PG No. 35 $C_{3v}(1)$ $3m$ (31m setting) [trigonal] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1
  $\vec{\mathbb{Q}}_{1}^{(1,0)}[g](A_1)$
  ** symmetry

  z

  ** expression

  $$\frac{\sqrt{2}G_{x}y}{2} - \frac{\sqrt{2}G_{y}x}{2}$$

  $\vec{\mathbb{Q}}_{1,1}^{(1,0)}[g](E)$, $\vec{\mathbb{Q}}_{1,2}^{(1,0)}[g](E)$
  ** symmetry

  x

  y

  ** expression

  $$\frac{\sqrt{2}G_{y}z}{2} - \frac{\sqrt{2}G_{z}y}{2}$$

  $$-\frac{\sqrt{2}G_{x}z}{2} + \frac{\sqrt{2}G_{z}x}{2}$$

* Harmonics for rank 2
  $\vec{\mathbb{Q}}_{2}^{(1,0)}[g](A_1)$
  ** symmetry

  $$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

  ** expression

  $$\frac{\sqrt{6}G_{x}yz}{2} - \frac{\sqrt{6}G_{y}xz}{2}$$

  $\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E,1)$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E,1)$
  ** symmetry

  $\sqrt{3}xz$

  $\sqrt{3}yz$

  ** expression

  $$\frac{\sqrt{2}G_{x}xy}{2} - \frac{\sqrt{2}G_{y}\left(x-z\right)\left(x+z\right)}{2} - \frac{\sqrt{2}G_{z}yz}{2}$$

  $$\frac{\sqrt{2}G_{x}\left(y-z\right)\left(y+z\right)}{2} - \frac{\sqrt{2}G_{y}xy}{2} + \frac{\sqrt{2}G_{z}xz}{2}$$

  $\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E,2)$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E,2)$
  ** symmetry

  $$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

  $-\sqrt{3}xy$

  ** expression

  $$\frac{\sqrt{2}G_{x}yz}{2} + \frac{\sqrt{2}G_{y}xz}{2} - \sqrt{2}G_{z}xy$$

  $$\frac{\sqrt{2}G_{x}xz}{2} - \frac{\sqrt{2}G_{y}yz}{2} - \frac{\sqrt{2}G_{z}\left(x-y\right)\left(x+y\right)}{2}$$

* Harmonics for rank 3
  $\vec{\mathbb{Q}}_{3}^{(1,0)}[g](A_1,1)$
  ** symmetry

  $$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}G_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}G_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](A_1,2)$
** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{30}G_x xyz}{4}+\frac{\sqrt{30}G_y z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}G_z y\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{Q}}_3^{(1,0)}[g](A_2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}G_x z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}G_y xyz}{4}+\frac{\sqrt{30}G_z x\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E,1)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{5\sqrt{2}G_x xyz}{4}-\frac{\sqrt{2}G_y z\left(11x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{2}G_z y\left(x^2+y^2-4z^2\right)}{8}$$

$$\frac{\sqrt{2}G_x z\left(x^2+11y^2-4z^2\right)}{8}-\frac{5\sqrt{2}G_y xyz}{4}-\frac{\sqrt{2}G_z x\left(x^2+y^2-4z^2\right)}{8}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E,2)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E,2)$
** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{5}G_x y\left(x^2-y^2+2z^2\right)}{4}+\frac{\sqrt{5}G_y x\left(x^2-y^2-2z^2\right)}{4}+\sqrt{5}G_z xyz$$

$$\frac{\sqrt{5}G_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}G_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}G_z z\left(x-y\right)\left(x+y\right)}{2}$$

* Harmonics for rank 4
$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_1,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{\sqrt{5}G_x yz\left(3x^2+3y^2-4z^2\right)}{4}+\frac{\sqrt{5}G_y xz\left(3x^2+3y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_1,2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}G_x xy\left(x^2-3y^2+6z^2\right)}{8}-\frac{\sqrt{14}G_y\left(x^4-3x^2y^2-3x^2z^2+3y^2z^2\right)}{8}-\frac{3\sqrt{14}G_z yz\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{Q}}_4^{(1,0)}[g](A_2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}G_x\left(3x^2y^2-3x^2z^2-y^4+3y^2z^2\right)}{8}-\frac{\sqrt{14}G_y xy\left(3x^2-y^2-6z^2\right)}{8}+\frac{3\sqrt{14}G_z xz\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E,1)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{2}G_x xy\left(x^2+y^2-6z^2\right)}{8}+\frac{\sqrt{2}G_y\left(3x^4+3x^2y^2-21x^2z^2-3y^2z^2+4z^4\right)}{8}+\frac{\sqrt{2}G_z yz\left(3x^2+3y^2-4z^2\right)}{8}$$

$$-\frac{\sqrt{2}G_x\left(3x^2y^2-3x^2z^2+3y^4-21y^2z^2+4z^4\right)}{8}+\frac{3\sqrt{2}G_y xy\left(x^2+y^2-6z^2\right)}{8}-\frac{\sqrt{2}G_z xz\left(3x^2+3y^2-4z^2\right)}{8}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E,2)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_x yz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}G_y xz\left(x^2-3y^2\right)}{4}-\sqrt{7}G_z xy\left(x-y\right)\left(x+y\right)$$

$$-\frac{\sqrt{7}G_x xz\left(x^2-3y^2\right)}{4}+\frac{\sqrt{7}G_y yz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}G_z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E,3)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{G_x yz\left(3x^2-4y^2+3z^2\right)}{2}-\frac{G_y xz\left(4x^2-3y^2-3z^2\right)}{2}+\frac{G_z xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{G_x xz\left(x^2+15y^2-6z^2\right)}{4}+\frac{G_y yz\left(15x^2+y^2-6z^2\right)}{4}+\frac{G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$