

PG No. 35 $C_{3v}(1)$ $3m$ (31m setting) [trigonal] (polar, internal axial octupole)

* Harmonics for rank 0

* Harmonics for rank 1

$$\bar{Q}_1^{(3,2)}[g](A_1)$$

** symmetry

$$z$$

** expression

$$-\frac{3\sqrt{70}G_1x(x^2-3y^2)}{56} + \frac{3\sqrt{70}G_2y(3x^2-y^2)}{56} - \frac{\sqrt{105}G_3z(x-y)(x+y)}{14} - \frac{\sqrt{42}G_{3x}y(x^2+y^2-4z^2)}{56} + \frac{\sqrt{42}G_{3y}x(x^2+y^2-4z^2)}{56} + \frac{\sqrt{105}G_{b_2}xyz}{7}$$

$$\bar{Q}_{1,1}^{(3,2)}[g](E), \bar{Q}_{1,2}^{(3,2)}[g](E)$$

** symmetry

$$x$$

$$y$$

** expression

$$\begin{aligned} & \frac{3\sqrt{70}G_1z(x-y)(x+y)}{56} - \frac{3\sqrt{70}G_2xyz}{28} - \frac{\sqrt{105}G_3x(y-z)(y+z)}{14} - \frac{5\sqrt{42}G_{3x}xyz}{28} \\ & - \frac{\sqrt{42}G_{3y}z(x^2+11y^2-4z^2)}{56} + \frac{3\sqrt{7}G_{a_2}y(x^2+y^2-4z^2)}{28} - \frac{\sqrt{105}G_{b_2}y(x^2-y^2+2z^2)}{28} \\ & - \frac{3\sqrt{70}G_1xyz}{28} - \frac{3\sqrt{70}G_2z(x-y)(x+y)}{56} + \frac{\sqrt{105}G_3y(x-z)(x+z)}{14} + \frac{\sqrt{42}G_{3x}z(11x^2+y^2-4z^2)}{56} \\ & + \frac{5\sqrt{42}G_{3y}xyz}{28} - \frac{3\sqrt{7}G_{a_2}x(x^2+y^2-4z^2)}{28} + \frac{\sqrt{105}G_{b_2}x(x^2-y^2-2z^2)}{28} \end{aligned}$$

* Harmonics for rank 2

$$\bar{Q}_2^{(3,0)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{210}G_3(x-y)(x+y)}{28} + \frac{\sqrt{21}G_{3x}yz}{7} - \frac{\sqrt{21}G_{3y}xz}{7} + \frac{\sqrt{210}G_{b_2}xy}{14}$$

$$\bar{Q}_2^{(3,2)}[g](A_1)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\begin{aligned} & -\frac{\sqrt{70}G_1xz(x^2-3y^2)}{8} + \frac{\sqrt{70}G_2yz(3x^2-y^2)}{8} + \frac{\sqrt{105}G_3(x-y)(x+y)(x^2+y^2-6z^2)}{42} \\ & - \frac{5\sqrt{42}G_{3x}yz(3x^2+3y^2-4z^2)}{168} + \frac{5\sqrt{42}G_{3y}xz(3x^2+3y^2-4z^2)}{168} - \frac{\sqrt{105}G_{b_2}xy(x^2+y^2-6z^2)}{21} \end{aligned}$$

$$\bar{Q}_{2,1}^{(3,0)}[g](E, 1), \bar{Q}_{2,2}^{(3,0)}[g](E, 1)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{105}G_1(x-y)(x+y)}{28} + \frac{\sqrt{105}G_2xy}{14} - \frac{3\sqrt{7}G_{3x}xy}{14} + \frac{\sqrt{7}G_{3y}(x^2-5y^2+4z^2)}{28} - \frac{\sqrt{42}G_{a_2}yz}{7}$$

$$\frac{\sqrt{105}G_1xy}{14} + \frac{\sqrt{105}G_2(x-y)(x+y)}{28} + \frac{\sqrt{7}G_{3x}(5x^2-y^2-4z^2)}{28} + \frac{3\sqrt{7}G_{3y}xy}{14} + \frac{\sqrt{42}G_{a_2}xz}{7}$$

$$\bar{Q}_{2,1}^{(3,0)}[g](E, 2), \bar{Q}_{2,2}^{(3,0)}[g](E, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{105}G_1xz}{14} - \frac{\sqrt{105}G_2yz}{14} - \frac{\sqrt{70}G_3(x^2 + y^2 - 2z^2)}{28} - \frac{3\sqrt{7}G_{3x}yz}{14} - \frac{3\sqrt{7}G_{3y}xz}{14} + \frac{\sqrt{42}G_{az}xy}{14}$$

$$\frac{\sqrt{105}G_1yz}{14} + \frac{\sqrt{105}G_2xz}{14} - \frac{3\sqrt{7}G_{3x}xz}{14} + \frac{3\sqrt{7}G_{3y}yz}{14} + \frac{\sqrt{42}G_{az}(x-y)(x+y)}{28} - \frac{\sqrt{70}G_{bz}(x^2 + y^2 - 2z^2)}{28}$$

$$\bar{Q}_{2,1}^{(3,2)}[g](E, 1), \bar{Q}_{2,2}^{(3,2)}[g](E, 1)$$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$-\frac{\sqrt{210}G_1(5x^4 - 21x^2y^2 - 9x^2z^2 + 2y^4 + 9y^2z^2)}{168} + \frac{\sqrt{210}G_2xy(17x^2 - 11y^2 - 18z^2)}{168} - \frac{\sqrt{35}G_3xz(x-z)(x+z)}{6} + \frac{5\sqrt{14}G_{3x}xy(x^2 + y^2 - 6z^2)}{168}$$

$$+ \frac{5\sqrt{14}G_{3y}(x^4 + 3x^2y^2 - 9x^2z^2 + 2y^4 - 15y^2z^2 + 4z^4)}{168} + \frac{5\sqrt{21}G_{az}yz(3x^2 + 3y^2 - 4z^2)}{84} + \frac{\sqrt{35}G_{bz}yz(3x^2 + y^2 - 2z^2)}{12}$$

$$-\frac{\sqrt{210}G_1xy(11x^2 - 17y^2 + 18z^2)}{168} - \frac{\sqrt{210}G_2(2x^4 - 21x^2y^2 + 9x^2z^2 + 5y^4 - 9y^2z^2)}{168}$$

$$+ \frac{\sqrt{35}G_3yz(y-z)(y+z)}{6} - \frac{5\sqrt{14}G_{3x}(2x^4 + 3x^2y^2 - 15x^2z^2 + y^4 - 9y^2z^2 + 4z^4)}{168}$$

$$-\frac{5\sqrt{14}G_{3y}xy(x^2 + y^2 - 6z^2)}{168} - \frac{5\sqrt{21}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{84} + \frac{\sqrt{35}G_{bz}xz(x^2 + 3y^2 - 2z^2)}{12}$$

$$\bar{Q}_{2,1}^{(3,2)}[g](E, 2), \bar{Q}_{2,2}^{(3,2)}[g](E, 2)$$

** symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{210}G_1xz(3x^2 + 3y^2 - 4z^2)}{168} - \frac{\sqrt{210}G_2yz(3x^2 + 3y^2 - 4z^2)}{168} + \frac{\sqrt{35}G_3(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{42}$$

$$-\frac{5\sqrt{14}G_{3x}yz(27x^2 - y^2 - 8z^2)}{168} + \frac{5\sqrt{14}G_{3y}xz(x^2 - 27y^2 + 8z^2)}{168} + \frac{5\sqrt{21}G_{az}xy(x^2 + y^2 - 6z^2)}{42} - \frac{\sqrt{35}G_{bz}xy(x-y)(x+y)}{6}$$

$$\frac{\sqrt{210}G_1yz(3x^2 + 3y^2 - 4z^2)}{168} + \frac{\sqrt{210}G_2xz(3x^2 + 3y^2 - 4z^2)}{168} - \frac{\sqrt{35}G_3xy(x-y)(x+y)}{6} - \frac{5\sqrt{14}G_{3x}xz(13x^2 - 15y^2 - 8z^2)}{168}$$

$$-\frac{5\sqrt{14}G_{3y}yz(15x^2 - 13y^2 + 8z^2)}{168} + \frac{5\sqrt{21}G_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{84} - \frac{\sqrt{35}G_{bz}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{84}$$

* Harmonics for rank 3

$$\bar{Q}_3^{(3,-2)}[g](A_1, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{2}G_{3xy}}{2} - \frac{\sqrt{2}G_{3yx}}{2}$$

$$\bar{Q}_3^{(3,-2)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{3}G_1z}{2} - \frac{\sqrt{2}G_3x}{4} - \frac{\sqrt{2}G_{bz}y}{4}$$

$$\bar{Q}_3^{(3,0)}[g](A_1, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{15}G_1x(x^2 - 3y^2)}{12} - \frac{\sqrt{15}G_2y(3x^2 - y^2)}{12} - \frac{\sqrt{10}G_3z(x-y)(x+y)}{4} - \frac{G_{3xy}(x^2 + y^2 - 4z^2)}{4} + \frac{G_{3yx}(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{10}G_{bxyz}}{2}$$

$$\bar{Q}_3^{(3,0)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{6}G_1z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_3x(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{10}G_{3xxyz}}{2} - \frac{\sqrt{10}G_{3yz}(x-y)(x+y)}{4} + \frac{\sqrt{15}G_{az}y(3x^2 - y^2)}{12} - \frac{G_{bzy}(x^2 + y^2 - 4z^2)}{4}$$

$$\bar{Q}_3^{(3,2)}[g](A_1, 1)$$

** symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{7\sqrt{330}G_1x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{528} - \frac{7\sqrt{330}G_2y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{528} + \frac{7\sqrt{55}G_3z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{44} + \frac{5\sqrt{22}G_{3xy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{5\sqrt{22}G_{3yx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{7\sqrt{55}G_{bxyz}(x^2 + y^2 - 2z^2)}{22}$$

$$\bar{Q}_3^{(3,2)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

** expression

$$\frac{\sqrt{33}G_1z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} + \frac{\sqrt{22}G_3x(4x^4 - 55x^2y^2 + 15x^2z^2 + 25y^4 + 15y^2z^2 - 10z^4)}{88} - \frac{7\sqrt{55}G_{3xxyz}(2x^2 - y^2 - z^2)}{22} + \frac{7\sqrt{55}G_{3yz}(x^4 - 18x^2y^2 + 4x^2z^2 + 5y^4 - 4y^2z^2)}{176} + \frac{7\sqrt{330}G_{az}y(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{528} - \frac{\sqrt{22}G_{bzy}(55x^4 - 100x^2y^2 - 30x^2z^2 + 13y^4 - 30y^2z^2 + 20z^4)}{176}$$

$$\bar{Q}_3^{(3,-2)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{3}G_2z}{2} - \frac{\sqrt{2}G_3y}{4} + \frac{\sqrt{2}G_{bzx}}{4}$$

$$\bar{Q}_3^{(3,0)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{6}G_2z(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_3y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{10}G_{3xz}(x-y)(x+y)}{4} - \frac{\sqrt{10}G_{3yxyz}}{2} - \frac{\sqrt{15}G_{az}x(x^2 - 3y^2)}{12} + \frac{G_{bzx}(x^2 + y^2 - 4z^2)}{4}$$

$$\bar{Q}_3^{(3,2)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{10}y(3x^2 - y^2)}{4}$$

** expression

$$-\frac{\sqrt{33}G_2z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} + \frac{\sqrt{22}G_3y(25x^4 - 55x^2y^2 + 15x^2z^2 + 4y^4 + 15y^2z^2 - 10z^4)}{88} + \frac{7\sqrt{55}G_{3xz}(5x^4 - 18x^2y^2 - 4x^2z^2 + y^4 + 4y^2z^2)}{176} + \frac{7\sqrt{55}G_{3yxyz}(x^2 - 2y^2 + z^2)}{22} - \frac{7\sqrt{330}G_{az}x(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{528} + \frac{\sqrt{22}G_{bzx}(13x^4 - 100x^2y^2 - 30x^2z^2 + 55y^4 - 30y^2z^2 + 20z^4)}{176}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E, 1), \bar{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E, 1)$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{30}G_3x}{12} + \frac{\sqrt{3}G_{3y}z}{6} - \frac{\sqrt{2}G_{az}y}{2} + \frac{\sqrt{30}G_{bz}y}{12}$$

$$\frac{\sqrt{30}G_3y}{12} - \frac{\sqrt{3}G_{3x}z}{6} + \frac{\sqrt{2}G_{az}x}{2} + \frac{\sqrt{30}G_{bz}x}{12}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,-2)}[g](E, 2), \bar{\mathbb{Q}}_{3,2}^{(3,-2)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{2}G_1x}{4} - \frac{\sqrt{2}G_2y}{4} - \frac{\sqrt{3}G_3z}{3} + \frac{\sqrt{30}G_{3xy}}{12} + \frac{\sqrt{30}G_{3yx}}{12}$$

$$\frac{\sqrt{2}G_1y}{4} + \frac{\sqrt{2}G_2x}{4} + \frac{\sqrt{30}G_{3xx}}{12} - \frac{\sqrt{30}G_{3yy}}{12} - \frac{\sqrt{3}G_{bz}z}{3}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,0)}[g](E, 1), \bar{\mathbb{Q}}_{3,2}^{(3,0)}[g](E, 1)$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$-\frac{\sqrt{10}G_1z(x-y)(x+y)}{4} + \frac{\sqrt{10}G_2xyz}{2} - \frac{\sqrt{15}G_3x(x^2 - 3y^2)}{12} - \frac{\sqrt{6}G_{3yz}(3x^2 + 3y^2 - 2z^2)}{12} + \frac{G_{az}y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{15}G_{bz}y(3x^2 - y^2)}{12}$$

$$\frac{\sqrt{10}G_1xyz}{2} + \frac{\sqrt{10}G_2z(x-y)(x+y)}{4} - \frac{\sqrt{15}G_3y(3x^2 - y^2)}{12} + \frac{\sqrt{6}G_{3xz}(3x^2 + 3y^2 - 2z^2)}{12} - \frac{G_{az}x(x^2 + y^2 - 4z^2)}{4} - \frac{\sqrt{15}G_{bz}x(x^2 - 3y^2)}{12}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,0)}[g](E, 2), \bar{\mathbb{Q}}_{3,2}^{(3,0)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{G_1x(x^2 + y^2 - 4z^2)}{4} - \frac{G_2y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{6}G_3z(3x^2 + 3y^2 - 2z^2)}{12} + \frac{\sqrt{15}G_{3xy}(3x^2 - y^2)}{12} - \frac{\sqrt{15}G_{3yx}(x^2 - 3y^2)}{12} + \frac{\sqrt{10}G_{az}xyz}{2}$$

$$\frac{G_1y(x^2 + y^2 - 4z^2)}{4} + \frac{G_2x(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{15}G_{3xx}(x^2 - 3y^2)}{12} + \frac{\sqrt{15}G_{3yy}(3x^2 - y^2)}{12} + \frac{\sqrt{10}G_{az}z(x-y)(x+y)}{4} + \frac{\sqrt{6}G_{bz}z(3x^2 + 3y^2 - 2z^2)}{12}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,2)}[g](E, 1), \bar{\mathbb{Q}}_{3,2}^{(3,2)}[g](E, 1)$$

** symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned}
& -\frac{7\sqrt{55}G_1z(5x^4 - 18x^2y^2 - 4x^2z^2 + y^4 + 4y^2z^2)}{176} + \frac{7\sqrt{55}G_2xyz(2x^2 - y^2 - z^2)}{22} \\
& + \frac{\sqrt{330}G_3x(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4)}{264} + \frac{5\sqrt{33}G_3yz(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} \\
& - \frac{5\sqrt{22}G_{az}y(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{\sqrt{330}G_{bz}y(15x^4 + 16x^2y^2 - 138x^2z^2 + y^4 - 26y^2z^2 + 36z^4)}{528} \\
& - \frac{7\sqrt{55}G_1xyz(x^2 - 2y^2 + z^2)}{22} - \frac{7\sqrt{55}G_2z(x^4 - 18x^2y^2 + 4x^2z^2 + 5y^4 - 4y^2z^2)}{176} \\
& + \frac{\sqrt{330}G_3y(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264} - \frac{5\sqrt{33}G_3xz(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{528} \\
& + \frac{5\sqrt{22}G_{az}x(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{176} - \frac{\sqrt{330}G_{bz}x(x^4 + 16x^2y^2 - 26x^2z^2 + 15y^4 - 138y^2z^2 + 36z^4)}{528}
\end{aligned}$$

$$\bar{\mathbb{Q}}_{3,1}^{(3,2)}[g](E, 2), \bar{\mathbb{Q}}_{3,2}^{(3,2)}[g](E, 2)$$

** symmetry

$$-\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\begin{aligned}
& \frac{\sqrt{22}G_1x(13x^4 - 100x^2y^2 - 30x^2z^2 + 55y^4 - 30y^2z^2 + 20z^4)}{176} - \frac{\sqrt{22}G_2y(55x^4 - 100x^2y^2 - 30x^2z^2 + 13y^4 - 30y^2z^2 + 20z^4)}{176} \\
& + \frac{\sqrt{33}G_3z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132} - \frac{\sqrt{330}G_3xy(15x^4 + 16x^2y^2 - 138x^2z^2 + y^4 - 26y^2z^2 + 36z^4)}{528} \\
& - \frac{\sqrt{330}G_3yx(x^4 + 16x^2y^2 - 26x^2z^2 + 15y^4 - 138y^2z^2 + 36z^4)}{528} - \frac{7\sqrt{55}G_{az}xyz(x^2 + y^2 - 2z^2)}{22} \\
& - \frac{\sqrt{22}G_1y(25x^4 - 55x^2y^2 + 15x^2z^2 + 4y^4 + 15y^2z^2 - 10z^4)}{88} - \frac{\sqrt{22}G_2x(4x^4 - 55x^2y^2 + 15x^2z^2 + 25y^4 + 15y^2z^2 - 10z^4)}{88} \\
& - \frac{\sqrt{330}G_3xx(4x^4 + x^2y^2 - 41x^2z^2 - 3y^4 + 15y^2z^2 + 18z^4)}{264} - \frac{\sqrt{330}G_3yy(3x^4 - x^2y^2 - 15x^2z^2 - 4y^4 + 41y^2z^2 - 18z^4)}{264} \\
& - \frac{7\sqrt{55}G_{az}z(x-y)(x+y)(x^2 + y^2 - 2z^2)}{44} + \frac{\sqrt{33}G_{bz}z(15x^4 + 30x^2y^2 - 40x^2z^2 + 15y^4 - 40y^2z^2 + 8z^4)}{132}
\end{aligned}$$

* Harmonics for rank 4

$$\bar{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\frac{\sqrt{21}G_3(x-y)(x+y)}{14} + \frac{\sqrt{210}G_3xyz}{14} - \frac{\sqrt{210}G_3yxz}{14} - \frac{\sqrt{21}G_{bz}xy}{7}$$

$$\bar{\mathbb{Q}}_4^{(3,-2)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$-\frac{3\sqrt{5}G_1(x^2 + y^2 - 2z^2)}{20} + \frac{\sqrt{30}G_3xz}{20} - \frac{\sqrt{3}G_{3x}xy}{2} - \frac{\sqrt{3}G_{3y}(x-y)(x+y)}{4} + \frac{\sqrt{30}G_{bz}yz}{20}$$

$$\bar{\mathbb{Q}}_4^{(3,0)}[g](A_1, 1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned}
& \frac{5\sqrt{77}G_1xz(x^2 - 3y^2)}{44} - \frac{5\sqrt{77}G_2yz(3x^2 - y^2)}{44} + \frac{5\sqrt{462}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{616} \\
& - \frac{3\sqrt{1155}G_3xyz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{3\sqrt{1155}G_3yxz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{5\sqrt{462}G_{bz}xy(x^2 + y^2 - 6z^2)}{308}
\end{aligned}$$

$$\bar{Q}_4^{(3,0)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & - \frac{\sqrt{110}G_1(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{\sqrt{165}G_3xz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{66}G_{3x}xy(3x^2 - 4y^2 + 3z^2)}{22} \\ & + \frac{\sqrt{66}G_{3y}(9x^4 - 42x^2y^2 - 12x^2z^2 + 5y^4 + 12y^2z^2)}{176} - \frac{7\sqrt{11}G_{az}yz(3x^2 - y^2)}{44} - \frac{\sqrt{165}G_{bz}yz(3x^2 + 3y^2 - 4z^2)}{44} \end{aligned}$$

$$\bar{Q}_4^{(3,2)}[g](A_1, 1)$$

** symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

** expression

$$\begin{aligned} & \frac{9\sqrt{10010}G_1xz(x^2 - 3y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{9\sqrt{10010}G_2yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} \\ & - \frac{\sqrt{15015}G_3(x-y)(x+y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} + \frac{5\sqrt{6006}G_{3x}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\ & - \frac{5\sqrt{6006}G_{3y}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{\sqrt{15015}G_{bz}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{286} \end{aligned}$$

$$\bar{Q}_4^{(3,2)}[g](A_1, 2)$$

** symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{143}G_1(19x^6 - 240x^4y^2 - 45x^4z^2 + 255x^2y^4 - 90x^2y^2z^2 + 60x^2z^4 - 14y^6 - 45y^4z^2 + 60y^2z^4 - 8z^6)}{2288} + \frac{9\sqrt{143}G_2xy(x^2 - 3y^2)(3x^2 - y^2)}{208} \\ & - \frac{\sqrt{858}G_3xz(13x^4 + 125x^2y^2 - 85x^2z^2 - 20y^4 - 85y^2z^2 + 34z^4)}{1144} + \frac{\sqrt{2145}G_{3x}xy(43x^4 + 26x^2y^2 - 508x^2z^2 - 17y^4 + 92y^2z^2 + 208z^4)}{2288} \\ & - \frac{\sqrt{2145}G_{3y}(x^6 - 44x^4y^2 + 29x^4z^2 - 31x^2y^4 + 450x^2y^2z^2 - 104x^2z^4 + 14y^6 - 179y^4z^2 + 104y^2z^4)}{2288} \\ & + \frac{27\sqrt{1430}G_{az}yz(3x^2 - y^2)(3x^2 + 3y^2 - 8z^2)}{2288} - \frac{\sqrt{858}G_{bz}yz(125x^4 - 80x^2y^2 - 170x^2z^2 + 59y^4 - 170y^2z^2 + 68z^4)}{2288} \end{aligned}$$

$$\bar{Q}_4^{(3,-2)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\frac{3\sqrt{5}G_2(x^2 + y^2 - 2z^2)}{20} + \frac{\sqrt{30}G_3yz}{20} + \frac{\sqrt{3}G_{3x}(x-y)(x+y)}{4} - \frac{\sqrt{3}G_{3y}xy}{2} - \frac{\sqrt{30}G_{bz}xz}{20}$$

$$\bar{Q}_4^{(3,0)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{\sqrt{110}G_2(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4)}{176} - \frac{\sqrt{165}G_3yz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{66}G_{3x}(5x^4 - 42x^2y^2 + 12x^2z^2 + 9y^4 - 12y^2z^2)}{176} \\ & + \frac{\sqrt{66}G_{3y}xy(4x^2 - 3y^2 - 3z^2)}{22} + \frac{7\sqrt{11}G_{az}xz(x^2 - 3y^2)}{44} + \frac{\sqrt{165}G_{bz}xz(3x^2 + 3y^2 - 4z^2)}{44} \end{aligned}$$

$$\bar{Q}_4^{(3,2)}[g](A_2)$$

** symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

** expression

$$\begin{aligned} & \frac{9\sqrt{143}G_1xy(x^2-3y^2)(3x^2-y^2)}{208} - \frac{3\sqrt{143}G_2(14x^6-255x^4y^2+45x^4z^2+240x^2y^4+90x^2y^2z^2-60x^2z^4-19y^6+45y^4z^2-60y^2z^4+8z^6)}{2288} \\ & + \frac{\sqrt{858}G_3yz(20x^4-125x^2y^2+85x^2z^2-13y^4+85y^2z^2-34z^4)}{1144} \\ & - \frac{\sqrt{2145}G_{3x}(14x^6-31x^4y^2-179x^4z^2-44x^2y^4+450x^2y^2z^2+104x^2z^4+y^6+29y^4z^2-104y^2z^4)}{2288} \\ & - \frac{\sqrt{2145}G_{3y}xy(17x^4-26x^2y^2-92x^2z^2-43y^4+508y^2z^2-208z^4)}{2288} - \frac{27\sqrt{1430}G_{az}xz(x^2-3y^2)(3x^2+3y^2-8z^2)}{2288} \\ & + \frac{\sqrt{858}G_{bz}xz(59x^4-80x^2y^2-170x^2z^2+125y^4-170y^2z^2+68z^4)}{2288} \end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,1), \bar{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,1)$$

** symmetry

$$\begin{aligned} & -\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4} \\ & -\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4} \end{aligned}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{35}G_1(x-y)(x+y)}{140} - \frac{3\sqrt{35}G_2xy}{70} - \frac{\sqrt{210}G_3xz}{20} + \frac{\sqrt{21}G_{3x}xy}{7} - \frac{\sqrt{21}G_{3y}(3x^2-y^2-2z^2)}{28} - \frac{3\sqrt{14}G_{az}yz}{14} + \frac{\sqrt{210}G_{bz}yz}{20} \\ & - \frac{3\sqrt{35}G_1xy}{70} - \frac{3\sqrt{35}G_2(x-y)(x+y)}{140} + \frac{\sqrt{210}G_3yz}{20} - \frac{\sqrt{21}G_{3x}(x^2-3y^2+2z^2)}{28} - \frac{\sqrt{21}G_{3y}xy}{7} + \frac{3\sqrt{14}G_{az}xz}{14} + \frac{\sqrt{210}G_{bz}xz}{20} \end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,2), \bar{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,2)$$

** symmetry

$$\begin{aligned} & \frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8} \\ & \frac{\sqrt{35}xy(x-y)(x+y)}{2} \end{aligned}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{10}G_1xz}{10} + \frac{3\sqrt{10}G_2yz}{10} - \frac{\sqrt{15}G_3(x-y)(x+y)}{10} - \frac{\sqrt{15}G_{bz}xy}{5} \\ & \frac{3\sqrt{10}G_1yz}{10} - \frac{3\sqrt{10}G_2xz}{10} - \frac{\sqrt{15}G_3xy}{5} + \frac{\sqrt{15}G_{bz}(x-y)(x+y)}{10} \end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,-2)}[g](E,3), \bar{\mathbb{Q}}_{4,2}^{(3,-2)}[g](E,3)$$

** symmetry

$$\begin{aligned} & -\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4} \\ & \frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2} \end{aligned}$$

** expression

$$\begin{aligned} & -\frac{9\sqrt{70}G_1xz}{140} + \frac{9\sqrt{70}G_2yz}{140} - \frac{\sqrt{105}G_3(x^2+y^2-2z^2)}{35} - \frac{\sqrt{42}G_{3x}yz}{28} - \frac{\sqrt{42}G_{3y}xz}{28} - \frac{3\sqrt{7}G_{az}xy}{7} \\ & -\frac{9\sqrt{70}G_1yz}{140} - \frac{9\sqrt{70}G_2xz}{140} - \frac{\sqrt{42}G_{3x}xz}{28} + \frac{\sqrt{42}G_{3y}yz}{28} - \frac{3\sqrt{7}G_{az}(x-y)(x+y)}{14} - \frac{\sqrt{105}G_{bz}(x^2+y^2-2z^2)}{35} \end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,0)}[g](E,1), \bar{\mathbb{Q}}_{4,2}^{(3,0)}[g](E,1)$$

** symmetry

$$\begin{aligned} & -\frac{\sqrt{10}xz(3x^2+3y^2-4z^2)}{4} \\ & -\frac{\sqrt{10}yz(3x^2+3y^2-4z^2)}{4} \end{aligned}$$

** expression

$$\begin{aligned} & \frac{\sqrt{770}G_1(17x^4 - 42x^2y^2 - 60x^2z^2 - 3y^4 + 60y^2z^2)}{1232} - \frac{\sqrt{770}G_2xy(6x^2 - y^2 - 15z^2)}{154} - \frac{\sqrt{1155}G_3xz(x^2 - 3y^2)}{44} - \frac{\sqrt{462}G_{3x}xy(x^2 + y^2 - 6z^2)}{154} \\ & + \frac{\sqrt{462}G_{3y}(13x^4 + 18x^2y^2 - 96x^2z^2 + 5y^4 - 48y^2z^2 + 24z^4)}{1232} + \frac{9\sqrt{77}G_{az}yz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{\sqrt{1155}G_{bz}yz(3x^2 - y^2)}{44} \\ & \frac{\sqrt{770}G_1xy(x^2 - 6y^2 + 15z^2)}{154} - \frac{\sqrt{770}G_2(3x^4 + 42x^2y^2 - 60x^2z^2 - 17y^4 + 60y^2z^2)}{1232} \\ & - \frac{\sqrt{1155}G_{3yz}(3x^2 - y^2)}{44} - \frac{\sqrt{462}G_{3x}(5x^4 + 18x^2y^2 - 48x^2z^2 + 13y^4 - 96y^2z^2 + 24z^4)}{1232} \\ & + \frac{\sqrt{462}G_{3y}xy(x^2 + y^2 - 6z^2)}{154} - \frac{9\sqrt{77}G_{az}xz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{1155}G_{bz}xz(x^2 - 3y^2)}{44} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 2), \tilde{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 2)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{55}G_1xz(3x^2 + 3y^2 - 4z^2)}{44} + \frac{\sqrt{55}G_2yz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{330}G_3(x-y)(x+y)(x^2 + y^2 - 6z^2)}{88} \\ & - \frac{7\sqrt{33}G_{3yz}(3x^2 - y^2)}{44} - \frac{7\sqrt{33}G_{3yxz}(x^2 - 3y^2)}{44} + \frac{7\sqrt{22}G_{az}xy(x-y)(x+y)}{22} - \frac{\sqrt{330}G_{bz}xy(x^2 + y^2 - 6z^2)}{44} \\ & \frac{\sqrt{55}G_1yz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{55}G_2xz(3x^2 + 3y^2 - 4z^2)}{44} - \frac{\sqrt{330}G_3xy(x^2 + y^2 - 6z^2)}{44} + \frac{7\sqrt{33}G_{3xz}(x^2 - 3y^2)}{44} \\ & - \frac{7\sqrt{33}G_{3yz}(3x^2 - y^2)}{44} - \frac{7\sqrt{22}G_{az}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{88} + \frac{\sqrt{330}G_{bz}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{88} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,0)}[g](E, 3), \tilde{\mathbb{Q}}_{4,2}^{(3,0)}[g](E, 3)$$

** symmetry

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned} & - \frac{5\sqrt{385}G_1xz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{5\sqrt{385}G_2yz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{\sqrt{2310}G_3(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{308} \\ & - \frac{\sqrt{231}G_{3yz}(27x^2 - y^2 - 8z^2)}{308} + \frac{\sqrt{231}G_{3yxz}(x^2 - 27y^2 + 8z^2)}{308} + \frac{13\sqrt{154}G_{az}xy(x^2 + y^2 - 6z^2)}{308} + \frac{\sqrt{2310}G_{bz}xy(x-y)(x+y)}{44} \\ & - \frac{5\sqrt{385}G_1yz(3x^2 + 3y^2 - 4z^2)}{308} - \frac{5\sqrt{385}G_2xz(3x^2 + 3y^2 - 4z^2)}{308} + \frac{\sqrt{2310}G_3xy(x-y)(x+y)}{44} - \frac{\sqrt{231}G_{3xz}(13x^2 - 15y^2 - 8z^2)}{308} \\ & - \frac{\sqrt{231}G_{3yz}(15x^2 - 13y^2 + 8z^2)}{308} + \frac{13\sqrt{154}G_{az}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{616} + \frac{\sqrt{2310}G_{bz}(5x^4 - 18x^2y^2 - 12x^2z^2 + 5y^4 - 12y^2z^2 + 4z^4)}{616} \end{aligned}$$

$$\tilde{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 1), \tilde{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 1)$$

** symmetry

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

** expression

$$\begin{aligned} & \frac{3\sqrt{1001}G_1(7x^6 - 20x^4y^2 - 85x^4z^2 - 25x^2y^4 + 270x^2y^2z^2 + 40x^2z^4 + 2y^6 - 5y^4z^2 - 40y^2z^4)}{2288} \\ & - \frac{3\sqrt{1001}G_2xy(23x^4 + 10x^2y^2 - 260x^2z^2 - 13y^4 + 100y^2z^2 + 80z^4)}{2288} \\ & + \frac{\sqrt{6006}G_3xz(25x^4 + 5x^2y^2 - 85x^2z^2 - 20y^4 + 35y^2z^2 + 22z^4)}{1144} + \frac{\sqrt{15015}G_{3x}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} \\ & - \frac{\sqrt{15015}G_{3y}(3x^6 + 8x^4y^2 - 53x^4z^2 + 7x^2y^4 - 90x^2y^2z^2 + 68x^2z^4 + 2y^6 - 37y^4z^2 + 52y^2z^4 - 8z^6)}{2288} \\ & - \frac{3\sqrt{10010}G_{az}yz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} - \frac{\sqrt{6006}G_{bz}yz(95x^4 + 100x^2y^2 - 290x^2z^2 + 5y^4 - 50y^2z^2 + 44z^4)}{2288} \end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{1001}G_1xy(13x^4 - 10x^2y^2 - 100x^2z^2 - 23y^4 + 260y^2z^2 - 80z^4)}{2288} \\
& + \frac{3\sqrt{1001}G_2(2x^6 - 25x^4y^2 - 5x^4z^2 - 20x^2y^4 + 270x^2y^2z^2 - 40x^2z^4 + 7y^6 - 85y^4z^2 + 40y^2z^4)}{2288} \\
& + \frac{\sqrt{6006}G_3yz(20x^4 - 5x^2y^2 - 35x^2z^2 - 25y^4 + 85y^2z^2 - 22z^4)}{1144} \\
& + \frac{\sqrt{15015}G_{3x}(2x^6 + 7x^4y^2 - 37x^4z^2 + 8x^2y^4 - 90x^2y^2z^2 + 52x^2z^4 + 3y^6 - 53y^4z^2 + 68y^2z^4 - 8z^6)}{2288} \\
& - \frac{\sqrt{15015}G_{3y}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{2288} + \frac{3\sqrt{10010}G_{az}xz(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& - \frac{\sqrt{6006}G_{bz}xz(5x^4 + 100x^2y^2 - 50x^2z^2 + 95y^4 - 290y^2z^2 + 44z^4)}{2288}
\end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 2), \bar{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 2)$$

** symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

** expression

$$\begin{aligned}
& \frac{3\sqrt{286}G_{1xz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} + \frac{3\sqrt{286}G_{2yz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& + \frac{\sqrt{429}G_3(x - y)(x + y)(7x^4 - 118x^2y^2 + 20x^2z^2 + 7y^4 + 20y^2z^2 - 20z^4)}{572} \\
& - \frac{3\sqrt{4290}G_{3xyz}(73x^4 - 98x^2y^2 - 48x^2z^2 + 5y^4 + 16y^2z^2)}{2288} + \frac{3\sqrt{4290}G_{3yxz}(5x^4 - 98x^2y^2 + 16x^2z^2 + 73y^4 - 48y^2z^2)}{2288} \\
& + \frac{9\sqrt{715}G_{az}xy(x - y)(x + y)(x^2 + y^2 - 10z^2)}{286} - \frac{\sqrt{429}G_{bz}xy(13x^4 - 40x^2y^2 - 10x^2z^2 + 13y^4 - 10y^2z^2 + 10z^4)}{143}
\end{aligned}$$

$$\begin{aligned}
& \frac{3\sqrt{286}G_{1yz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} - \frac{3\sqrt{286}G_{2xz}(5x^4 + 10x^2y^2 - 20x^2z^2 + 5y^4 - 20y^2z^2 + 8z^4)}{2288} \\
& + \frac{\sqrt{429}G_3xy(47x^4 - 170x^2y^2 + 40x^2z^2 + 47y^4 + 40y^2z^2 - 40z^4)}{572} + \frac{3\sqrt{4290}G_{3x}xz(17x^4 - 122x^2y^2 - 16x^2z^2 + 37y^4 + 48y^2z^2)}{2288} \\
& + \frac{3\sqrt{4290}G_{3y}yz(37x^4 - 122x^2y^2 + 48x^2z^2 + 17y^4 - 16y^2z^2)}{2288} - \frac{9\sqrt{715}G_{az}(x^2 + y^2 - 10z^2)(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{1144} \\
& + \frac{\sqrt{429}G_{bz}(x - y)(x + y)(19x^4 - 226x^2y^2 - 40x^2z^2 + 19y^4 - 40y^2z^2 + 40z^4)}{1144}
\end{aligned}$$

$$\bar{\mathbb{Q}}_{4,1}^{(3,2)}[g](E, 3), \bar{\mathbb{Q}}_{4,2}^{(3,2)}[g](E, 3)$$

** symmetry

$$-\frac{\sqrt{5}(x - y)(x + y)(x^2 + y^2 - 6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$\begin{aligned}
& -\frac{9\sqrt{2002}G_{1xz}(4x^4 - 25x^2y^2 - 5x^2z^2 + 15y^4 - 5y^2z^2 + 2z^4)}{1144} + \frac{9\sqrt{2002}G_{2yz}(15x^4 - 25x^2y^2 - 5x^2z^2 + 4y^4 - 5y^2z^2 + 2z^4)}{1144} \\
& + \frac{\sqrt{3003}G_3(x^6 - 15x^4z^2 + 15x^2z^4 + y^6 - 15y^4z^2 + 15y^2z^4 - 2z^6)}{286} \\
& + \frac{\sqrt{30030}G_{3xyz}(13x^4 + 17x^2y^2 - 43x^2z^2 + 4y^4 - 19y^2z^2 + 10z^4)}{1144} + \frac{\sqrt{30030}G_{3yxz}(4x^4 + 17x^2y^2 - 19x^2z^2 + 13y^4 - 43y^2z^2 + 10z^4)}{1144} \\
& - \frac{3\sqrt{5005}G_{az}xy(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{572} - \frac{3\sqrt{3003}G_{bz}xy(x - y)(x + y)(x^2 + y^2 - 10z^2)}{572}
\end{aligned}$$

$$\begin{aligned}
& \frac{9\sqrt{2002}G_{1yz}(25x^4 - 60x^2y^2 + 10x^2z^2 + 3y^4 + 10y^2z^2 - 4z^4)}{2288} + \frac{9\sqrt{2002}G_{2xz}(3x^4 - 60x^2y^2 + 10x^2z^2 + 25y^4 + 10y^2z^2 - 4z^4)}{2288} \\
& - \frac{3\sqrt{3003}G_3xy(x - y)(x + y)(x^2 + y^2 - 10z^2)}{572} + \frac{\sqrt{30030}G_{3x}xz(17x^4 + 16x^2y^2 - 62x^2z^2 - y^4 - 14y^2z^2 + 20z^4)}{2288} \\
& + \frac{\sqrt{30030}G_{3y}yz(x^4 - 16x^2y^2 + 14x^2z^2 - 17y^4 + 62y^2z^2 - 20z^4)}{2288} - \frac{3\sqrt{5005}G_{az}(x - y)(x + y)(x^4 + 2x^2y^2 - 16x^2z^2 + y^4 - 16y^2z^2 + 16z^4)}{1144} \\
& + \frac{\sqrt{3003}G_{bz}(x^6 + 15x^4y^2 - 30x^4z^2 + 15x^2y^4 - 180x^2y^2z^2 + 60x^2z^4 + y^6 - 30y^4z^2 + 60y^2z^4 - 8z^6)}{1144}
\end{aligned}$$