PG No. 5 $C_{2\mathrm{h}}$ $2/m$ (b-axis setting) [monoclinic] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,0)}[q](A_g)$

** symmetry

y

** expression

$$-\frac{\sqrt{2}Q_x z}{2}+\frac{\sqrt{2}Q_z x}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B_g,1)$

** symmetry

x

** expression

$$\frac{\sqrt{2}Q_y z}{2}-\frac{\sqrt{2}Q_z y}{2}$$

$\vec{\mathbb{G}}_1^{(1,0)}[q](B_g,2)$

** symmetry

z

** expression

$$\frac{\sqrt{2}Q_x y}{2}-\frac{\sqrt{2}Q_y x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_u,1)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\frac{\sqrt{6}Q_x yz}{2}-\frac{\sqrt{6}Q_y xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_u,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x yz}{2}+\frac{\sqrt{2}Q_y xz}{2}-\sqrt{2}Q_z xy$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A_u,3)$

** symmetry

$\sqrt{3}xz$

** expression

$$\frac{\sqrt{2}Q_x xy}{2}-\frac{\sqrt{2}Q_y\,(x-z)\,(x+z)}{2}-\frac{\sqrt{2}Q_z yz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B_u,1)$

** symmetry

$\sqrt{3}yz$

** expression

$$\frac{\sqrt{2}Q_x\,(y-z)\,(y+z)}{2}-\frac{\sqrt{2}Q_y xy}{2}+\frac{\sqrt{2}Q_z xz}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B_u,2)$

** symmetry

$\sqrt{3}xy$

** expression

$$-\frac{\sqrt{2}Q_x xz}{2}+\frac{\sqrt{2}Q_y yz}{2}+\frac{\sqrt{2}Q_z\left(x-y\right)\left(x+y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,0)}[q](A_g,1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_z z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A_g,2)$

** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_x z\left(x^2-4y^2+z^2\right)}{4}-\frac{\sqrt{3}Q_z x\left(x^2-4y^2+z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A_g,3)$

** symmetry

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x z\left(x^2+2y^2-z^2\right)}{4}-\sqrt{5}Q_y xyz-\frac{\sqrt{5}Q_z x\left(x^2-2y^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_y z\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{3}Q_z y\left(4x^2-y^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,3)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$-\sqrt{5}Q_x xyz+\frac{\sqrt{5}Q_y z\left(2x^2+y^2-z^2\right)}{4}+\frac{\sqrt{5}Q_z y\left(2x^2-y^2+z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,4)$

** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_z xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u, 1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_x yz\,(y-z)\,(y+z)}{6} + \frac{\sqrt{105}Q_y xz\,(x-z)\,(x+z)}{6} - \frac{\sqrt{105}Q_z xy\,(x-y)\,(x+y)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u, 2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{3}Q_x yz\left(9x^2 + 2y^2 - 5z^2\right)}{6} + \frac{\sqrt{3}Q_y xz\left(2x^2 + 9y^2 - 5z^2\right)}{6} + \frac{7\sqrt{3}Q_z xy\,(x-y)\,(x+y)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u, 3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$-\frac{Q_x yz\left(3x^2 - 4y^2 + 3z^2\right)}{2} + \frac{Q_y xz\left(4x^2 - 3y^2 - 3z^2\right)}{2} - \frac{Q_z xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u, 4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_x xy\left(x^2 - 3z^2\right)}{4} + \frac{\sqrt{7}Q_y\left(x^2 - 2xz - z^2\right)\left(x^2 + 2xz - z^2\right)}{4} + \frac{\sqrt{7}Q_z yz\left(3x^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u, 5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$-\frac{Q_x xy\left(x^2 - 6y^2 + 15z^2\right)}{4} + \frac{Q_y\,(x-z)\,(x+z)\left(x^2 - 6y^2 + z^2\right)}{4} + \frac{Q_z yz\left(15x^2 - 6y^2 + z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u, 1)$

** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x\left(y^2 - 2yz - z^2\right)\left(y^2 + 2yz - z^2\right)}{4} - \frac{\sqrt{7}Q_y xy\left(y^2 - 3z^2\right)}{4} + \frac{\sqrt{7}Q_z xz\left(3y^2 - z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u, 2)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_x xz\left(x^2 - 3y^2\right)}{4} + \frac{\sqrt{7}Q_y yz\left(3x^2 - y^2\right)}{4} + \frac{\sqrt{7}Q_z\left(x^2 - 2xy - y^2\right)\left(x^2 + 2xy - y^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u, 3)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2 - y^2 - z^2\right)}{2}$$

** expression

$$\frac{Q_x\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{4}-\frac{Q_yxy\left(6x^2-y^2-15z^2\right)}{4}+\frac{Q_zxz\left(6x^2-15y^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u,4)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{Q_xxz\left(x^2+15y^2-6z^2\right)}{4}-\frac{Q_yyz\left(15x^2+y^2-6z^2\right)}{4}-\frac{Q_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$