PG No. 20 D_{3d} $\bar{3}m$ (-3m1 setting) [trigonal] (polar, internal axial quadrupole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}_1^{(2,1)}[g](A_{2u})$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{30}G_v xy}{5} - \frac{\sqrt{30}G_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}G_{xz}yz}{10} - \frac{\sqrt{30}G_{yz}xz}{10}$$

$\vec{\mathbb{Q}}_{1,1}^{(2,1)}[g](E_u)$, $\vec{\mathbb{Q}}_{1,2}^{(2,1)}[g](E_u)$

** symmetry

$$x$$

$$y$$

** expression

$$-\frac{3\sqrt{10}G_u yz}{10} - \frac{\sqrt{30}G_v yz}{10} + \frac{\sqrt{30}G_{xy}xz}{10} - \frac{\sqrt{30}G_{xz}xy}{10} - \frac{\sqrt{30}G_{yz}(y-z)(y+z)}{10}$$

$$\frac{3\sqrt{10}G_u xz}{10} - \frac{\sqrt{30}G_v xz}{10} - \frac{\sqrt{30}G_{xy}yz}{10} + \frac{\sqrt{30}G_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}G_{yz}xy}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_2^{(2,-1)}[g](A_{1g})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{2}G_{xz}y}{2} - \frac{\sqrt{2}G_{yz}x}{2}$$

$\vec{\mathbb{Q}}_2^{(2,1)}[g](A_{1g})$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{5\sqrt{42}G_v xyz}{14} - \frac{5\sqrt{42}G_{xy}z(x-y)(x+y)}{28} - \frac{\sqrt{42}G_{xz}y\left(x^2+y^2-4z^2\right)}{28} + \frac{\sqrt{42}G_{yz}x\left(x^2+y^2-4z^2\right)}{28}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E_g,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_u x}{2} + \frac{\sqrt{6}G_v x}{6} + \frac{\sqrt{6}G_{xy}y}{6} - \frac{\sqrt{6}G_{xz}z}{6}$$

$$\frac{\sqrt{2}G_u y}{2} - \frac{\sqrt{6}G_v y}{6} + \frac{\sqrt{6}G_{xy}x}{6} - \frac{\sqrt{6}G_{yz}z}{6}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-1)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-1)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{6}G_{xy}z}{3} - \frac{\sqrt{6}G_{xz}y}{6} - \frac{\sqrt{6}G_{yz}x}{6}$$

$$\frac{\sqrt{6}G_{v}z}{3} - \frac{\sqrt{6}G_{xz}x}{6} + \frac{\sqrt{6}G_{yz}y}{6}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E_g, 1)$, $\vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E_g, 1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{42}G_{u}x\left(x^{2} + y^{2} - 4z^{2}\right)}{28} - \frac{\sqrt{14}G_{v}x\left(x^{2} - 9y^{2} + 6z^{2}\right)}{28} - \frac{\sqrt{14}G_{xy}y\left(3x^{2} - 2y^{2} + 3z^{2}\right)}{14} + \frac{\sqrt{14}G_{xz}z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{14}$$

$$-\frac{\sqrt{42}G_{u}y\left(x^{2} + y^{2} - 4z^{2}\right)}{28} - \frac{\sqrt{14}G_{v}y\left(9x^{2} - y^{2} - 6z^{2}\right)}{28} + \frac{\sqrt{14}G_{xy}x\left(2x^{2} - 3y^{2} - 3z^{2}\right)}{14} + \frac{\sqrt{14}G_{yz}z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,1)}[g](E_g, 2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,1)}[g](E_g, 2)$
** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}G_{u}xyz}{14} + \frac{\sqrt{14}G_{xy}z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{28} - \frac{\sqrt{14}G_{xz}y\left(9x^{2} - y^{2} - 6z^{2}\right)}{28} + \frac{\sqrt{14}G_{yz}x\left(x^{2} - 9y^{2} + 6z^{2}\right)}{28}$$

$$-\frac{5\sqrt{42}G_{u}z\left(x - y\right)\left(x + y\right)}{28} + \frac{\sqrt{14}G_{v}z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{28} - \frac{\sqrt{14}G_{xz}x\left(2x^{2} - 3y^{2} - 3z^{2}\right)}{14} - \frac{\sqrt{14}G_{yz}y\left(3x^{2} - 2y^{2} + 3z^{2}\right)}{14}$$

* Harmonics for rank 3
$\vec{\mathbb{Q}}_{3}^{(2,-1)}[g](A_{1u})$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2} - 3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{3}G_{v}yz}{2} + \frac{\sqrt{3}G_{xy}xz}{2} - \frac{\sqrt{3}G_{xz}xy}{2} - \frac{\sqrt{3}G_{yz}\left(x - y\right)\left(x + y\right)}{4}$$

$\vec{\mathbb{Q}}_{3}^{(2,1)}[g](A_{1u})$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2} - 3y^{2}\right)}{4}$$

** expression

$$-\frac{7\sqrt{2}G_{u}yz\left(3x^{2} - y^{2}\right)}{8} + \frac{\sqrt{6}G_{v}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{24} + \frac{\sqrt{6}G_{xy}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{24}$$
$$- \frac{\sqrt{6}G_{xz}xy\left(17x^{2} - 11y^{2} - 18z^{2}\right)}{24} + \frac{\sqrt{6}G_{yz}\left(2x^{4} - 21x^{2}y^{2} + 9x^{2}z^{2} + 5y^{4} - 9y^{2}z^{2}\right)}{24}$$

$\vec{\mathbb{Q}}_{3}^{(2,-1)}[g](A_{2u}, 1)$
** symmetry

$$-\frac{z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{30}G_{v}xy}{10} + \frac{\sqrt{30}G_{xy}\left(x - y\right)\left(x + y\right)}{20} + \frac{\sqrt{30}G_{xz}yz}{5} - \frac{\sqrt{30}G_{yz}xz}{5}$$

$\vec{\mathbb{Q}}_{3}^{(2,-1)}[g](A_{2u}, 2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2} - y^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{3}G_{v}xz}{2} + \frac{\sqrt{3}G_{xy}yz}{2} + \frac{\sqrt{3}G_{xz}\left(x - y\right)\left(x + y\right)}{4} - \frac{\sqrt{3}G_{yz}xy}{2}$$

$\vec{\mathbb{Q}}_{3}^{(2,1)}[g](A_{2u},1)$
** symmetry

$$-\frac{z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_{v}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{6} + \frac{\sqrt{15}G_{xy}\left(x - y\right)\left(x + y\right)\left(x^{2} + y^{2} - 6z^{2}\right)}{12} - \frac{\sqrt{15}G_{xz}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{12} + \frac{\sqrt{15}G_{yz}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3}^{(2,1)}[g](A_{2u},2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2} - y^{2}\right)}{4}$$

** expression

$$\frac{7\sqrt{2}G_{u}xz\left(x^{2} - 3y^{2}\right)}{8} - \frac{\sqrt{6}G_{v}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{24} + \frac{\sqrt{6}G_{xy}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{24} + \frac{\sqrt{6}G_{xz}\left(5x^{4} - 21x^{2}y^{2} - 9x^{2}z^{2} + 2y^{4} + 9y^{2}z^{2}\right)}{24} + \frac{\sqrt{6}G_{yz}xy\left(11x^{2} - 17y^{2} + 18z^{2}\right)}{24}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E_{u},1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E_{u},1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^{2} + y^{2} - 4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2} + y^{2} - 4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{15}G_{u}yz}{5} + \frac{3\sqrt{5}G_{v}yz}{10} - \frac{3\sqrt{5}G_{xy}xz}{10} + \frac{3\sqrt{5}G_{xz}xy}{10} - \frac{\sqrt{5}G_{yz}\left(5x^{2} - y^{2} - 4z^{2}\right)}{20}$$

$$\frac{\sqrt{15}G_{u}xz}{5} + \frac{3\sqrt{5}G_{v}xz}{10} + \frac{3\sqrt{5}G_{xy}yz}{10} - \frac{\sqrt{5}G_{xz}\left(x^{2} - 5y^{2} + 4z^{2}\right)}{20} - \frac{3\sqrt{5}G_{yz}xy}{10}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-1)}[g](E_{u},2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-1)}[g](E_{u},2)$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{6}G_{u}\left(x - y\right)\left(x + y\right)}{4} + \frac{\sqrt{2}G_{v}\left(x^{2} + y^{2} - 2z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}G_{u}xy}{2} - \frac{\sqrt{2}G_{xy}\left(x^{2} + y^{2} - 2z^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](E_{u},1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](E_{u},1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^{2} + y^{2} - 4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2} + y^{2} - 4z^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{30}G_{u}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{24} + \frac{\sqrt{10}G_{v}yz\left(27x^{2} - y^{2} - 8z^{2}\right)}{24} - \frac{\sqrt{10}G_{xy}xz\left(13x^{2} - 15y^{2} - 8z^{2}\right)}{24} - \frac{\sqrt{10}G_{xz}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{24} + \frac{\sqrt{10}G_{yz}\left(2x^{4} + 3x^{2}y^{2} - 15x^{2}z^{2} + y^{4} - 9y^{2}z^{2} + 4z^{4}\right)}{24}$$

$$-\frac{\sqrt{30}G_{u}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{24}-\frac{\sqrt{10}G_{v}xz\left(x^{2}-27y^{2}+8z^{2}\right)}{24}-\frac{\sqrt{10}G_{xy}yz\left(15x^{2}-13y^{2}+8z^{2}\right)}{24}-\frac{\sqrt{10}G_{xz}\left(x^{4}+3x^{2}y^{2}-9x^{2}z^{2}+2y^{4}-15y^{2}z^{2}+4z^{4}\right)}{24}+\frac{\sqrt{10}G_{yz}xy\left(x^{2}+y^{2}-6z^{2}\right)}{24}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,1)}[g](E_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,1)}[g](E_u,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{3}G_{u}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{6}-\frac{G_{v}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{6}-\frac{7G_{xy}xy\left(x-y\right)\left(x+y\right)}{6}+\frac{7G_{xz}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{7G_{yz}yz\left(y-z\right)\left(y+z\right)}{6}$$

$$\frac{\sqrt{3}G_{u}xy\left(x^{2}+y^{2}-6z^{2}\right)}{3}+\frac{7G_{v}xy\left(x-y\right)\left(x+y\right)}{6}-\frac{G_{xy}\left(5x^{4}-18x^{2}y^{2}-12x^{2}z^{2}+5y^{4}-12y^{2}z^{2}+4z^{4}\right)}{12}-\frac{7G_{xz}yz\left(3x^{2}+y^{2}-2z^{2}\right)}{12}-\frac{7G_{yz}xz\left(x^{2}+3y^{2}-2z^{2}\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_{4}^{(2,-1)}[g](A_{1g},1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$-\frac{\sqrt{105}G_{v}xyz}{7}+\frac{\sqrt{105}G_{xy}z\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{105}G_{xz}y\left(x^{2}+y^{2}-4z^{2}\right)}{28}+\frac{\sqrt{105}G_{yz}x\left(x^{2}+y^{2}-4z^{2}\right)}{28}$$

$\vec{\mathbb{Q}}_{4}^{(2,-1)}[g](A_{1g},2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{3\sqrt{2}G_{u}x\left(x^{2}-3y^{2}\right)}{8}+\frac{\sqrt{6}G_{v}x\left(x^{2}+y^{2}-4z^{2}\right)}{8}-\frac{\sqrt{6}G_{xy}y\left(x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{\sqrt{6}G_{xz}z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{6}G_{yz}xyz}{4}$$

$\vec{\mathbb{Q}}_{4}^{(2,1)}[g](A_{1g},1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$-\frac{7\sqrt{330}G_{v}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{7\sqrt{330}G_{xy}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{330}G_{xz}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}-\frac{\sqrt{330}G_{yz}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}$$

$\vec{\mathbb{Q}}_{4}^{(2,1)}[g](A_{1g},2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{3\sqrt{77}G_{u}x\left(x^{2}-3y^{2}\right)\left(x^{2}+y^{2}-8z^{2}\right)}{88}-\frac{\sqrt{231}G_{v}x\left(x^{4}-16x^{2}y^{2}+6x^{2}z^{2}+7y^{4}+6y^{2}z^{2}-4z^{4}\right)}{88}-\frac{\sqrt{231}G_{xy}y\left(4x^{4}-7x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+2z^{4}\right)}{44}+\frac{\sqrt{231}G_{xz}z\left(4x^{4}-9x^{2}y^{2}-5x^{2}z^{2}-y^{4}+5y^{2}z^{2}\right)}{44}+\frac{\sqrt{231}G_{yz}xyz\left(x^{2}-11y^{2}+10z^{2}\right)}{44}$$

$\vec{\mathbb{Q}}_{4}^{(2,-1)}[g](A_{2g})$

** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{3\sqrt{2}G_u y \left(3x^2-y^2\right)}{8}-\frac{\sqrt{6}G_v y \left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{6}G_{xy} x \left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{6}G_{xz} xyz}{4}-\frac{\sqrt{6}G_{yz} z \left(x-y\right)\left(x+y\right)}{8}$$

$\vec{\mathbb{Q}}_4^{(2,1)}[g](A_{2g})$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{77}G_u y \left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{88}+\frac{\sqrt{231}G_v y \left(7x^4-16x^2y^2+6x^2z^2+y^4+6y^2z^2-4z^4\right)}{88}$$
$$-\frac{\sqrt{231}G_{xy} x \left(x^4-7x^2y^2-3x^2z^2+4y^4-3y^2z^2+2z^4\right)}{44}-\frac{\sqrt{231}G_{xz} xyz\left(11x^2-y^2-10z^2\right)}{44}-\frac{\sqrt{231}G_{yz} z\left(x^4+9x^2y^2-5x^2z^2-4y^4+5y^2z^2\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{14}G_u x\left(x^2+y^2-4z^2\right)}{56}-\frac{\sqrt{42}G_v x\left(x^2+5y^2-8z^2\right)}{56}+\frac{\sqrt{42}G_{xy} y\left(x^2-3y^2+8z^2\right)}{56}-\frac{\sqrt{42}G_{xz} z\left(x^2-13y^2+4z^2\right)}{56}-\frac{\sqrt{42}G_{yz} xyz}{4}$$

$$-\frac{3\sqrt{14}G_u y\left(x^2+y^2-4z^2\right)}{56}+\frac{\sqrt{42}G_v y\left(5x^2+y^2-8z^2\right)}{56}-\frac{\sqrt{42}G_{xy} x\left(3x^2-y^2-8z^2\right)}{56}-\frac{\sqrt{42}G_{xz} xyz}{4}+\frac{\sqrt{42}G_{yz} z\left(13x^2-y^2-4z^2\right)}{56}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E_g,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\sqrt{3}G_v xyz+\frac{\sqrt{3}G_{xy} z\left(x-y\right)\left(x+y\right)}{2}-\frac{\sqrt{3}G_{xz} y\left(3x^2-y^2\right)}{4}-\frac{\sqrt{3}G_{yz} x\left(x^2-3y^2\right)}{4}$$

$$-\frac{\sqrt{3}G_v z\left(x-y\right)\left(x+y\right)}{2}+\sqrt{3}G_{xy} xyz+\frac{\sqrt{3}G_{xz} x\left(x^2-3y^2\right)}{4}-\frac{\sqrt{3}G_{yz} y\left(3x^2-y^2\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-1)}[g](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-1)}[g](E_g,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{6\sqrt{7}G_u xyz}{7}-\frac{\sqrt{21}G_{xy} z\left(3x^2+3y^2-2z^2\right)}{14}+\frac{\sqrt{21}G_{xz} y\left(2x^2-y^2+z^2\right)}{14}-\frac{\sqrt{21}G_{yz} x\left(x^2-2y^2-z^2\right)}{14}$$

$$-\frac{3\sqrt{7}G_u z\left(x-y\right)\left(x+y\right)}{7}-\frac{\sqrt{21}G_v z\left(3x^2+3y^2-2z^2\right)}{14}+\frac{\sqrt{21}G_{xz} x\left(x^2-5y^2+2z^2\right)}{28}+\frac{\sqrt{21}G_{yz} y\left(5x^2-y^2-2z^2\right)}{28}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E_g,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{11}G_{u}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}+\frac{\sqrt{33}G_{v}x\left(x^{4}-12x^{2}y^{2}+2x^{2}z^{2}-13y^{4}+114y^{2}z^{2}-20z^{4}\right)}{88}+\frac{\sqrt{33}G_{xy}y\left(4x^{4}+x^{2}y^{2}-27x^{2}z^{2}-3y^{4}+29y^{2}z^{2}-10z^{4}\right)}{44}-\frac{\sqrt{33}G_{xz}z\left(4x^{4}+15x^{2}y^{2}-13x^{2}z^{2}+11y^{4}-27y^{2}z^{2}+4z^{4}\right)}{44}+\frac{7\sqrt{33}G_{yz}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}$$

$$\frac{3\sqrt{11}G_{u}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}+\frac{\sqrt{33}G_{v}y\left(13x^{4}+12x^{2}y^{2}-114x^{2}z^{2}-y^{4}-2y^{2}z^{2}+20z^{4}\right)}{88}-\frac{\sqrt{33}G_{xy}x\left(3x^{4}-x^{2}y^{2}-29x^{2}z^{2}-4y^{4}+27y^{2}z^{2}+10z^{4}\right)}{44}+\frac{7\sqrt{33}G_{xz}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}-\frac{\sqrt{33}G_{yz}z\left(11x^{4}+15x^{2}y^{2}-27x^{2}z^{2}+4y^{4}-13y^{2}z^{2}+4z^{4}\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E_g,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{9\sqrt{154}G_{u}xyz\left(x-y\right)\left(x+y\right)}{22}+\frac{\sqrt{462}G_{v}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{\sqrt{462}G_{xy}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}-\frac{\sqrt{462}G_{xz}y\left(9x^{4}-14x^{2}y^{2}-12x^{2}z^{2}+y^{4}+4y^{2}z^{2}\right)}{88}+\frac{\sqrt{462}G_{yz}x\left(x^{4}-14x^{2}y^{2}+4x^{2}z^{2}+9y^{4}-12y^{2}z^{2}\right)}{88}$$

$$\frac{9\sqrt{154}G_{u}z\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{88}-\frac{\sqrt{462}G_{v}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{462}G_{xy}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{\sqrt{462}G_{xz}x\left(x^{4}-8x^{2}y^{2}-2x^{2}z^{2}+3y^{4}+6y^{2}z^{2}\right)}{44}+\frac{\sqrt{462}G_{yz}y\left(3x^{4}-8x^{2}y^{2}+6x^{2}z^{2}+y^{4}-2y^{2}z^{2}\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,1)}[g](E_g,3)$, $\vec{\mathbb{Q}}_{4,2}^{(2,1)}[g](E_g,3)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{21\sqrt{22}G_{u}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{21\sqrt{66}G_{v}xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}G_{xy}z\left(9x^{4}-24x^{2}y^{2}-10x^{2}z^{2}+9y^{4}-10y^{2}z^{2}+2z^{4}\right)}{44}+\frac{\sqrt{66}G_{xz}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}+\frac{\sqrt{66}G_{yz}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}$$

$$\frac{21\sqrt{22}G_{u}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}+\frac{\sqrt{66}G_{v}z\left(3x^{4}-78x^{2}y^{2}+20x^{2}z^{2}+3y^{4}+20y^{2}z^{2}-4z^{4}\right)}{88}+\frac{21\sqrt{66}G_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}+\frac{\sqrt{66}G_{xz}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}-\frac{\sqrt{66}G_{yz}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{44}$$