PG No. 35 $C_{3\mathrm{v}}(1)$ $3m$ (31m setting) [trigonal] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,0)}[q](A_2)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E)$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E)$

** symmetry

$$-y$$

$$x$$

** expression

$$\frac{\sqrt{2}Q_x z}{2} - \frac{\sqrt{2}Q_z x}{2}$$

$$\frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_x yz}{2} - \frac{\sqrt{6}Q_y xz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_x\left(y-z\right)\left(y+z\right)}{2} - \frac{\sqrt{2}Q_y xy}{2} + \frac{\sqrt{2}Q_z xz}{2}$$

$$-\frac{\sqrt{2}Q_x xy}{2} + \frac{\sqrt{2}Q_y\left(x-z\right)\left(x+z\right)}{2} + \frac{\sqrt{2}Q_z yz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E,2)$

** symmetry

$$\sqrt{3}xy$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{2}Q_x xz}{2} + \frac{\sqrt{2}Q_y yz}{2} + \frac{\sqrt{2}Q_z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{2}Q_x yz}{2} + \frac{\sqrt{2}Q_y xz}{2} - \sqrt{2}Q_z xy$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_x z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}Q_y xyz}{4}+\frac{\sqrt{30}Q_z x\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A_2,2)$
** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{30}Q_x xyz}{4}+\frac{\sqrt{30}Q_y z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}Q_z y\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E,1)$
** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_x z\left(x^2+11y^2-4z^2\right)}{8}+\frac{5\sqrt{2}Q_y xyz}{4}+\frac{\sqrt{2}Q_z x\left(x^2+y^2-4z^2\right)}{8}$$

$$\frac{5\sqrt{2}Q_x xyz}{4}-\frac{\sqrt{2}Q_y z\left(11x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{2}Q_z y\left(x^2+y^2-4z^2\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E,2)$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_z z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{5}Q_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_z xyz$$

* Harmonics for rank 4
$\vec{\mathbb{G}}_4^{(1,0)}[q](A_1)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x\left(3x^2y^2-3x^2z^2-y^4+3y^2z^2\right)}{8}-\frac{\sqrt{14}Q_y xy\left(3x^2-y^2-6z^2\right)}{8}+\frac{3\sqrt{14}Q_z xz\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_2,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{\sqrt{5}Q_{x}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{5}Q_{y}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A_2,2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_{x}xy\left(x^{2}-3y^{2}+6z^{2}\right)}{8}-\frac{\sqrt{14}Q_{y}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+3y^{2}z^{2}\right)}{8}-\frac{3\sqrt{14}Q_{z}yz\left(3x^{2}-y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_{x}\left(3x^{2}y^{2}-3x^{2}z^{2}+3y^{4}-21y^{2}z^{2}+4z^{4}\right)}{8}+\frac{3\sqrt{2}Q_{y}xy\left(x^{2}+y^{2}-6z^{2}\right)}{8}-\frac{\sqrt{2}Q_{z}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{8}$$

$$\frac{3\sqrt{2}Q_{x}xy\left(x^{2}+y^{2}-6z^{2}\right)}{8}-\frac{\sqrt{2}Q_{y}\left(3x^{4}+3x^{2}y^{2}-21x^{2}z^{2}-3y^{2}z^{2}+4z^{4}\right)}{8}-\frac{\sqrt{2}Q_{z}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E,2)$
** symmetry

$$-\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{35}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{8}$$

** expression

$$\frac{\sqrt{7}Q_{x}xz\left(x^{2}-3y^{2}\right)}{4}-\frac{\sqrt{7}Q_{y}yz\left(3x^{2}-y^{2}\right)}{4}-\frac{\sqrt{7}Q_{z}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{4}$$

$$\frac{\sqrt{7}Q_{x}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}Q_{y}xz\left(x^{2}-3y^{2}\right)}{4}-\sqrt{7}Q_{z}xy\left(x-y\right)\left(x+y\right)$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E,3)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$\frac{Q_{x}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{4}-\frac{Q_{y}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{4}-\frac{Q_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$$\frac{Q_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}-\frac{Q_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}+\frac{Q_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$