

* Harmonics for rank 0

* Harmonics for rank 1

$$\bar{\mathbb{G}}_{1,1}^{(1,0)}[q](T_g), \bar{\mathbb{G}}_{1,2}^{(1,0)}[q](T_g), \bar{\mathbb{G}}_{1,3}^{(1,0)}[q](T_g)$$

** symmetry

x

y

z

** expression

$$\begin{aligned} & \frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2} \\ & - \frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2} \\ & \frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2} \end{aligned}$$

* Harmonics for rank 2

$$\bar{\mathbb{G}}_{2,1}^{(1,0)}[q](E_u), \bar{\mathbb{G}}_{2,2}^{(1,0)}[q](E_u)$$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

** expression

$$\begin{aligned} & \frac{\sqrt{6}Q_x y z}{2} - \frac{\sqrt{6}Q_y x z}{2} \\ & \frac{\sqrt{2}Q_x y z}{2} + \frac{\sqrt{2}Q_y x z}{2} - \sqrt{2}Q_z x y \end{aligned}$$

$$\bar{\mathbb{G}}_{2,1}^{(1,0)}[q](T_u), \bar{\mathbb{G}}_{2,2}^{(1,0)}[q](T_u), \bar{\mathbb{G}}_{2,3}^{(1,0)}[q](T_u)$$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\begin{aligned} & \frac{\sqrt{2}Q_x (y-z)(y+z)}{2} - \frac{\sqrt{2}Q_y x y}{2} + \frac{\sqrt{2}Q_z x z}{2} \\ & \frac{\sqrt{2}Q_x x y}{2} - \frac{\sqrt{2}Q_y (x-z)(x+z)}{2} - \frac{\sqrt{2}Q_z y z}{2} \\ & - \frac{\sqrt{2}Q_x x z}{2} + \frac{\sqrt{2}Q_y y z}{2} + \frac{\sqrt{2}Q_z (x-y)(x+y)}{2} \end{aligned}$$

* Harmonics for rank 3

$$\bar{\mathbb{G}}_3^{(1,0)}[q](A_g)$$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_x x (y-z)(y+z)}{2} - \frac{\sqrt{5}Q_y y (x-z)(x+z)}{2} + \frac{\sqrt{5}Q_z z (x-y)(x+y)}{2}$$

$$\bar{\mathbb{G}}_{3,1}^{(1,0)}[q](T_g, 1), \bar{\mathbb{G}}_{3,2}^{(1,0)}[q](T_g, 1), \bar{\mathbb{G}}_{3,3}^{(1,0)}[q](T_g, 1)$$

** symmetry

$$\frac{x(2x^2 - 3y^2 - 3z^2)}{2}$$

$$- \frac{y(3x^2 - 2y^2 + 3z^2)}{2}$$

$$- \frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

** expression

$$\frac{\sqrt{3}Q_y z(4x^2 - y^2 - z^2)}{4} - \frac{\sqrt{3}Q_z y(4x^2 - y^2 - z^2)}{4}$$

$$\frac{\sqrt{3}Q_x z(x^2 - 4y^2 + z^2)}{4} - \frac{\sqrt{3}Q_z x(x^2 - 4y^2 + z^2)}{4}$$

$$- \frac{\sqrt{3}Q_x y(x^2 + y^2 - 4z^2)}{4} + \frac{\sqrt{3}Q_y x(x^2 + y^2 - 4z^2)}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](T_g, 2), \vec{\mathbb{G}}_{3,2}^{(1,0)}[q](T_g, 2), \vec{\mathbb{G}}_{3,3}^{(1,0)}[q](T_g, 2)$$

** symmetry

$$\frac{\sqrt{15}x(y-z)(y+z)}{2}$$

$$- \frac{\sqrt{15}y(x-z)(x+z)}{2}$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

** expression

$$-\sqrt{5}Q_x xyz + \frac{\sqrt{5}Q_y z(2x^2 + y^2 - z^2)}{4} + \frac{\sqrt{5}Q_z y(2x^2 - y^2 + z^2)}{4}$$

$$\frac{\sqrt{5}Q_x z(x^2 + 2y^2 - z^2)}{4} - \sqrt{5}Q_y xyz - \frac{\sqrt{5}Q_z x(x^2 - 2y^2 - z^2)}{4}$$

$$\frac{\sqrt{5}Q_x y(x^2 - y^2 + 2z^2)}{4} - \frac{\sqrt{5}Q_y x(x^2 - y^2 - 2z^2)}{4} - \sqrt{5}Q_z xyz$$

* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u)$$

** symmetry

$$\frac{\sqrt{21}(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

** expression

$$- \frac{\sqrt{105}Q_x y z(y-z)(y+z)}{6} + \frac{\sqrt{105}Q_y x z(x-z)(x+z)}{6} - \frac{\sqrt{105}Q_z x y(x-y)(x+y)}{6}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_u), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_u)$$

** symmetry

$$- \frac{\sqrt{15}(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4)}{12}$$

$$\frac{\sqrt{5}(x-y)(x+y)(x^2 + y^2 - 6z^2)}{4}$$

** expression

$$- \frac{\sqrt{3}Q_x y z(9x^2 + 2y^2 - 5z^2)}{6} + \frac{\sqrt{3}Q_y x z(2x^2 + 9y^2 - 5z^2)}{6} + \frac{7\sqrt{3}Q_z x y(x-y)(x+y)}{6}$$

$$- \frac{Q_x y z(3x^2 - 4y^2 + 3z^2)}{2} + \frac{Q_y x z(4x^2 - 3y^2 - 3z^2)}{2} - \frac{Q_z x y(x^2 + y^2 - 6z^2)}{2}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](T_u, 1), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](T_u, 1), \vec{\mathbb{G}}_{4,3}^{(1,0)}[q](T_u, 1)$$

** symmetry

$$\frac{\sqrt{35}yz(y-z)(y+z)}{2}$$

$$-\frac{\sqrt{35}xz(x-z)(x+z)}{2}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x(y^2-2yz-z^2)(y^2+2yz-z^2)}{4} - \frac{\sqrt{7}Q_yxy(y^2-3z^2)}{4} + \frac{\sqrt{7}Q_zxz(3y^2-z^2)}{4}$$

$$-\frac{\sqrt{7}Q_xxy(x^2-3z^2)}{4} + \frac{\sqrt{7}Q_y(x^2-2xz-z^2)(x^2+2xz-z^2)}{4} + \frac{\sqrt{7}Q_zyz(3x^2-z^2)}{4}$$

$$-\frac{\sqrt{7}Q_xxz(x^2-3y^2)}{4} + \frac{\sqrt{7}Q_yyz(3x^2-y^2)}{4} + \frac{\sqrt{7}Q_z(x^2-2xy-y^2)(x^2+2xy-y^2)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](T_u, 2), \vec{\mathbb{G}}_{4,2}^{(1,0)}[q](T_u, 2), \vec{\mathbb{G}}_{4,3}^{(1,0)}[q](T_u, 2)$$

** symmetry

$$\frac{\sqrt{5}yz(6x^2-y^2-z^2)}{2}$$

$$-\frac{\sqrt{5}xz(x^2-6y^2+z^2)}{2}$$

$$-\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

** expression

$$\frac{Q_x(y-z)(y+z)(6x^2-y^2-z^2)}{4} - \frac{Q_yxy(6x^2-y^2-15z^2)}{4} + \frac{Q_zxz(6x^2-15y^2-z^2)}{4}$$

$$-\frac{Q_xxy(x^2-6y^2+15z^2)}{4} + \frac{Q_y(x-z)(x+z)(x^2-6y^2+z^2)}{4} + \frac{Q_zyz(15x^2-6y^2+z^2)}{4}$$

$$\frac{Q_xxz(x^2+15y^2-6z^2)}{4} - \frac{Q_yyz(15x^2+y^2-6z^2)}{4} - \frac{Q_z(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$