{2}z^{2}-46y^{4}+121y^{2}z^{2}+2z^{4}\right)}{1144}-\frac{\sqrt{4290}Q_{2}yz\left(46x^{4}+29x^{2}y^{2}-121x^{2}z^{2}-17y^{4}+47y^{2}z^{2}-2z^{4}\right)}{1144}-\frac{3\sqrt{715}Q_{3}\left(x-y\right)\left(x+y\right)\left(x-z\right)\left(x+z\right)\left(y-z\right)\left(y+z\right)}{26}-\frac{5\sqrt{286}Q_{3x}yz\left(46x^{4}-91x^{2}y^{2}-x^{2}z^{2}-5y^{4}+47y^{2}z^{2}-14z^{4}\right)}{1144}-\frac{5\sqrt{286}Q_{3y}xz\left(5x^{4}+91x^{2}y^{2}-47x^{2}z^{2}-46y^{4}+y^{2}z^{2}+14z^{4}\right)}{1144}+\frac{15\sqrt{429}Q_{az}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}-\frac{\sqrt{715}Q_{bz}xy\left(19x^{4}-94x^{2}y^{2}+92x^{2}z^{2}+19y^{4}+92y^{2}z^{2}-92z^{4}\right)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](E_{u})$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](E_{u})$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{210}Q_{1}xz}{20}-\frac{\sqrt{210}Q_{2}yz}{20}+\frac{3\sqrt{35}Q_{3}\left(x-y\right)\left(x+y\right)}{35}+\frac{5\sqrt{14}Q_{3x}yz}{28}-\frac{5\sqrt{14}Q_{3y}xz}{28}+\frac{\sqrt{35}Q_{bz}xy}{35}$$

$$\frac{9\sqrt{70}Q_{1}xz}{140}-\frac{9\sqrt{70}Q_{2}yz}{140}+\frac{\sqrt{105}Q_{3}\left(x^{2}+y^{2}-2z^{2}\right)}{35}+\frac{\sqrt{42}Q_{3x}yz}{28}+\frac{\sqrt{42}Q_{3y}xz}{28}+\frac{3\sqrt{7}Q_{az}xy}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](E_u)$, $\vec{\mathbb{G}}_{4,2}^{(3,0)}[q](E_u)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{1155}Q_{1}xz\left(x^{2}-9y^{2}+2z^{2}\right)}{132}-\frac{\sqrt{1155}Q_{2}yz\left(9x^{2}-y^{2}-2z^{2}\right)}{132}+\frac{3\sqrt{770}Q_{3}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{308}+\frac{\sqrt{77}Q_{3x}yz\left(51x^{2}-47y^{2}+30z^{2}\right)}{308}+\frac{\sqrt{77}Q_{3y}xz\left(47x^{2}-51y^{2}-30z^{2}\right)}{308}-\frac{7\sqrt{462}Q_{az}xy\left(x-y\right)\left(x+y\right)}{132}+\frac{\sqrt{770}Q_{bz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{308}$$

$$\frac{5\sqrt{385}Q_{1}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}-\frac{5\sqrt{385}Q_{2}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}+\frac{\sqrt{2310}Q_{3}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{308}+\frac{\sqrt{231}Q_{3x}yz\left(27x^{2}-y^{2}-8z^{2}\right)}{308}-\frac{\sqrt{231}Q_{3y}xz\left(x^{2}-27y^{2}+8z^{2}\right)}{308}-\frac{13\sqrt{154}Q_{az}xy\left(x^{2}+y^{2}-6z^{2}\right)}{308}-\frac{\sqrt{2310}Q_{bz}xy\left(x-y\right)\left(x+y\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](E_u)$, $\vec{\mathbb{G}}_{4,2}^{(3,2)}[q](E_u)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{6006}Q_{1}xz\left(10x^{4}-25x^{2}y^{2}-25x^{2}z^{2}-35y^{4}+95y^{2}z^{2}-2z^{4}\right)}{1144}-\frac{\sqrt{6006}Q_{2}yz\left(35x^{4}+25x^{2}y^{2}-95x^{2}z^{2}-10y^{4}+25y^{2}z^{2}+2z^{4}\right)}{1144}-\frac{3\sqrt{1001}Q_{3}\left(x-y\right)\left(x+y\right)\left(x^{4}-9x^{2}y^{2}-5x^{2}z^{2}+y^{4}-5y^{2}z^{2}+5z^{4}\right)}{286}+\frac{\sqrt{10010}Q_{3x}yz\left(61x^{4}-61x^{2}y^{2}-61x^{2}z^{2}+10y^{4}-13y^{2}z^{2}+10z^{4}\right)}{1144}-\frac{\sqrt{10010}Q_{3y}xz\left(10x^{4}-61x^{2}y^{2}-13x^{2}z^{2}+61y^{4}-61y^{2}z^{2}+10z^{4}\right)}{1144}-\frac{3\sqrt{15015}Q_{az}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}+\frac{\sqrt{1001}Q_{bz}xy\left(31x^{4}-70x^{2}y^{2}-100x^{2}z^{2}+31y^{4}-100y^{2}z^{2}+100z^{4}\right)}{572}$$

$$\frac{9\sqrt{2002}Q_{1}xz\left(4x^{4}-25x^{2}y^{2}-5x^{2}z^{2}+15y^{4}-5y^{2}z^{2}+2z^{4}\right)}{1144}-\frac{9\sqrt{2002}Q_{2}yz\left(15x^{4}-25x^{2}y^{2}-5x^{2}z^{2}+4y^{4}-5y^{2}z^{2}+2z^{4}\right)}{1144}-\frac{\sqrt{3003}Q_{3}\left(x^{6}-15x^{4}z^{2}+15x^{2}z^{4}+y^{6}-15y^{4}z^{2}+15y^{2}z^{4}-2z^{6}\right)}{286}-\frac{\sqrt{30030}Q_{3x}yz\left(13x^{4}+17x^{2}y^{2}-43x^{2}z^{2}+4y^{4}-19y^{2}z^{2}+10z^{4}\right)}{1144}-\frac{\sqrt{30030}Q_{3y}xz\left(4x^{4}+17x^{2}y^{2}-19x^{2}z^{2}+13y^{4}-43y^{2}z^{2}+10z^{4}\right)}{1144}+\frac{3\sqrt{5005}Q_{az}xy\left(x^{4}+2x^{2}y^{2}-16x^{2}z^{2}+y^{4}-16y^{2}z^{2}+16z^{4}\right)}{572}+\frac{3\sqrt{3003}Q_{bz}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(3,-2)}[q](T_{1u})$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{10}Q_1xy}{40} - \frac{3\sqrt{10}Q_2\left(y-z\right)\left(y+z\right)}{40} - \frac{\sqrt{15}Q_3yz}{5} - \frac{\sqrt{6}Q_{3x}\left(y-z\right)\left(y+z\right)}{8} + \frac{3\sqrt{6}Q_{3y}xy}{8} - \frac{3Q_{az}xz}{4} - \frac{3\sqrt{15}Q_{bz}xz}{20}$$

$$\frac{3\sqrt{10}Q_1\left(x-z\right)\left(x+z\right)}{40} - \frac{3\sqrt{10}Q_2xy}{40} - \frac{\sqrt{15}Q_3xz}{5} + \frac{3\sqrt{6}Q_{3x}xy}{8} - \frac{\sqrt{6}Q_{3y}\left(x-z\right)\left(x+z\right)}{8} - \frac{3Q_{az}yz}{4} + \frac{3\sqrt{15}Q_{bz}yz}{20}$$

$$\frac{3\sqrt{10}Q_1yz}{10} - \frac{3\sqrt{10}Q_2xz}{10} - \frac{\sqrt{15}Q_3xy}{5} + \frac{\sqrt{15}Q_{bz}\left(x-y\right)\left(x+y\right)}{10}$$

$\vec{\mathbb{G}}^{(3,0)}_{4,1}[q](T_{1u})$, $\vec{\mathbb{G}}^{(3,0)}_{4,2}[q](T_{1u})$, $\vec{\mathbb{G}}^{(3,0)}_{4,3}[q](T_{1u})$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{55}Q_1xy\left(x^2-6y^2+15z^2\right)}{44} + \frac{\sqrt{55}Q_2\left(9x^2y^2-9x^2z^2-5y^4+21y^2z^2-2z^4\right)}{88} + \frac{\sqrt{330}Q_3yz\left(6x^2-y^2-z^2\right)}{44} + \frac{\sqrt{33}Q_{3x}\left(15x^2y^2-15x^2z^2+y^4-21y^2z^2+6z^4\right)}{88} - \frac{\sqrt{33}Q_{3y}xy\left(5x^2-2y^2-9z^2\right)}{44} + \frac{\sqrt{22}Q_{az}xz\left(5x^2+12y^2-9z^2\right)}{44} + \frac{\sqrt{330}Q_{bz}xz\left(x^2-6y^2+z^2\right)}{44}$$

$$\frac{\sqrt{55}Q_1\left(5x^4-9x^2y^2-21x^2z^2+9y^2z^2+2z^4\right)}{88} - \frac{\sqrt{55}Q_2xy\left(6x^2-y^2-15z^2\right)}{44} - \frac{\sqrt{330}Q_3xz\left(x^2-6y^2+z^2\right)}{44} + \frac{\sqrt{33}Q_{3x}xy\left(2x^2-5y^2+9z^2\right)}{44} + \frac{\sqrt{33}Q_{3y}\left(x^4+15x^2y^2-21x^2z^2-15y^2z^2+6z^4\right)}{88} + \frac{\sqrt{22}Q_{az}yz\left(12x^2+5y^2-9z^2\right)}{44} + \frac{\sqrt{330}Q_{bz}yz\left(6x^2-y^2-z^2\right)}{44}$$

$$\frac{\sqrt{55}Q_1yz\left(3x^2+3y^2-4z^2\right)}{44} - \frac{\sqrt{55}Q_2xz\left(3x^2+3y^2-4z^2\right)}{44} - \frac{\sqrt{330}Q_3xy\left(x^2+y^2-6z^2\right)}{44} + \frac{7\sqrt{33}Q_{3x}xz\left(x^2-3y^2\right)}{44} - \frac{7\sqrt{33}Q_{3y}yz\left(3x^2-y^2\right)}{44} - \frac{7\sqrt{22}Q_{az}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88} + \frac{\sqrt{330}Q_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{88}$$

$\vec{\mathbb{G}}^{(3,2)}_{4,1}[q](T_{1u})$, $\vec{\mathbb{G}}^{(3,2)}_{4,2}[q](T_{1u})$, $\vec{\mathbb{G}}^{(3,2)}_{4,3}[q](T_{1u})$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{286}Q_1xy\left(2x^4-65x^2y^2+175x^2z^2+65y^4-455y^2z^2+140z^4\right)}{2288} - \frac{3\sqrt{286}Q_2\left(20x^4y^2-20x^4z^2-95x^2y^4+450x^2y^2z^2-55x^2z^4+17y^6-160y^4z^2+85y^2z^4-2z^6\right)}{2288} - \frac{\sqrt{429}Q_3yz\left(40x^4-40x^2y^2-40x^2z^2-47y^4+170y^2z^2-47z^4\right)}{572} - \frac{\sqrt{4290}Q_{3x}\left(20x^4y^2-20x^4z^2+25x^2y^4-270x^2y^2z^2+65x^2z^4+5y^6-100y^4z^2+145y^2z^4-14z^6\right)}{2288} + \frac{3\sqrt{4290}Q_{3y}xy\left(2x^4-x^2y^2-17x^2z^2-3y^4+33y^2z^2-8z^4\right)}{2288} - \frac{3\sqrt{715}Q_{az}xz\left(2x^4+31x^2y^2-17x^2z^2+29y^4-89y^2z^2+14z^4\right)}{1144} - \frac{3\sqrt{429}Q_{bz}xz\left(2x^4-65x^2y^2+15x^2z^2-45y^4+155y^2z^2-20z^4\right)}{1144}$$

$$\frac{3\sqrt{286}Q_1\left(17x^6-95x^4y^2-160x^4z^2+20x^2y^4+450x^2y^2z^2+85x^2z^4-20y^4z^2-55y^2z^4-2z^6\right)}{2288} - \frac{3\sqrt{286}Q_2xy\left(65x^4-65x^2y^2-455x^2z^2+2y^4+175y^2z^2+140z^4\right)}{2288} + \frac{\sqrt{429}Q_3xz\left(47x^4+40x^2y^2-170x^2z^2-40y^4+40y^2z^2+47z^4\right)}{572} - \frac{3\sqrt{4290}Q_{3x}xy\left(3x^4+x^2y^2-33x^2z^2-2y^4+17y^2z^2+8z^4\right)}{2288} - \frac{\sqrt{4290}Q_{3y}\left(5x^6+25x^4y^2-100x^4z^2+20x^2y^4-270x^2y^2z^2+145x^2z^4-20y^4z^2+65y^2z^4-14z^6\right)}{2288} - \frac{3\sqrt{715}Q_{az}yz\left(29x^4+31x^2y^2-89x^2z^2+2y^4-17y^2z^2+14z^4\right)}{1144} - \frac{3\sqrt{429}Q_{bz}yz\left(45x^4+65x^2y^2-155x^2z^2-2y^4-15y^2z^2+20z^4\right)}{1144}$$

$$\frac{3\sqrt{286}Q_{1}yz\left(5x^{4}+10x^{2}y^{2}-20x^{2}z^{2}+5y^{4}-20y^{2}z^{2}+8z^{4}\right)}{2288}-\frac{3\sqrt{286}Q_{2}xz\left(5x^{4}+10x^{2}y^{2}-20x^{2}z^{2}+5y^{4}-20y^{2}z^{2}+8z^{4}\right)}{2288}+\frac{\sqrt{429}Q_{3}xy\left(47x^{4}-170x^{2}y^{2}+40x^{2}z^{2}+47y^{4}+40y^{2}z^{2}-40z^{4}\right)}{572}+\frac{3\sqrt{4290}Q_{3x}xz\left(17x^{4}-122x^{2}y^{2}-16x^{2}z^{2}+37y^{4}+48y^{2}z^{2}\right)}{2288}+\frac{3\sqrt{4290}Q_{3y}yz\left(37x^{4}-122x^{2}y^{2}+48x^{2}z^{2}+17y^{4}-16y^{2}z^{2}\right)}{2288}-\frac{9\sqrt{715}Q_{az}\left(x^{2}+y^{2}-10z^{2}\right)\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{1144}+\frac{\sqrt{429}Q_{bz}\left(x-y\right)\left(x+y\right)\left(19x^{4}-226x^{2}y^{2}-40x^{2}z^{2}+19y^{4}-40y^{2}z^{2}+40z^{4}\right)}{1144}$$

$\vec{\mathbb{G}}_{4,1}^{(3,-2)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(3,-2)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(3,-2)}[q](T_{2u})$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{3\sqrt{70}Q_{1}xy}{280}+\frac{3\sqrt{70}Q_{2}\left(4x^{2}+3y^{2}-7z^{2}\right)}{280}+\frac{\sqrt{42}Q_{3x}\left(4x^{2}-5y^{2}+z^{2}\right)}{56}-\frac{5\sqrt{42}Q_{3y}xy}{56}-\frac{3\sqrt{7}Q_{az}xz}{28}-\frac{\sqrt{105}Q_{bz}xz}{20}$$

$$\frac{3\sqrt{70}Q_{1}\left(3x^{2}+4y^{2}-7z^{2}\right)}{280}+\frac{3\sqrt{70}Q_{2}xy}{280}+\frac{5\sqrt{42}Q_{3x}xy}{56}+\frac{\sqrt{42}Q_{3y}\left(5x^{2}-4y^{2}-z^{2}\right)}{56}+\frac{3\sqrt{7}Q_{az}yz}{28}-\frac{\sqrt{105}Q_{bz}yz}{20}$$

$$\frac{9\sqrt{70}Q_{1}yz}{140}+\frac{9\sqrt{70}Q_{2}xz}{140}+\frac{\sqrt{42}Q_{3x}xz}{28}-\frac{\sqrt{42}Q_{3y}yz}{28}+\frac{3\sqrt{7}Q_{az}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}Q_{bz}\left(x^{2}+y^{2}-2z^{2}\right)}{35}$$

$\vec{\mathbb{G}}_{4,1}^{(3,0)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(3,0)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(3,0)}[q](T_{2u})$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{385}Q_{1}xy\left(x^{2}-6y^{2}+15z^{2}\right)}{308}+\frac{\sqrt{385}Q_{2}\left(6x^{4}+21x^{2}y^{2}-57x^{2}z^{2}+y^{4}-27y^{2}z^{2}+14z^{4}\right)}{616}-\frac{\sqrt{2310}Q_{3}yz\left(y-z\right)\left(y+z\right)}{44}+\frac{\sqrt{231}Q_{3x}\left(10x^{4}-69x^{2}y^{2}+9x^{2}z^{2}+19y^{4}-45y^{2}z^{2}+6z^{4}\right)}{616}+\frac{\sqrt{231}Q_{3y}xy\left(27x^{2}-22y^{2}-15z^{2}\right)}{308}+\frac{\sqrt{154}Q_{az}xz\left(19x^{2}-30y^{2}-9z^{2}\right)}{308}+\frac{\sqrt{2310}Q_{bz}xz\left(x-z\right)\left(x+z\right)}{44}$$

$$\frac{\sqrt{385}Q_{1}\left(x^{4}+21x^{2}y^{2}-27x^{2}z^{2}+6y^{4}-57y^{2}z^{2}+14z^{4}\right)}{616}+\frac{\sqrt{385}Q_{2}xy\left(6x^{2}-y^{2}-15z^{2}\right)}{308}+\frac{\sqrt{2310}Q_{3}xz\left(x-z\right)\left(x+z\right)}{44}+\frac{\sqrt{231}Q_{3x}xy\left(22x^{2}-27y^{2}+15z^{2}\right)}{308}-\frac{\sqrt{231}Q_{3y}\left(19x^{4}-69x^{2}y^{2}-45x^{2}z^{2}+10y^{4}+9y^{2}z^{2}+6z^{4}\right)}{616}+\frac{\sqrt{154}Q_{az}yz\left(30x^{2}-19y^{2}+9z^{2}\right)}{308}+\frac{\sqrt{2310}Q_{bz}yz\left(y-z\right)\left(y+z\right)}{44}$$

$$\frac{5\sqrt{385}Q_{1}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}+\frac{5\sqrt{385}Q_{2}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{308}-\frac{\sqrt{2310}Q_{3}xy\left(x-y\right)\left(x+y\right)}{44}+\frac{\sqrt{231}Q_{3x}xz\left(13x^{2}-15y^{2}-8z^{2}\right)}{308}+\frac{\sqrt{231}Q_{3y}yz\left(15x^{2}-13y^{2}+8z^{2}\right)}{308}-\frac{13\sqrt{154}Q_{az}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{616}-\frac{\sqrt{2310}Q_{bz}\left(5x^{4}-18x^{2}y^{2}-12x^{2}z^{2}+5y^{4}-12y^{2}z^{2}+4z^{4}\right)}{616}$$

$\vec{\mathbb{G}}_{4,1}^{(3,2)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(3,2)}[q](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(3,2)}[q](T_{2u})$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{3\sqrt{2002}Q_1xy\left(28x^4-85x^2y^2-25x^2z^2+19y^4+65y^2z^2-20z^4\right)}{2288} - \frac{3\sqrt{2002}Q_2\left(4x^6-70x^4y^2+10x^4z^2+55x^2y^4+90x^2y^2z^2-25x^2z^4-3y^6-10y^4z^2-5y^2z^4+2z^6\right)}{2288} - \frac{3\sqrt{3003}Q_3yz\left(y-z\right)\left(y+z\right)\left(10x^2-y^2-z^2\right)}{572} - \frac{\sqrt{30030}Q_{3x}\left(4x^6-6x^4y^2-54x^4z^2-9x^2y^4+90x^2y^2z^2+39x^2z^4+y^6-6y^4z^2-9y^2z^4-2z^6\right)}{2288} - \frac{\sqrt{30030}Q_{3y}xy\left(4x^4-7x^2y^2-19x^2z^2-11y^4+131y^2z^2-56z^4\right)}{2288} - \frac{3\sqrt{5005}Q_{az}xz\left(8x^4-11x^2y^2-23x^2z^2-19y^4+49y^2z^2+2z^4\right)}{1144} + \frac{\sqrt{3003}Q_{bz}xz\left(16x^4+5x^2y^2-55x^2z^2+55y^4-115y^2z^2+28z^4\right)}{1144}$$

$$\frac{3\sqrt{2002}Q_1\left(3x^6-55x^4y^2+10x^4z^2+70x^2y^4-90x^2y^2z^2+5x^2z^4-4y^6-10y^4z^2+25y^2z^4-2z^6\right)}{2288} - \frac{3\sqrt{2002}Q_2xy\left(19x^4-85x^2y^2+65x^2z^2+28y^4-25y^2z^2-20z^4\right)}{2288} - \frac{3\sqrt{3003}Q_3xz\left(x-z\right)\left(x+z\right)\left(x^2-10y^2+z^2\right)}{572} - \frac{\sqrt{30030}Q_{3x}xy\left(11x^4+7x^2y^2-131x^2z^2-4y^4+19y^2z^2+56z^4\right)}{2288} + \frac{\sqrt{30030}Q_{3y}\left(x^6-9x^4y^2-6x^4z^2-6x^2y^4+90x^2y^2z^2-9x^2z^4+4y^6-54y^4z^2+39y^2z^4-2z^6\right)}{2288} - \frac{3\sqrt{5005}Q_{az}yz\left(19x^4+11x^2y^2-49x^2z^2-8y^4+23y^2z^2-2z^4\right)}{1144} + \frac{\sqrt{3003}Q_{bz}yz\left(55x^4+5x^2y^2-115x^2z^2+16y^4-55y^2z^2+28z^4\right)}{1144}$$

$$-\frac{9\sqrt{2002}Q_1yz\left(25x^4-60x^2y^2+10x^2z^2+3y^4+10y^2z^2-4z^4\right)}{2288} - \frac{9\sqrt{2002}Q_2xz\left(3x^4-60x^2y^2+10x^2z^2+25y^4+10y^2z^2-4z^4\right)}{2288} + \frac{3\sqrt{3003}Q_3xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572} - \frac{\sqrt{30030}Q_{3x}xz\left(17x^4+16x^2y^2-62x^2z^2-y^4-14y^2z^2+20z^4\right)}{2288} - \frac{\sqrt{30030}Q_{3y}yz\left(x^4-16x^2y^2+14x^2z^2-17y^4+62y^2z^2-20z^4\right)}{2288} + \frac{3\sqrt{5005}Q_{az}\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144} - \frac{\sqrt{3003}Q_{bz}\left(x^6+15x^4y^2-30x^4z^2+15x^2y^4-180x^2y^2z^2+60x^2z^4+y^6-30y^4z^2+60y^2z^4-8z^6\right)}{1144}$$