PG No. 17 $C_{3\mathrm{i}}$ $\bar{3}$ [trigonal] (polar, internal axial octupole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(3,2)}[g](A_u)$

** symmetry

z

** expression

$$-\frac{3\sqrt{70}G_1x\left(x^2-3y^2\right)}{56}+\frac{3\sqrt{70}G_2y\left(3x^2-y^2\right)}{56}-\frac{\sqrt{105}G_3z\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}G_{3x}y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{42}G_{3y}x\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{105}G_{bz}xyz}{7}$$

$\vec{\mathbb{Q}}_{1,1}^{(3,2)}[g](E_u)$, $\vec{\mathbb{Q}}_{1,2}^{(3,2)}[g](E_u)$

** symmetry

x

y

** expression

$$\frac{3\sqrt{70}G_1z\left(x-y\right)\left(x+y\right)}{56}-\frac{3\sqrt{70}G_2xyz}{28}-\frac{\sqrt{105}G_3x\left(y-z\right)\left(y+z\right)}{14}-\frac{5\sqrt{42}G_{3x}xyz}{28}$$
$$-\frac{\sqrt{42}G_{3y}z\left(x^2+11y^2-4z^2\right)}{56}+\frac{3\sqrt{7}G_{az}y\left(x^2+y^2-4z^2\right)}{28}-\frac{\sqrt{105}G_{bz}y\left(x^2-y^2+2z^2\right)}{28}$$

$$-\frac{3\sqrt{70}G_1xyz}{28}-\frac{3\sqrt{70}G_2z\left(x-y\right)\left(x+y\right)}{56}+\frac{\sqrt{105}G_3y\left(x-z\right)\left(x+z\right)}{14}+\frac{\sqrt{42}G_{3x}z\left(11x^2+y^2-4z^2\right)}{56}$$
$$+\frac{5\sqrt{42}G_{3y}xyz}{28}-\frac{3\sqrt{7}G_{az}x\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{105}G_{bz}x\left(x^2-y^2-2z^2\right)}{28}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(3,0)}[g](A_g)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{210}G_3\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{21}G_{3x}yz}{7}-\frac{\sqrt{21}G_{3y}xz}{7}+\frac{\sqrt{210}G_{bz}xy}{14}$$

$\vec{\mathbb{Q}}_2^{(3,2)}[g](A_g)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{70}G_1xz\left(x^2-3y^2\right)}{8}+\frac{\sqrt{70}G_2yz\left(3x^2-y^2\right)}{8}+\frac{\sqrt{105}G_3\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{42}$$
$$-\frac{5\sqrt{42}G_{3x}yz\left(3x^2+3y^2-4z^2\right)}{168}+\frac{5\sqrt{42}G_{3y}xz\left(3x^2+3y^2-4z^2\right)}{168}-\frac{\sqrt{105}G_{bz}xy\left(x^2+y^2-6z^2\right)}{21}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,0)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(3,0)}[g](E_g,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{105}G_1xy}{14}+\frac{\sqrt{105}G_2\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{7}G_{3x}\left(5x^2-y^2-4z^2\right)}{28}+\frac{3\sqrt{7}G_{3y}xy}{14}+\frac{\sqrt{42}G_{az}xz}{7}$$

$$\frac{\sqrt{105}G_1\left(x-y\right)\left(x+y\right)}{28}-\frac{\sqrt{105}G_2xy}{14}+\frac{3\sqrt{7}G_{3x}xy}{14}-\frac{\sqrt{7}G_{3y}\left(x^2-5y^2+4z^2\right)}{28}+\frac{\sqrt{42}G_{az}yz}{7}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,0)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(3,0)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{105}G_1xz}{14} - \frac{\sqrt{105}G_2yz}{14} - \frac{\sqrt{70}G_3\left(x^2+y^2-2z^2\right)}{28} - \frac{3\sqrt{7}G_{3x}yz}{14} - \frac{3\sqrt{7}G_{3y}xz}{14} + \frac{\sqrt{42}G_{az}xy}{14}$$

$$\frac{\sqrt{105}G_1yz}{14} + \frac{\sqrt{105}G_2xz}{14} - \frac{3\sqrt{7}G_{3x}xz}{14} + \frac{3\sqrt{7}G_{3y}yz}{14} + \frac{\sqrt{42}G_{az}(x-y)(x+y)}{28} - \frac{\sqrt{70}G_{bz}\left(x^2+y^2-2z^2\right)}{28}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,2)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(3,2)}[g](E_g,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{210}G_1xy\left(11x^2-17y^2+18z^2\right)}{168} - \frac{\sqrt{210}G_2\left(2x^4-21x^2y^2+9x^2z^2+5y^4-9y^2z^2\right)}{168}$$
$$+\frac{\sqrt{35}G_3yz(y-z)(y+z)}{6} - \frac{5\sqrt{14}G_{3x}\left(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4\right)}{168}$$
$$-\frac{5\sqrt{14}G_{3y}xy\left(x^2+y^2-6z^2\right)}{168} - \frac{5\sqrt{21}G_{az}xz\left(3x^2+3y^2-4z^2\right)}{84} + \frac{\sqrt{35}G_{bz}xz\left(x^2+3y^2-2z^2\right)}{12}$$

$$\frac{\sqrt{210}G_1\left(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2\right)}{168} - \frac{\sqrt{210}G_2xy\left(17x^2-11y^2-18z^2\right)}{168} + \frac{\sqrt{35}G_3xz(x-z)(x+z)}{6} - \frac{5\sqrt{14}G_{3x}xy\left(x^2+y^2-6z^2\right)}{168}$$
$$-\frac{5\sqrt{14}G_{3y}\left(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4\right)}{168} - \frac{5\sqrt{21}G_{az}yz\left(3x^2+3y^2-4z^2\right)}{84} - \frac{\sqrt{35}G_{bz}yz\left(3x^2+y^2-2z^2\right)}{12}$$

$\vec{\mathbb{Q}}_{2,1}^{(3,2)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(3,2)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{210}G_1xz\left(3x^2+3y^2-4z^2\right)}{168} - \frac{\sqrt{210}G_2yz\left(3x^2+3y^2-4z^2\right)}{168} + \frac{\sqrt{35}G_3\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{42}$$
$$-\frac{5\sqrt{14}G_{3x}yz\left(27x^2-y^2-8z^2\right)}{168} + \frac{5\sqrt{14}G_{3y}xz\left(x^2-27y^2+8z^2\right)}{168} + \frac{5\sqrt{21}G_{az}xy\left(x^2+y^2-6z^2\right)}{42} - \frac{\sqrt{35}G_{bz}xy(x-y)(x+y)}{6}$$

$$\frac{\sqrt{210}G_1yz\left(3x^2+3y^2-4z^2\right)}{168} + \frac{\sqrt{210}G_2xz\left(3x^2+3y^2-4z^2\right)}{168} - \frac{\sqrt{35}G_3xy(x-y)(x+y)}{6} - \frac{5\sqrt{14}G_{3x}xz\left(13x^2-15y^2-8z^2\right)}{168}$$
$$-\frac{5\sqrt{14}G_{3y}yz\left(15x^2-13y^2+8z^2\right)}{168} + \frac{5\sqrt{21}G_{az}(x-y)(x+y)\left(x^2+y^2-6z^2\right)}{84} - \frac{\sqrt{35}G_{bz}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{84}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_u,1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{2}G_{3x}y}{2} - \frac{\sqrt{2}G_{3y}x}{2}$$

$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{3}G_2z}{2} - \frac{\sqrt{2}G_3y}{4} + \frac{\sqrt{2}G_{bz}x}{4}$$

$\vec{\mathbb{Q}}_3^{(3,-2)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{3}G_1z}{2}-\frac{\sqrt{2}G_3x}{4}-\frac{\sqrt{2}G_{bz}y}{4}$$

$\vec{\mathbb{Q}}_3^{(3,0)}[g](A_u,1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_1x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{15}G_2y\left(3x^2-y^2\right)}{12}-\frac{\sqrt{10}G_3z\left(x-y\right)\left(x+y\right)}{4}-\frac{G_{3x}y\left(x^2+y^2-4z^2\right)}{4}+\frac{G_{3y}x\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{10}G_{bz}xyz}{2}$$

$\vec{\mathbb{Q}}_3^{(3,0)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{6}G_2z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{G_3y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{10}G_{3x}z\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{10}G_{3y}xyz}{2}-\frac{\sqrt{15}G_{az}x\left(x^2-3y^2\right)}{12}+\frac{G_{bz}x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(3,0)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{6}G_1z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{G_3x\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{10}G_{3x}xyz}{2}-\frac{\sqrt{10}G_{3y}z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{15}G_{az}y\left(3x^2-y^2\right)}{12}-\frac{G_{bz}y\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(3,2)}[g](A_u,1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{7\sqrt{330}G_1x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{528}-\frac{7\sqrt{330}G_2y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{528}+\frac{7\sqrt{55}G_3z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{5\sqrt{22}G_{3x}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{5\sqrt{22}G_{3y}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{7\sqrt{55}G_{bz}xyz\left(x^2+y^2-2z^2\right)}{22}$$

$\vec{\mathbb{Q}}_3^{(3,2)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{33}G_2z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}+\frac{\sqrt{22}G_3y\left(25x^4-55x^2y^2+15x^2z^2+4y^4+15y^2z^2-10z^4\right)}{88}$$
$$+\frac{7\sqrt{55}G_{3x}z\left(5x^4-18x^2y^2-4x^2z^2+y^4+4y^2z^2\right)}{176}+\frac{7\sqrt{55}G_{3y}xyz\left(x^2-2y^2+z^2\right)}{22}$$
$$-\frac{7\sqrt{330}G_{az}x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{528}+\frac{\sqrt{22}G_{bz}x\left(13x^4-100x^2y^2-30x^2z^2+55y^4-30y^2z^2+20z^4\right)}{176}$$

$\vec{\mathbb{Q}}_3^{(3,2)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{33}G_1z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}+\frac{\sqrt{22}G_3x\left(4x^4-55x^2y^2+15x^2z^2+25y^4+15y^2z^2-10z^4\right)}{88}$$
$$-\frac{7\sqrt{55}G_{3x}xyz\left(2x^2-y^2-z^2\right)}{22}+\frac{7\sqrt{55}G_{3y}z\left(x^4-18x^2y^2+4x^2z^2+5y^4-4y^2z^2\right)}{176}$$
$$+\frac{7\sqrt{330}G_{az}y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{528}-\frac{\sqrt{22}G_{bz}y\left(55x^4-100x^2y^2-30x^2z^2+13y^4-30y^2z^2+20z^4\right)}{176}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E_u,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}G_3x}{12}+\frac{\sqrt{3}G_{3y}z}{6}-\frac{\sqrt{2}G_{az}y}{2}+\frac{\sqrt{30}G_{bz}y}{12}$$

$$\frac{\sqrt{30}G_3y}{12}-\frac{\sqrt{3}G_{3x}z}{6}+\frac{\sqrt{2}G_{az}x}{2}+\frac{\sqrt{30}G_{bz}x}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E_u,2)$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{2}G_1y}{4}+\frac{\sqrt{2}G_2x}{4}+\frac{\sqrt{30}G_{3x}x}{12}-\frac{\sqrt{30}G_{3y}y}{12}-\frac{\sqrt{3}G_{bz}z}{3}$$

$$-\frac{\sqrt{2}G_1x}{4}+\frac{\sqrt{2}G_2y}{4}+\frac{\sqrt{3}G_3z}{3}-\frac{\sqrt{30}G_{3x}y}{12}-\frac{\sqrt{30}G_{3y}x}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E_u,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{10}G_1z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{10}G_2xyz}{2}-\frac{\sqrt{15}G_3x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{6}G_{3y}z\left(3x^2+3y^2-2z^2\right)}{12}+\frac{G_{az}y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{15}G_{bz}y\left(3x^2-y^2\right)}{12}$$

$$\frac{\sqrt{10}G_1xyz}{2}+\frac{\sqrt{10}G_2z\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{15}G_3y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{6}G_{3x}z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{G_{az}x\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{15}G_{bz}x\left(x^2-3y^2\right)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,0)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(3,0)}[g](E_u,2)$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{G_1y\left(x^2+y^2-4z^2\right)}{4}+\frac{G_2x\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{15}G_{3x}x\left(x^2-3y^2\right)}{12}+\frac{\sqrt{15}G_{3y}y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{10}G_{az}z\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{6}G_{bz}z\left(3x^2+3y^2-2z^2\right)}{12}$$

$$-\frac{G_1x\left(x^2+y^2-4z^2\right)}{4}+\frac{G_2y\left(x^2+y^2-4z^2\right)}{4}-\frac{\sqrt{6}G_3z\left(3x^2+3y^2-2z^2\right)}{12}-\frac{\sqrt{15}G_{3x}y\left(3x^2-y^2\right)}{12}+\frac{\sqrt{15}G_{3y}x\left(x^2-3y^2\right)}{12}-\frac{\sqrt{10}G_{az}xyz}{2}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E_u,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{55}G_1 z\left(5x^4-18x^2y^2-4x^2z^2+y^4+4y^2z^2\right)}{176}+\frac{7\sqrt{55}G_2 xyz\left(2x^2-y^2-z^2\right)}{22}$$
$$+\frac{\sqrt{330}G_3 x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264}+\frac{5\sqrt{33}G_{3y} z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}$$
$$-\frac{5\sqrt{22}G_{az} y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{\sqrt{330}G_{bz} y\left(15x^4+16x^2y^2-138x^2z^2+y^4-26y^2z^2+36z^4\right)}{528}$$

$$-\frac{7\sqrt{55}G_1 xyz\left(x^2-2y^2+z^2\right)}{22}-\frac{7\sqrt{55}G_2 z\left(x^4-18x^2y^2+4x^2z^2+5y^4-4y^2z^2\right)}{176}$$
$$+\frac{\sqrt{330}G_3 y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264}-\frac{5\sqrt{33}G_{3x} z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{528}$$
$$+\frac{5\sqrt{22}G_{az} x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{176}-\frac{\sqrt{330}G_{bz} x\left(x^4+16x^2y^2-26x^2z^2+15y^4-138y^2z^2+36z^4\right)}{528}$$

$\vec{\mathbb{Q}}_{3,1}^{(3,2)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(3,2)}[g](E_u,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{22}G_1 y\left(25x^4-55x^2y^2+15x^2z^2+4y^4+15y^2z^2-10z^4\right)}{88}-\frac{\sqrt{22}G_2 x\left(4x^4-55x^2y^2+15x^2z^2+25y^4+15y^2z^2-10z^4\right)}{88}$$
$$-\frac{\sqrt{330}G_{3x} x\left(4x^4+x^2y^2-41x^2z^2-3y^4+15y^2z^2+18z^4\right)}{264}-\frac{\sqrt{330}G_{3y} y\left(3x^4-x^2y^2-15x^2z^2-4y^4+41y^2z^2-18z^4\right)}{264}$$
$$-\frac{7\sqrt{55}G_{az} z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{33}G_{bz} z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{132}$$

$$-\frac{\sqrt{22}G_1 x\left(13x^4-100x^2y^2-30x^2z^2+55y^4-30y^2z^2+20z^4\right)}{176}+\frac{\sqrt{22}G_2 y\left(55x^4-100x^2y^2-30x^2z^2+13y^4-30y^2z^2+20z^4\right)}{176}$$
$$-\frac{\sqrt{33}G_3 z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{132}+\frac{\sqrt{330}G_{3x} y\left(15x^4+16x^2y^2-138x^2z^2+y^4-26y^2z^2+36z^4\right)}{528}$$
$$+\frac{\sqrt{330}G_{3y} x\left(x^4+16x^2y^2-26x^2z^2+15y^4-138y^2z^2+36z^4\right)}{528}+\frac{7\sqrt{55}G_{az} xyz\left(x^2+y^2-2z^2\right)}{22}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_g,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{\sqrt{21}G_3\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{210}G_{3x} yz}{14}-\frac{\sqrt{210}G_{3y} xz}{14}-\frac{\sqrt{21}G_{bz} xy}{7}$$

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_g,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{5}G_1\left(x^2+y^2-2z^2\right)}{20}+\frac{\sqrt{30}G_3 xz}{20}-\frac{\sqrt{3}G_{3x} xy}{2}-\frac{\sqrt{3}G_{3y}\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{30}G_{bz} yz}{20}$$

$\vec{\mathbb{Q}}_4^{(3,-2)}[g](A_g,3)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{5}G_2\left(x^2+y^2-2z^2\right)}{20}+\frac{\sqrt{30}G_3 yz}{20}+\frac{\sqrt{3}G_{3x}\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{3}G_{3y} xy}{2}-\frac{\sqrt{30}G_{bz} xz}{20}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_g,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{5\sqrt{77}G_1xz\left(x^2-3y^2\right)}{44}-\frac{5\sqrt{77}G_2yz\left(3x^2-y^2\right)}{44}+\frac{5\sqrt{462}G_3\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{616}$$
$$-\frac{3\sqrt{1155}G_{3x}yz\left(3x^2+3y^2-4z^2\right)}{308}+\frac{3\sqrt{1155}G_{3y}xz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{5\sqrt{462}G_{bz}xy\left(x^2+y^2-6z^2\right)}{308}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_g,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{110}G_1\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{176}-\frac{\sqrt{165}G_3xz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{66}G_{3x}xy\left(3x^2-4y^2+3z^2\right)}{22}$$
$$+\frac{\sqrt{66}G_{3y}\left(9x^4-42x^2y^2-12x^2z^2+5y^4+12y^2z^2\right)}{176}-\frac{7\sqrt{11}G_{az}yz\left(3x^2-y^2\right)}{44}-\frac{\sqrt{165}G_{bz}yz\left(3x^2+3y^2-4z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(3,0)}[g](A_g,3)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{110}G_2\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{176}-\frac{\sqrt{165}G_3yz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{66}G_{3x}\left(5x^4-42x^2y^2+12x^2z^2+9y^4-12y^2z^2\right)}{176}$$
$$+\frac{\sqrt{66}G_{3y}xy\left(4x^2-3y^2-3z^2\right)}{22}+\frac{7\sqrt{11}G_{az}xz\left(x^2-3y^2\right)}{44}+\frac{\sqrt{165}G_{bz}xz\left(3x^2+3y^2-4z^2\right)}{44}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](A_g,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{9\sqrt{10010}G_1xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{9\sqrt{10010}G_2yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}$$
$$-\frac{\sqrt{15015}G_3\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}+\frac{5\sqrt{6006}G_{3x}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$-\frac{5\sqrt{6006}G_{3y}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{\sqrt{15015}G_{bz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{286}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](A_g,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{143}G_1\left(19x^6-240x^4y^2-45x^4z^2+255x^2y^4-90x^2y^2z^2+60x^2z^4-14y^6-45y^4z^2+60y^2z^4-8z^6\right)}{2288}+\frac{9\sqrt{143}G_2xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{208}$$
$$-\frac{\sqrt{858}G_3xz\left(13x^4+125x^2y^2-85x^2z^2-20y^4-85y^2z^2+34z^4\right)}{1144}+\frac{\sqrt{2145}G_{3x}xy\left(43x^4+26x^2y^2-508x^2z^2-17y^4+92y^2z^2+208z^4\right)}{2288}$$
$$-\frac{\sqrt{2145}G_{3y}\left(x^6-44x^4y^2+29x^4z^2-31x^2y^4+450x^2y^2z^2-104x^2z^4+14y^6-179y^4z^2+104y^2z^4\right)}{2288}$$
$$+\frac{27\sqrt{1430}G_{az}yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}-\frac{\sqrt{858}G_{bz}yz\left(125x^4-80x^2y^2-170x^2z^2+59y^4-170y^2z^2+68z^4\right)}{2288}$$

$\vec{\mathbb{Q}}_4^{(3,2)}[g](A_g,3)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{143}G_1xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{208}-\frac{3\sqrt{143}G_2\left(14x^6-255x^4y^2+45x^4z^2+240x^2y^4+90x^2y^2z^2-60x^2z^4-19y^6+45y^4z^2-60y^2z^4+8z^6\right)}{2288}$$
$$+\frac{\sqrt{858}G_3yz\left(20x^4-125x^2y^2+85x^2z^2-13y^4+85y^2z^2-34z^4\right)}{1144}$$
$$-\frac{\sqrt{2145}G_{3x}\left(14x^6-31x^4y^2-179x^4z^2-44x^2y^4+450x^2y^2z^2+104x^2z^4+y^6+29y^4z^2-104y^2z^4\right)}{2288}$$
$$-\frac{\sqrt{2145}G_{3y}xy\left(17x^4-26x^2y^2-92x^2z^2-43y^4+508y^2z^2-208z^4\right)}{2288}-\frac{27\sqrt{1430}G_{az}xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{2288}$$
$$+\frac{\sqrt{858}G_{bz}xz\left(59x^4-80x^2y^2-170x^2z^2+125y^4-170y^2z^2+68z^4\right)}{2288}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{35}G_1xy}{70}-\frac{3\sqrt{35}G_2\left(x-y\right)\left(x+y\right)}{140}+\frac{\sqrt{210}G_3yz}{20}-\frac{\sqrt{21}G_{3x}\left(x^2-3y^2+2z^2\right)}{28}-\frac{\sqrt{21}G_{3y}xy}{7}+\frac{3\sqrt{14}G_{az}xz}{14}+\frac{\sqrt{210}G_{bz}xz}{20}$$

$$-\frac{3\sqrt{35}G_1\left(x-y\right)\left(x+y\right)}{140}+\frac{3\sqrt{35}G_2xy}{70}+\frac{\sqrt{210}G_3xz}{20}-\frac{\sqrt{21}G_{3x}xy}{7}+\frac{\sqrt{21}G_{3y}\left(3x^2-y^2-2z^2\right)}{28}+\frac{3\sqrt{14}G_{az}yz}{14}-\frac{\sqrt{210}G_{bz}yz}{20}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E_g,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{10}G_1xz}{10}+\frac{3\sqrt{10}G_2yz}{10}-\frac{\sqrt{15}G_3\left(x-y\right)\left(x+y\right)}{10}-\frac{\sqrt{15}G_{bz}xy}{5}$$

$$\frac{3\sqrt{10}G_1yz}{10}-\frac{3\sqrt{10}G_2xz}{10}-\frac{\sqrt{15}G_3xy}{5}+\frac{\sqrt{15}G_{bz}\left(x-y\right)\left(x+y\right)}{10}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E_g,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{9\sqrt{70}G_1xz}{140}+\frac{9\sqrt{70}G_2yz}{140}-\frac{\sqrt{105}G_3\left(x^2+y^2-2z^2\right)}{35}-\frac{\sqrt{42}G_{3x}yz}{28}-\frac{\sqrt{42}G_{3y}xz}{28}-\frac{3\sqrt{7}G_{az}xy}{7}$$

$$-\frac{9\sqrt{70}G_1yz}{140}-\frac{9\sqrt{70}G_2xz}{140}-\frac{\sqrt{42}G_{3x}xz}{28}+\frac{\sqrt{42}G_{3y}yz}{28}-\frac{3\sqrt{7}G_{az}\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{105}G_{bz}\left(x^2+y^2-2z^2\right)}{35}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{\sqrt{770}G_1xy\left(x^2-6y^2+15z^2\right)}{154}-\frac{\sqrt{770}G_2\left(3x^4+42x^2y^2-60x^2z^2-17y^4+60y^2z^2\right)}{1232}$$
$$-\frac{\sqrt{1155}G_3yz\left(3x^2-y^2\right)}{44}-\frac{\sqrt{462}G_{3x}\left(5x^4+18x^2y^2-48x^2z^2+13y^4-96y^2z^2+24z^4\right)}{1232}$$
$$+\frac{\sqrt{462}G_{3y}xy\left(x^2+y^2-6z^2\right)}{154}-\frac{9\sqrt{77}G_{az}xz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{1155}G_{bz}xz\left(x^2-3y^2\right)}{44}$$

$$-\frac{\sqrt{770}G_1\left(17x^4-42x^2y^2-60x^2z^2-3y^4+60y^2z^2\right)}{1232}+\frac{\sqrt{770}G_2xy\left(6x^2-y^2-15z^2\right)}{154}+\frac{\sqrt{1155}G_3xz\left(x^2-3y^2\right)}{44}+\frac{\sqrt{462}G_{3x}xy\left(x^2+y^2-6z^2\right)}{154}$$
$$-\frac{\sqrt{462}G_{3y}\left(13x^4+18x^2y^2-96x^2z^2+5y^4-48y^2z^2+24z^4\right)}{1232}-\frac{9\sqrt{77}G_{az}yz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{1155}G_{bz}yz\left(3x^2-y^2\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{55}G_1xz\left(3x^2+3y^2-4z^2\right)}{44}+\frac{\sqrt{55}G_2yz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{330}G_3\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{88}$$
$$-\frac{7\sqrt{33}G_{3x}yz\left(3x^2-y^2\right)}{44}-\frac{7\sqrt{33}G_{3y}xz\left(x^2-3y^2\right)}{44}+\frac{7\sqrt{22}G_{az}xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{330}G_{bz}xy\left(x^2+y^2-6z^2\right)}{44}$$

$$\frac{\sqrt{55}G_1yz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{55}G_2xz\left(3x^2+3y^2-4z^2\right)}{44}-\frac{\sqrt{330}G_3xy\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}G_{3x}xz\left(x^2-3y^2\right)}{44}$$
$$-\frac{7\sqrt{33}G_{3y}yz\left(3x^2-y^2\right)}{44}-\frac{7\sqrt{22}G_{az}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}+\frac{\sqrt{330}G_{bz}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{88}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,0)}[g](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(3,0)}[g](E_g,3)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{5\sqrt{385}G_1xz\left(3x^2+3y^2-4z^2\right)}{308}+\frac{5\sqrt{385}G_2yz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{2310}G_3\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{308}$$
$$-\frac{\sqrt{231}G_{3x}yz\left(27x^2-y^2-8z^2\right)}{308}+\frac{\sqrt{231}G_{3y}xz\left(x^2-27y^2+8z^2\right)}{308}+\frac{13\sqrt{154}G_{az}xy\left(x^2+y^2-6z^2\right)}{308}+\frac{\sqrt{2310}G_{bz}xy\left(x-y\right)\left(x+y\right)}{44}$$

$$-\frac{5\sqrt{385}G_1yz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{5\sqrt{385}G_2xz\left(3x^2+3y^2-4z^2\right)}{308}+\frac{\sqrt{2310}G_3xy\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{231}G_{3x}xz\left(13x^2-15y^2-8z^2\right)}{308}$$
$$-\frac{\sqrt{231}G_{3y}yz\left(15x^2-13y^2+8z^2\right)}{308}+\frac{13\sqrt{154}G_{az}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{616}+\frac{\sqrt{2310}G_{bz}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{616}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E_g,1)$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{1001}G_1xy\left(13x^4-10x^2y^2-100x^2z^2-23y^4+260y^2z^2-80z^4\right)}{2288}$$
$$+\frac{3\sqrt{1001}G_2\left(2x^6-25x^4y^2-5x^4z^2-20x^2y^4+270x^2y^2z^2-40x^2z^4+7y^6-85y^4z^2+40y^2z^4\right)}{2288}$$
$$+\frac{\sqrt{6006}G_3yz\left(20x^4-5x^2y^2-35x^2z^2-25y^4+85y^2z^2-22z^4\right)}{1144}$$
$$+\frac{\sqrt{15015}G_{3x}\left(2x^6+7x^4y^2-37x^4z^2+8x^2y^4-90x^2y^2z^2+52x^2z^4+3y^6-53y^4z^2+68y^2z^4-8z^6\right)}{2288}$$
$$-\frac{\sqrt{15015}G_{3y}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}+\frac{3\sqrt{10010}G_{az}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$-\frac{\sqrt{6006}G_{bz}xz\left(5x^4+100x^2y^2-50x^2z^2+95y^4-290y^2z^2+44z^4\right)}{2288}$$

$$-\frac{3\sqrt{1001}G_1\left(7x^6-20x^4y^2-85x^4z^2-25x^2y^4+270x^2y^2z^2+40x^2z^4+2y^6-5y^4z^2-40y^2z^4\right)}{2288}$$
$$+\frac{3\sqrt{1001}G_2xy\left(23x^4+10x^2y^2-260x^2z^2-13y^4+100y^2z^2+80z^4\right)}{2288}$$
$$-\frac{\sqrt{6006}G_3xz\left(25x^4+5x^2y^2-85x^2z^2-20y^4+35y^2z^2+22z^4\right)}{1144}-\frac{\sqrt{15015}G_{3x}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}$$
$$+\frac{\sqrt{15015}G_{3y}\left(3x^6+8x^4y^2-53x^4z^2+7x^2y^4-90x^2y^2z^2+68x^2z^4+2y^6-37y^4z^2+52y^2z^4-8z^6\right)}{2288}$$
$$+\frac{3\sqrt{10010}G_{az}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{\sqrt{6006}G_{bz}yz\left(95x^4+100x^2y^2-290x^2z^2+5y^4-50y^2z^2+44z^4\right)}{2288}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{3\sqrt{286}G_1xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}+\frac{3\sqrt{286}G_2yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$+\frac{\sqrt{429}G_3\left(x-y\right)\left(x+y\right)\left(7x^4-118x^2y^2+20x^2z^2+7y^4+20y^2z^2-20z^4\right)}{572}$$
$$-\frac{3\sqrt{4290}G_{3x}yz\left(73x^4-98x^2y^2-48x^2z^2+5y^4+16y^2z^2\right)}{2288}+\frac{3\sqrt{4290}G_{3y}xz\left(5x^4-98x^2y^2+16x^2z^2+73y^4-48y^2z^2\right)}{2288}$$
$$+\frac{9\sqrt{715}G_{az}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{286}-\frac{\sqrt{429}G_{bz}xy\left(13x^4-40x^2y^2-10x^2z^2+13y^4-10y^2z^2+10z^4\right)}{143}$$

$$\frac{3\sqrt{286}G_1yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}-\frac{3\sqrt{286}G_2xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{2288}$$
$$+\frac{\sqrt{429}G_3xy\left(47x^4-170x^2y^2+40x^2z^2+47y^4+40y^2z^2-40z^4\right)}{572}+\frac{3\sqrt{4290}G_{3x}xz\left(17x^4-122x^2y^2-16x^2z^2+37y^4+48y^2z^2\right)}{2288}$$
$$+\frac{3\sqrt{4290}G_{3y}yz\left(37x^4-122x^2y^2+48x^2z^2+17y^4-16y^2z^2\right)}{2288}-\frac{9\sqrt{715}G_{az}\left(x^2+y^2-10z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{1144}$$
$$+\frac{\sqrt{429}G_{bz}\left(x-y\right)\left(x+y\right)\left(19x^4-226x^2y^2-40x^2z^2+19y^4-40y^2z^2+40z^4\right)}{1144}$$

$\vec{\mathbb{Q}}_{4,1}^{(3,2)}[g](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(3,2)}[g](E_g,3)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{9\sqrt{2002}G_1xz\left(4x^4-25x^2y^2-5x^2z^2+15y^4-5y^2z^2+2z^4\right)}{1144}+\frac{9\sqrt{2002}G_2yz\left(15x^4-25x^2y^2-5x^2z^2+4y^4-5y^2z^2+2z^4\right)}{1144}$$
$$+\frac{\sqrt{3003}G_3\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{286}$$
$$+\frac{\sqrt{30030}G_{3x}yz\left(13x^4+17x^2y^2-43x^2z^2+4y^4-19y^2z^2+10z^4\right)}{1144}+\frac{\sqrt{30030}G_{3y}xz\left(4x^4+17x^2y^2-19x^2z^2+13y^4-43y^2z^2+10z^4\right)}{1144}$$
$$-\frac{3\sqrt{5005}G_{az}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{3\sqrt{3003}G_{bz}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}$$

$$\frac{9\sqrt{2002}G_1yz\left(25x^4-60x^2y^2+10x^2z^2+3y^4+10y^2z^2-4z^4\right)}{2288}+\frac{9\sqrt{2002}G_2xz\left(3x^4-60x^2y^2+10x^2z^2+25y^4+10y^2z^2-4z^4\right)}{2288}$$
$$-\frac{3\sqrt{3003}G_3xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{30030}G_{3x}xz\left(17x^4+16x^2y^2-62x^2z^2-y^4-14y^2z^2+20z^4\right)}{2288}$$
$$+\frac{\sqrt{30030}G_{3y}yz\left(x^4-16x^2y^2+14x^2z^2-17y^4+62y^2z^2-20z^4\right)}{2288}-\frac{3\sqrt{5005}G_{az}\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{\sqrt{3003}G_{bz}\left(x^6+15x^4y^2-30x^4z^2+15x^2y^4-180x^2y^2z^2+60x^2z^4+y^6-30y^4z^2+60y^2z^4-8z^6\right)}{1144}$$