

PG No. 37 $D_{3h}(1)$ $\bar{6}m2$ (-62m setting) [hexagonal] (axial, internal axial quadrupole)

* Harmonics for rank 0

$$\bar{G}_0^{(2,2)}[g](A_1'')$$

** symmetry

1

** expression

$$-\frac{\sqrt{5}G_u(x^2 + y^2 - 2z^2)}{10} + \frac{\sqrt{15}G_v(x - y)(x + y)}{10} + \frac{\sqrt{15}G_{xy}xy}{5} + \frac{\sqrt{15}G_{xz}xz}{5} + \frac{\sqrt{15}G_{yz}yz}{5}$$

* Harmonics for rank 1

$$\bar{G}_1^{(2,0)}[g](A_2')$$

** symmetry

z

** expression

$$\frac{\sqrt{10}G_u z}{5} + \frac{\sqrt{30}G_{xz}x}{10} + \frac{\sqrt{30}G_{yz}y}{10}$$

$$\bar{G}_1^{(2,2)}[g](A_2')$$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}G_u z(3x^2 + 3y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_v z(x - y)(x + y)}{14} + \frac{\sqrt{105}G_{xy}xyz}{7} - \frac{\sqrt{105}G_{xz}x(x^2 + y^2 - 4z^2)}{35} - \frac{\sqrt{105}G_{yz}y(x^2 + y^2 - 4z^2)}{35}$$

$$\bar{G}_{1,1}^{(2,0)}[g](E''), \bar{G}_{1,2}^{(2,0)}[g](E'')$$

** symmetry

x

y

** expression

$$-\frac{\sqrt{10}G_u x}{10} + \frac{\sqrt{30}G_v x}{10} + \frac{\sqrt{30}G_{xy}y}{10} + \frac{\sqrt{30}G_{xz}z}{10}$$

$$-\frac{\sqrt{10}G_u y}{10} - \frac{\sqrt{30}G_v y}{10} + \frac{\sqrt{30}G_{xy}x}{10} + \frac{\sqrt{30}G_{yz}z}{10}$$

$$\bar{G}_{1,1}^{(2,2)}[g](E''), \bar{G}_{1,2}^{(2,2)}[g](E'')$$

** symmetry

x

y

** expression

$$-\frac{3\sqrt{35}G_u x(x^2 + y^2 - 4z^2)}{70} + \frac{\sqrt{105}G_v x(3x^2 - 7y^2 - 2z^2)}{70} + \frac{\sqrt{105}G_{xy}y(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{xz}z(4x^2 - y^2 - z^2)}{35} + \frac{\sqrt{105}G_{yz}xyz}{7}$$

$$-\frac{3\sqrt{35}G_u y(x^2 + y^2 - 4z^2)}{70} + \frac{\sqrt{105}G_v y(7x^2 - 3y^2 + 2z^2)}{70} - \frac{\sqrt{105}G_{xy}x(x^2 - 4y^2 + z^2)}{35} + \frac{\sqrt{105}G_{xz}xyz}{7} - \frac{\sqrt{105}G_{yz}z(x^2 - 4y^2 + z^2)}{35}$$

* Harmonics for rank 2

$$\bar{G}_2^{(2,-2)}[g](A_1'')$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$G_u$$

$$\bar{G}_2^{(2,0)}[g](A_1'')$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{14}G_u(x^2+y^2-2z^2)}{14} - \frac{\sqrt{42}G_v(x-y)(x+y)}{14} - \frac{\sqrt{42}G_{xy}xy}{7} + \frac{\sqrt{42}G_{xz}xz}{14} + \frac{\sqrt{42}G_{yz}yz}{14}$$

$$\vec{\mathbb{G}}_2^{(2,2)}[g](A_1')$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{14}G_u(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4)}{14} - \frac{5\sqrt{42}G_v(x-y)(x+y)(x^2+y^2-6z^2)}{168} - \frac{5\sqrt{42}G_{xy}xy(x^2+y^2-6z^2)}{84} - \frac{5\sqrt{42}G_{xz}xz(3x^2+3y^2-4z^2)}{84} - \frac{5\sqrt{42}G_{yz}yz(3x^2+3y^2-4z^2)}{84}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E'), \vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E')$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$G_{yz}$$

$$-G_{xz}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E'), \vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E')$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}G_u yz}{14} - \frac{3\sqrt{14}G_v yz}{14} + \frac{3\sqrt{14}G_{xy}xz}{14} + \frac{3\sqrt{14}G_{xz}xy}{14} - \frac{\sqrt{14}G_{yz}(2x^2-y^2-z^2)}{14}$$

$$-\frac{\sqrt{42}G_u xz}{14} - \frac{3\sqrt{14}G_v xz}{14} - \frac{3\sqrt{14}G_{xy}yz}{14} - \frac{\sqrt{14}G_{xz}(x^2-2y^2+z^2)}{14} - \frac{3\sqrt{14}G_{yz}xy}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E'), \vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E')$$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{5\sqrt{42}G_u yz(3x^2+3y^2-4z^2)}{84} + \frac{5\sqrt{14}G_v yz(9x^2-5y^2+2z^2)}{84} - \frac{5\sqrt{14}G_{xy}xz(x^2-6y^2+z^2)}{42} - \frac{5\sqrt{14}G_{xz}xy(x^2+y^2-6z^2)}{42} + \frac{\sqrt{14}G_{yz}(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4)}{42}$$

$$\frac{5\sqrt{42}G_u xz(3x^2+3y^2-4z^2)}{84} - \frac{5\sqrt{14}G_v xz(5x^2-9y^2-2z^2)}{84} - \frac{5\sqrt{14}G_{xy}yz(6x^2-y^2-z^2)}{42} + \frac{\sqrt{14}G_{xz}(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4)}{42} + \frac{5\sqrt{14}G_{yz}xy(x^2+y^2-6z^2)}{42}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$G_v$$

$$-G_{xy}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}G_u(x-y)(x+y)}{14} + \frac{\sqrt{14}G_v(x^2+y^2-2z^2)}{14} + \frac{3\sqrt{14}G_{xz}xz}{14} - \frac{3\sqrt{14}G_{yz}yz}{14}$$

$$\frac{\sqrt{42}G_{uxy}}{7} - \frac{\sqrt{14}G_{xy}(x^2+y^2-2z^2)}{14} - \frac{3\sqrt{14}G_{xzyz}}{14} - \frac{3\sqrt{14}G_{yzxz}}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E''), \vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E'')$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}G_u(x-y)(x+y)(x^2+y^2-6z^2)}{168} + \frac{\sqrt{14}G_v(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4)}{168}$$

$$+ \frac{5\sqrt{14}G_{xyxy}(x-y)(x+y)}{12} + \frac{5\sqrt{14}G_{xzxz}(5x^2-9y^2-2z^2)}{84} + \frac{5\sqrt{14}G_{yzyz}(9x^2-5y^2+2z^2)}{84}$$

$$\frac{5\sqrt{42}G_{uxy}(x^2+y^2-6z^2)}{84} - \frac{5\sqrt{14}G_{vxy}(x-y)(x+y)}{12} + \frac{\sqrt{14}G_{xy}(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4)}{42}$$

$$- \frac{5\sqrt{14}G_{xzyz}(6x^2-y^2-z^2)}{42} + \frac{5\sqrt{14}G_{yzxz}(x^2-6y^2+z^2)}{42}$$

* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A_1'')$$

** symmetry

$$\frac{\sqrt{10}x(x^2-3y^2)}{4}$$

** expression

$$\frac{\sqrt{2}G_vx}{2} - \frac{\sqrt{2}G_{xyy}}{2}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A_1'')$$

** symmetry

$$\frac{\sqrt{10}x(x^2-3y^2)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_{ux}(x^2-3y^2)}{24} + \frac{\sqrt{2}G_vx(x^2+y^2-4z^2)}{8} - \frac{\sqrt{2}G_{xyy}(x^2+y^2-4z^2)}{8} + \frac{5\sqrt{2}G_{xzz}(x-y)(x+y)}{8} - \frac{5\sqrt{2}G_{yzxyz}}{4}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A_1'')$$

** symmetry

$$\frac{\sqrt{10}x(x^2-3y^2)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_{ux}(x^2-3y^2)(x^2+y^2-8z^2)}{264} + \frac{\sqrt{11}G_vx(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4)}{88}$$

$$+ \frac{\sqrt{11}G_{xyy}(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4)}{44} + \frac{7\sqrt{11}G_{xzz}(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2)}{44} + \frac{7\sqrt{11}G_{yzxyz}(5x^2-7y^2+2z^2)}{44}$$

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A'_2)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_{uz}}{5} - \frac{\sqrt{5}G_{zxz}}{5} - \frac{\sqrt{5}G_{yzy}}{5}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A'_2)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$-\frac{\sqrt{15}G_{uz}(3x^2 + 3y^2 - 2z^2)}{15} - \frac{\sqrt{5}G_{vz}(x-y)(x+y)}{2} - \sqrt{5}G_{xyxyz} - \frac{\sqrt{5}G_{xzx}(x^2 + y^2 - 4z^2)}{20} - \frac{\sqrt{5}G_{yzy}(x^2 + y^2 - 4z^2)}{20}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A'_2)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{330}G_{uz}(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{264} - \frac{7\sqrt{110}G_{vz}(x-y)(x+y)(x^2 + y^2 - 2z^2)}{88} - \frac{7\sqrt{110}G_{xyxyz}(x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{110}G_{xzx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44} + \frac{\sqrt{110}G_{yzy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{44}$$

$$\vec{\mathbb{G}}_3^{(2,-2)}[g](A''_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{2}G_{vy}}{2} + \frac{\sqrt{2}G_{xyx}}{2}$$

$$\vec{\mathbb{G}}_3^{(2,0)}[g](A''_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_{uy}(3x^2 - y^2)}{24} + \frac{\sqrt{2}G_{vy}(x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{2}G_{xyx}(x^2 + y^2 - 4z^2)}{8} + \frac{5\sqrt{2}G_{xzx}xyz}{4} + \frac{5\sqrt{2}G_{yzy}(x-y)(x+y)}{8}$$

$$\vec{\mathbb{G}}_3^{(2,2)}[g](A''_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_{uy}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{264} + \frac{\sqrt{11}G_{vy}(53x^4 - 104x^2y^2 - 6x^2z^2 + 11y^4 - 6y^2z^2 + 4z^4)}{88} - \frac{\sqrt{11}G_{xyx}(5x^4 - 53x^2y^2 + 3x^2z^2 + 26y^4 + 3y^2z^2 - 2z^4)}{44} + \frac{7\sqrt{11}G_{xzx}xyz(7x^2 - 5y^2 - 2z^2)}{44} - \frac{7\sqrt{11}G_{yzy}(x^4 - 9x^2y^2 + x^2z^2 + 2y^4 - y^2z^2)}{44}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_{xyz}}{3} + \frac{\sqrt{3}G_{xzy}}{3} + \frac{\sqrt{3}G_{yzx}}{3}$$

$$\frac{\sqrt{3}G_v z}{3} + \frac{\sqrt{3}G_{xz}x}{3} - \frac{\sqrt{3}G_{yz}y}{3}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{3}G_{xy}z(3x^2+3y^2-2z^2)}{6} + \frac{\sqrt{3}G_{xz}y(3x^2-2y^2+3z^2)}{6} - \frac{\sqrt{3}G_{yz}x(2x^2-3y^2-3z^2)}{6}$$

$$\frac{\sqrt{3}G_v z(3x^2+3y^2-2z^2)}{6} + \frac{\sqrt{3}G_{xz}x(x^2-9y^2+6z^2)}{12} + \frac{\sqrt{3}G_{yz}y(9x^2-y^2-6z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E'), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E')$$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\frac{21\sqrt{22}G_uxyz(x^2+y^2-2z^2)}{44} + \frac{21\sqrt{66}G_vxyz(x-y)(x+y)}{44} - \frac{\sqrt{66}G_{xy}z(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4)}{66}$$

$$-\frac{\sqrt{66}G_{xz}y(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4)}{66} + \frac{\sqrt{66}G_{yz}x(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4)}{66}$$

$$-\frac{21\sqrt{22}G_u z(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{66}G_v z(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4)}{264} + \frac{21\sqrt{66}G_{xy}xyz(x-y)(x+y)}{44}$$

$$-\frac{\sqrt{66}G_{xz}x(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4)}{132} - \frac{\sqrt{66}G_{yz}y(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4)}{132}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

** expression

$$\frac{\sqrt{10}G_u x}{5} - \frac{\sqrt{30}G_v x}{30} - \frac{\sqrt{30}G_{xy}y}{30} + \frac{2\sqrt{30}G_{xz}z}{15}$$

$$\frac{\sqrt{10}G_u y}{5} + \frac{\sqrt{30}G_v y}{30} - \frac{\sqrt{30}G_{xy}x}{30} + \frac{2\sqrt{30}G_{yz}z}{15}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

** expression

$$-\frac{3\sqrt{10}G_u x(x^2+y^2-4z^2)}{40} - \frac{\sqrt{30}G_v x(11x^2-9y^2-24z^2)}{120} - \frac{\sqrt{30}G_{xy}y(21x^2+y^2-24z^2)}{120} + \frac{\sqrt{30}G_{xz}z(9x^2-21y^2+4z^2)}{120} + \frac{\sqrt{30}G_{yz}xyz}{4}$$

$$-\frac{3\sqrt{10}G_u y(x^2+y^2-4z^2)}{40} - \frac{\sqrt{30}G_v y(9x^2-11y^2+24z^2)}{120} - \frac{\sqrt{30}G_{xy}x(x^2+21y^2-24z^2)}{120} + \frac{\sqrt{30}G_{xz}xyz}{4} - \frac{\sqrt{30}G_{yz}z(21x^2-9y^2-4z^2)}{120}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E''), \vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E'')$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\frac{3\sqrt{55}G_{ux}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{165}G_vx(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{264}$$

$$-\frac{\sqrt{165}G_{xyy}(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4)}{132}$$

$$-\frac{\sqrt{165}G_{xzz}(18x^4 + 15x^2y^2 - 41x^2z^2 - 3y^4 + y^2z^2 + 4z^4)}{132} - \frac{7\sqrt{165}G_{yzxyz}(x^2 + y^2 - 2z^2)}{44}$$

$$\frac{3\sqrt{55}G_{uy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{165}G_vy(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{264}$$

$$+\frac{\sqrt{165}G_{xyx}(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{132} - \frac{7\sqrt{165}G_{xxyz}(x^2 + y^2 - 2z^2)}{44}$$

$$+\frac{\sqrt{165}G_{yzz}(3x^4 - 15x^2y^2 - x^2z^2 - 18y^4 + 41y^2z^2 - 4z^4)}{132}$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(2,-2)}[g](A'_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_vyz}{2} + \frac{\sqrt{3}G_{xyxz}}{2} + \frac{\sqrt{3}G_{xzyx}}{2} + \frac{\sqrt{3}G_{yz}(x-y)(x+y)}{4}$$

$$\vec{\mathbb{G}}_4^{(2,0)}[g](A'_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$-\frac{7\sqrt{22}G_{uyz}(3x^2 - y^2)}{88} + \frac{3\sqrt{66}G_vyz(3x^2 + 3y^2 - 4z^2)}{88} + \frac{3\sqrt{66}G_{xyxz}(3x^2 + 3y^2 - 4z^2)}{88}$$

$$+\frac{\sqrt{66}G_{xzyx}(9x^2 - 19y^2 + 30z^2)}{88} - \frac{\sqrt{66}G_{yz}(6x^4 - 21x^2y^2 - 15x^2z^2 + y^4 + 15y^2z^2)}{88}$$

$$\vec{\mathbb{G}}_4^{(2,2)}[g](A'_1)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$-\frac{9\sqrt{2002}G_{uyz}(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{1144} + \frac{\sqrt{6006}G_vyz(85x^4 - 160x^2y^2 - 10x^2z^2 + 19y^4 - 10y^2z^2 + 4z^4)}{1144}$$

$$-\frac{\sqrt{6006}G_{xyxz}(7x^4 - 85x^2y^2 + 5x^2z^2 + 40y^4 + 5y^2z^2 - 2z^4)}{572} - \frac{\sqrt{6006}G_{xzyx}(7x^4 + 2x^2y^2 - 76x^2z^2 - 5y^4 + 44y^2z^2 + 16z^4)}{572}$$

$$+\frac{\sqrt{6006}G_{yz}(x^6 - 8x^4y^2 - 7x^4z^2 - 7x^2y^4 + 90x^2y^2z^2 - 8x^2z^4 + 2y^6 - 23y^4z^2 + 8y^2z^4)}{572}$$

$$\vec{\mathbb{G}}_4^{(2,-2)}[g](A''_1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$-\frac{3\sqrt{70}G_u(x^2 + y^2 - 2z^2)}{70} + \frac{\sqrt{210}G_v(x-y)(x+y)}{140} + \frac{\sqrt{210}G_{xyxy}}{70} - \frac{2\sqrt{210}G_{xzxz}}{35} - \frac{2\sqrt{210}G_{yzyz}}{35}$$

$$\vec{\mathbb{G}}_4^{(2,0)}[g](A''_1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{\sqrt{385}G_u(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{308} + \frac{3\sqrt{1155}G_v(x-y)(x+y)(x^2 + y^2 - 6z^2)}{308} \\ + \frac{3\sqrt{1155}G_{xy}xy(x^2 + y^2 - 6z^2)}{154} - \frac{\sqrt{1155}G_{xz}xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{1155}G_{yz}yz(3x^2 + 3y^2 - 4z^2)}{308}$$

$$\vec{\mathbb{G}}_4^{(2,2)}[g](A_1'')$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{3\sqrt{715}G_u(5x^6 + 15x^4y^2 - 90x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 120x^2z^4 + 5y^6 - 90y^4z^2 + 120y^2z^4 - 16z^6)}{2288} \\ + \frac{7\sqrt{2145}G_v(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} + \frac{7\sqrt{2145}G_{xy}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\ + \frac{7\sqrt{2145}G_{xz}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144} + \frac{7\sqrt{2145}G_{yz}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144}$$

$$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_2')$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_vxz}{2} - \frac{\sqrt{3}G_{xy}yz}{2} + \frac{\sqrt{3}G_{xz}(x-y)(x+y)}{4} - \frac{\sqrt{3}G_{yz}xy}{2}$$

$$\vec{\mathbb{G}}_4^{(2,0)}[g](A_2'')$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{7\sqrt{22}G_{uxz}(x^2 - 3y^2)}{88} + \frac{3\sqrt{66}G_vxz(3x^2 + 3y^2 - 4z^2)}{88} - \frac{3\sqrt{66}G_{xy}yz(3x^2 + 3y^2 - 4z^2)}{88} \\ + \frac{\sqrt{66}G_{xz}(x^4 - 21x^2y^2 + 15x^2z^2 + 6y^4 - 15y^2z^2)}{88} + \frac{\sqrt{66}G_{yz}xy(19x^2 - 9y^2 - 30z^2)}{88}$$

$$\vec{\mathbb{G}}_4^{(2,2)}[g](A_2''')$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{9\sqrt{2002}G_{uxz}(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{1144} + \frac{\sqrt{6006}G_vxz(19x^4 - 160x^2y^2 - 10x^2z^2 + 85y^4 - 10y^2z^2 + 4z^4)}{1144} \\ + \frac{\sqrt{6006}G_{xy}yz(40x^4 - 85x^2y^2 + 5x^2z^2 + 7y^4 + 5y^2z^2 - 2z^4)}{572} \\ - \frac{\sqrt{6006}G_{xz}(2x^6 - 7x^4y^2 - 23x^4z^2 - 8x^2y^4 + 90x^2y^2z^2 + 8x^2z^4 + y^6 - 7y^4z^2 - 8y^2z^4)}{572} \\ - \frac{\sqrt{6006}G_{yz}xy(5x^4 - 2x^2y^2 - 44x^2z^2 - 7y^4 + 76y^2z^2 - 16z^4)}{572}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E'), \vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E')$$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\frac{3\sqrt{7}G_u yz}{7} + \frac{\sqrt{21}G_v yz}{14} - \frac{\sqrt{21}G_{xy}xz}{14} - \frac{\sqrt{21}G_{xz}xy}{14} - \frac{\sqrt{21}G_{yz}(x^2 + 3y^2 - 4z^2)}{28}$$

$$-\frac{3\sqrt{7}G_u xz}{7} + \frac{\sqrt{21}G_v xz}{14} + \frac{\sqrt{21}G_{xy}yz}{14} + \frac{\sqrt{21}G_{xz}(3x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{21}G_{yz}xy}{14}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E'), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E')$$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$-\frac{17\sqrt{154}G_u yz(3x^2 + 3y^2 - 4z^2)}{616} - \frac{\sqrt{462}G_v yz(33x^2 - 51y^2 + 40z^2)}{616} - \frac{\sqrt{462}G_{xy}xz(9x^2 + 93y^2 - 40z^2)}{616}$$

$$-\frac{9\sqrt{462}G_{xz}xy(x^2 + y^2 - 6z^2)}{616} + \frac{\sqrt{462}G_{yz}(6x^4 + 3x^2y^2 - 39x^2z^2 - 3y^4 + 15y^2z^2 + 4z^4)}{616}$$

$$\frac{17\sqrt{154}G_u xz(3x^2 + 3y^2 - 4z^2)}{616} + \frac{\sqrt{462}G_v xz(51x^2 - 33y^2 - 40z^2)}{616} + \frac{\sqrt{462}G_{xy}yz(93x^2 + 9y^2 - 40z^2)}{616}$$

$$+ \frac{\sqrt{462}G_{xz}(3x^4 - 3x^2y^2 - 15x^2z^2 - 6y^4 + 39y^2z^2 - 4z^4)}{616} + \frac{9\sqrt{462}G_{yz}xy(x^2 + y^2 - 6z^2)}{616}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E'), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E')$$

** symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\frac{21\sqrt{286}G_u yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144} - \frac{7\sqrt{858}G_v yz(11x^4 + 4x^2y^2 - 26x^2z^2 - 7y^4 + 22y^2z^2 - 4z^4)}{1144}$$

$$+ \frac{7\sqrt{858}G_{xy}xz(x^4 - 7x^2y^2 - x^2z^2 - 8y^4 + 23y^2z^2 - 2z^4)}{572} + \frac{7\sqrt{858}G_{xz}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572}$$

$$-\frac{\sqrt{858}G_{yz}(x^6 - 4x^4y^2 - 11x^4z^2 - 11x^2y^4 + 90x^2y^2z^2 - 4x^2z^4 - 6y^6 + 101y^4z^2 - 116y^2z^4 + 8z^6)}{572}$$

$$-\frac{21\sqrt{286}G_u xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{1144}$$

$$+ \frac{7\sqrt{858}G_v xz(7x^4 - 4x^2y^2 - 22x^2z^2 - 11y^4 + 26y^2z^2 + 4z^4)}{1144} + \frac{7\sqrt{858}G_{xy}yz(8x^4 + 7x^2y^2 - 23x^2z^2 - y^4 + y^2z^2 + 2z^4)}{572}$$

$$-\frac{\sqrt{858}G_{xz}(6x^6 + 11x^4y^2 - 101x^4z^2 + 4x^2y^4 - 90x^2y^2z^2 + 116x^2z^4 - y^6 + 11y^4z^2 + 4y^2z^4 - 8z^6)}{572}$$

$$-\frac{7\sqrt{858}G_{yz}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E'', 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}G_v(x-y)(x+y)}{4} - \frac{\sqrt{6}G_{xy}xy}{2}$$

$$\frac{\sqrt{6}G_vxy}{2} + \frac{\sqrt{6}G_{xy}(x-y)(x+y)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E'', 2)$$

** symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\frac{3\sqrt{14}G_u(x-y)(x+y)}{28} - \frac{\sqrt{42}G_v(x^2 + y^2 - 2z^2)}{28} + \frac{\sqrt{42}G_{xz}xz}{7} - \frac{\sqrt{42}G_{yz}yz}{7}$$

$$- \frac{3\sqrt{14}G_{uxy}}{14} + \frac{\sqrt{42}G_{xy}(x^2 + y^2 - 2z^2)}{28} - \frac{\sqrt{42}G_{xz}yz}{7} - \frac{\sqrt{42}G_{yz}xz}{7}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E'', 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$- \frac{7\sqrt{11}G_u(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{44} + \frac{\sqrt{33}G_v(x-y)(x+y)(x^2 + y^2 - 6z^2)}{44}$$

$$- \frac{\sqrt{33}G_{xy}xy(x^2 + y^2 - 6z^2)}{22} + \frac{7\sqrt{33}G_{xz}xz(x^2 - 3y^2)}{44} - \frac{7\sqrt{33}G_{yz}yz(3x^2 - y^2)}{44}$$

$$- \frac{7\sqrt{11}G_{uxy}(x-y)(x+y)}{11} + \frac{\sqrt{33}G_{vxy}(x^2 + y^2 - 6z^2)}{22}$$

$$+ \frac{\sqrt{33}G_{xy}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{44} + \frac{7\sqrt{33}G_{xzyz}(3x^2 - y^2)}{44} + \frac{7\sqrt{33}G_{yzxz}(x^2 - 3y^2)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E'', 2)$$

** symmetry

$$- \frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$- \frac{\sqrt{77}G_u(x-y)(x+y)(x^2 + y^2 - 6z^2)}{77} - \frac{\sqrt{231}G_v(2x^4 - 3x^2y^2 - 9x^2z^2 + 2y^4 - 9y^2z^2 + 3z^4)}{154}$$

$$- \frac{\sqrt{231}G_{xy}xy(x-y)(x+y)}{22} + \frac{\sqrt{231}G_{xz}xz(2x^2 - 3y^2 + 9z^2)}{154} + \frac{\sqrt{231}G_{yz}yz(33x^2 - 2y^2 - 9z^2)}{154}$$

$$\frac{2\sqrt{77}G_{uxy}(x^2 + y^2 - 6z^2)}{77} + \frac{\sqrt{231}G_{vxy}(x-y)(x+y)}{22} + \frac{\sqrt{231}G_{xy}(x^4 + 30x^2y^2 - 36x^2z^2 + y^4 - 36y^2z^2 + 12z^4)}{308}$$

$$- \frac{\sqrt{231}G_{xzyz}(39x^2 - 31y^2 + 18z^2)}{308} + \frac{\sqrt{231}G_{yzxz}(31x^2 - 39y^2 - 18z^2)}{308}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E'', 1)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$- \frac{9\sqrt{1001}G_u(x^2 + y^2 - 10z^2)(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{2288} + \frac{\sqrt{3003}G_v(x-y)(x+y)(17x^4 - 230x^2y^2 - 8x^2z^2 + 17y^4 - 8y^2z^2 + 8z^4)}{2288}$$

$$+ \frac{\sqrt{3003}G_{xy}xy(49x^4 - 166x^2y^2 + 8x^2z^2 + 49y^4 + 8y^2z^2 - 8z^4)}{1144}$$

$$+ \frac{3\sqrt{3003}G_{xz}xz(7x^4 - 58x^2y^2 - 4x^2z^2 + 23y^4 + 12y^2z^2)}{1144} + \frac{3\sqrt{3003}G_{yz}yz(23x^4 - 58x^2y^2 + 12x^2z^2 + 7y^4 - 4y^2z^2)}{1144}$$

$$- \frac{9\sqrt{1001}G_{uxy}(x-y)(x+y)(x^2 + y^2 - 10z^2)}{572} + \frac{\sqrt{3003}G_{vxy}(25x^4 - 82x^2y^2 - 4x^2z^2 + 25y^4 - 4y^2z^2 + 4z^4)}{572}$$

$$- \frac{\sqrt{3003}G_{xy}(x-y)(x+y)(2x^4 - 29x^2y^2 + x^2z^2 + 2y^4 + y^2z^2 - z^4)}{286}$$

$$+ \frac{3\sqrt{3003}G_{xzyz}(8x^4 - 13x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{286} - \frac{3\sqrt{3003}G_{yzxz}(x^4 - 13x^2y^2 + x^2z^2 + 8y^4 - 3y^2z^2)}{286}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E'', 2)$$

** symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\begin{aligned} & \frac{21\sqrt{143}G_u(x-y)(x+y)(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{1144} \\ & - \frac{\sqrt{429}G_v(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6)}{1144} \\ & - \frac{21\sqrt{429}G_{xy}xy(x-y)(x+y)(x^2+y^2-10z^2)}{572} - \frac{7\sqrt{429}G_{xz}xz(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4)}{572} \\ & - \frac{7\sqrt{429}G_{yz}yz(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4)}{572} \\ & - \frac{21\sqrt{143}G_uxy(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4)}{572} + \frac{21\sqrt{429}G_vxy(x-y)(x+y)(x^2+y^2-10z^2)}{572} \\ & - \frac{\sqrt{429}G_{xy}(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6)}{286} \\ & + \frac{7\sqrt{429}G_{xz}yz(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4)}{286} - \frac{7\sqrt{429}G_{yz}xz(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4)}{286} \end{aligned}$$