PG No. 17 $C_{3\mathrm{i}}$ $\bar{3}$ [trigonal]

Table 1 Harmonics for rank 0.

No.	multipole	expression
1	$\mathbb{Q}_0(A_g)$	1

Table 2 Harmonics for rank 1.

No.	multipole	expression
2	$\mathbb{Q}_1(A_u)$	z
3	$\mathbb{Q}_{1,1}(E_u)$	x
4	$\mathbb{Q}_{1,2}(E_u)$	y

Table 3 Harmonics for rank 2.

No.	multipole	expression
5	$\mathbb{Q}_2(A_g)$	$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$
6	$\mathbb{Q}_{2,1}(E_g, 1)$	$\sqrt{3}yz$
7	$\mathbb{Q}_{2,2}(E_g, 1)$	$-\sqrt{3}xz$
8	$\mathbb{Q}_{2,1}(E_g, 2)$	$\frac{\sqrt{3}(x-y)(x+y)}{2}$
9	$\mathbb{Q}_{2,2}(E_g, 2)$	$-\sqrt{3}xy$

Table 4 Harmonics for rank 3.

No.	multipole	expression
10	$\mathbb{Q}_3(A_u, 1)$	$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$
11	$\mathbb{Q}_3(A_u, 2)$	$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$
12	$\mathbb{Q}_3(A_u, 3)$	$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$
13	$\mathbb{Q}_{3,1}(E_u, 1)$	$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$
14	$\mathbb{Q}_{3,2}(E_u, 1)$	$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$
15	$\mathbb{Q}_{3,1}(E_u, 2)$	$\sqrt{15}xyz$
16	$\mathbb{Q}_{3,2}(E_u, 2)$	$\frac{\sqrt{15}z(x-y)(x+y)}{2}$

Table 5 Harmonics for rank 4.

No.	multipole	expression
17	$\mathbb{Q}_4(A_g,1)$	$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$
18	$\mathbb{Q}_4(A_g,2)$	$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$
19	$\mathbb{Q}_4(A_g,3)$	$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$
20	$\mathbb{Q}_{4,1}(E_g,1)$	$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$
21	$\mathbb{Q}_{4,2}(E_g,1)$	$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$
22	$\mathbb{Q}_{4,1}(E_g,2)$	$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$
23	$\mathbb{Q}_{4,2}(E_g,2)$	$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$
24	$\mathbb{Q}_{4,1}(E_g,3)$	$-\frac{\sqrt{5}(x-y)(x+y)\left(x^2+y^2-6z^2\right)}{4}$
25	$\mathbb{Q}_{4,2}(E_g,3)$	$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$

Table 6 Harmonics for rank 5.

No.	multipole	expression
26	$\mathbb{Q}_5(A_u,1)$	$\frac{z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{8}$
27	$\mathbb{Q}_5(A_u,2)$	$-\frac{\sqrt{70}y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{16}$
28	$\mathbb{Q}_5(A_u,3)$	$-\frac{\sqrt{70}x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{16}$
29	$\mathbb{Q}_{5,1}(E_u,1)$	$\frac{3\sqrt{14}x\left(x^4-10x^2y^2+5y^4\right)}{16}$
30	$\mathbb{Q}_{5,2}(E_u,1)$	$-\frac{3\sqrt{14}y\left(5x^4-10x^2y^2+y^4\right)}{16}$
31	$\mathbb{Q}_{5,1}(E_u,2)$	$\frac{\sqrt{15}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{8}$
32	$\mathbb{Q}_{5,2}(E_u,2)$	$\frac{\sqrt{15}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{8}$
33	$\mathbb{Q}_{5,1}(E_u,3)$	$-\frac{3\sqrt{35}xyz(x-y)(x+y)}{2}$
34	$\mathbb{Q}_{5,2}(E_u,3)$	$\frac{3\sqrt{35}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$
35	$\mathbb{Q}_{5,1}(E_u,4)$	$-\frac{\sqrt{105}xyz\left(x^2+y^2-2z^2\right)}{2}$
36	$\mathbb{Q}_{5,2}(E_u,4)$	$-\frac{\sqrt{105}z(x-y)(x+y)\left(x^2+y^2-2z^2\right)}{4}$

Table 7 Harmonics for rank 6.

No.	multipole	expression
37	$\mathbb{Q}_6(A_g,1)$	$-\frac{5x^6}{16}-\frac{15x^4y^2}{16}+\frac{45x^4z^2}{8}-\frac{15x^2y^4}{16}+\frac{45x^2y^2z^2}{4}-\frac{15x^2z^4}{2}-\frac{5y^6}{16}+\frac{45y^4z^2}{8}-\frac{15y^2z^4}{2}+z^6$
38	$\mathbb{Q}_6(A_g,2)$	$\frac{\sqrt{462}(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)}{32}$
39	$\mathbb{Q}_6(A_g,3)$	$\frac{\sqrt{462}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{16}$
40	$\mathbb{Q}_6(A_g,4)$	$-\frac{\sqrt{210}xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{16}$
41	$\mathbb{Q}_6(A_g,5)$	$-\frac{\sqrt{210}yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{16}$
42	$\mathbb{Q}_{6,1}(E_g,1)$	$\frac{3\sqrt{154}yz\left(5x^4-10x^2y^2+y^4\right)}{16}$
43	$\mathbb{Q}_{6,2}(E_g,1)$	$\frac{3\sqrt{154}xz\left(x^4-10x^2y^2+5y^4\right)}{16}$
44	$\mathbb{Q}_{6,1}(E_g,2)$	$\frac{\sqrt{21}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{8}$
45	$\mathbb{Q}_{6,2}(E_g,2)$	$-\frac{\sqrt{21}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{8}$
46	$\mathbb{Q}_{6,1}(E_g,3)$	$-\frac{3\sqrt{7}\left(x^2+y^2-10z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{16}$
47	$\mathbb{Q}_{6,2}(E_g,3)$	$-\frac{3\sqrt{7}xy(x-y)(x+y)\left(x^2+y^2-10z^2\right)}{4}$
48	$\mathbb{Q}_{6,1}(E_g,4)$	$\frac{\sqrt{210}(x-y)(x+y)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{32}$
49	$\mathbb{Q}_{6,2}(E_g,4)$	$-\frac{\sqrt{210}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{16}$

Table 8 Harmonics for rank 7.

No.	multipole	expression
50	$\mathbb{Q}_7(A_u,1)$	$\frac{\sqrt{6006}xyz\left(x^2-3y^2\right)\left(3x^2-y^2\right)}{16}$
51	$\mathbb{Q}_7(A_u,2)$	$-\frac{z\left(35x^6+105x^4y^2-210x^4z^2+105x^2y^4-420x^2y^2z^2+168x^2z^4+35y^6-210y^4z^2+168y^2z^4-16z^6\right)}{16}$
52	$\mathbb{Q}_7(A_u,3)$	$\frac{\sqrt{6006}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)}{32}$
53	$\mathbb{Q}_7(A_u,4)$	$\frac{\sqrt{21}y\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{32}$
54	$\mathbb{Q}_7(A_u,5)$	$\frac{\sqrt{21}x\left(x^2-3y^2\right)\left(3x^4+6x^2y^2-60x^2z^2+3y^4-60y^2z^2+80z^4\right)}{32}$
55	$\mathbb{Q}_{7,1}(E_u,1)$	$\frac{\sqrt{429}x\left(x^6-21x^4y^2+35x^2y^4-7y^6\right)}{32}$
56	$\mathbb{Q}_{7,2}(E_u,1)$	$\frac{\sqrt{429}y\left(7x^6-35x^4y^2+21x^2y^4-y^6\right)}{32}$
57	$\mathbb{Q}_{7,1}(E_u,2)$	$-\frac{\sqrt{231}x\left(x^2+y^2-12z^2\right)\left(x^4-10x^2y^2+5y^4\right)}{32}$
58	$\mathbb{Q}_{7,2}(E_u,2)$	$\frac{\sqrt{231}y\left(x^2+y^2-12z^2\right)\left(5x^4-10x^2y^2+y^4\right)}{32}$
59	$\mathbb{Q}_{7,1}(E_u,3)$	$-\frac{\sqrt{7}x\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{32}$
60	$\mathbb{Q}_{7,2}(E_u,3)$	$-\frac{\sqrt{7}y\left(5x^6+15x^4y^2-120x^4z^2+15x^2y^4-240x^2y^2z^2+240x^2z^4+5y^6-120y^4z^2+240y^2z^4-64z^6\right)}{32}$
61	$\mathbb{Q}_{7,1}(E_u,4)$	$\frac{\sqrt{231}xyz(x-y)(x+y)\left(3x^2+3y^2-10z^2\right)}{4}$
62	$\mathbb{Q}_{7,2}(E_u,4)$	$-\frac{\sqrt{231}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(3x^2+3y^2-10z^2\right)}{16}$
63	$\mathbb{Q}_{7,1}(E_u,5)$	$\frac{\sqrt{42}xyz\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{16}$
64	$\mathbb{Q}_{7,2}(E_u,5)$	$\frac{\sqrt{42}z(x-y)(x+y)\left(15x^4+30x^2y^2-80x^2z^2+15y^4-80y^2z^2+48z^4\right)}{32}$

Table 9 Harmonics for rank 8.

No.	multipole	expression
65	$\mathbb{Q}_8(A_g,1)$	$\frac{35x^8}{128}+\frac{35x^6y^2}{32}-\frac{35x^6z^2}{4}+\frac{105x^4y^4}{64}-\frac{105x^4y^2z^2}{4}+\frac{105x^4z^4}{4}+\frac{35x^2y^6}{32}-\frac{105x^2y^4z^2}{4}+\frac{105x^2y^2z^4}{2}-14x^2z^6+\frac{35y^8}{128}-\frac{35y^6z^2}{4}+\frac{105y^4z^4}{4}-14y^2z^6+z^8$
66	$\mathbb{Q}_8(A_g,2)$	$-\frac{\sqrt{858}(x-y)(x+y)\left(x^2+y^2-14z^2\right)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)}{64}$
67	$\mathbb{Q}_8(A_g,3)$	$-\frac{\sqrt{858}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(x^2+y^2-14z^2\right)}{32}$
68	$\mathbb{Q}_8(A_g,4)$	$\frac{\sqrt{1155}xz\left(x^2-3y^2\right)\left(3x^4+6x^2y^2-20x^2z^2+3y^4-20y^2z^2+16z^4\right)}{32}$
69	$\mathbb{Q}_8(A_g,5)$	$\frac{\sqrt{1155}yz\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-20x^2z^2+3y^4-20y^2z^2+16z^4\right)}{32}$
70	$\mathbb{Q}_{8,1}(E_g,1)$	$\frac{3\sqrt{715}yz\left(7x^6-35x^4y^2+21x^2y^4-y^6\right)}{32}$
71	$\mathbb{Q}_{8,2}(E_g,1)$	$-\frac{3\sqrt{715}xz\left(x^6-21x^4y^2+35x^2y^4-7y^6\right)}{32}$
72	$\mathbb{Q}_{8,1}(E_g,2)$	$-\frac{3\sqrt{1001}yz\left(x^2+y^2-4z^2\right)\left(5x^4-10x^2y^2+y^4\right)}{32}$
73	$\mathbb{Q}_{8,2}(E_g,2)$	$-\frac{3\sqrt{1001}xz\left(x^2+y^2-4z^2\right)\left(x^4-10x^2y^2+5y^4\right)}{32}$
74	$\mathbb{Q}_{8,1}(E_g,3)$	$-\frac{3yz\left(35x^6+105x^4y^2-280x^4z^2+105x^2y^4-560x^2y^2z^2+336x^2z^4+35y^6-280y^4z^2+336y^2z^4-64z^6\right)}{32}$
75	$\mathbb{Q}_{8,2}(E_g,3)$	$\frac{3xz\left(35x^6+105x^4y^2-280x^4z^2+105x^2y^4-560x^2y^2z^2+336x^2z^4+35y^6-280y^4z^2+336y^2z^4-64z^6\right)}{32}$
76	$\mathbb{Q}_{8,1}(E_g,4)$	$\frac{3\sqrt{715}\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{128}$
77	$\mathbb{Q}_{8,2}(E_g,4)$	$-\frac{3\sqrt{715}xy(x-y)(x+y)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{16}$
78	$\mathbb{Q}_{8,1}(E_g,5)$	$\frac{3\sqrt{77}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(x^4+2x^2y^2-24x^2z^2+y^4-24y^2z^2+40z^4\right)}{64}$
79	$\mathbb{Q}_{8,2}(E_g,5)$	$\frac{3\sqrt{77}xy(x-y)(x+y)\left(x^4+2x^2y^2-24x^2z^2+y^4-24y^2z^2+40z^4\right)}{16}$
80	$\mathbb{Q}_{8,1}(E_g,6)$	$-\frac{3\sqrt{70}(x-y)(x+y)\left(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6\right)}{64}$
81	$\mathbb{Q}_{8,2}(E_g,6)$	$\frac{3\sqrt{70}xy\left(x^6+3x^4y^2-30x^4z^2+3x^2y^4-60x^2y^2z^2+80x^2z^4+y^6-30y^4z^2+80y^2z^4-32z^6\right)}{32}$

Table 10 Harmonics for rank 9.

No.	multipole	expression
82	$\mathbb{Q}_9(A_u,1)$	$-\frac{\sqrt{4290}xyz\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(3x^2+3y^2-14z^2\right)}{32}$
83	$\mathbb{Q}_9(A_u,2)$	$\frac{z\left(315x^8+1260x^6y^2-3360x^6z^2+1890x^4y^4-10080x^4y^2z^2+6048x^4z^4+1260x^2y^6-10080x^2y^4z^2+12096x^2y^2z^4-2304x^2z^6+315y^8-3360y^6z^2+6048y^4z^4-2304y^2z^6+128z^8\right)}{128}$
84	$\mathbb{Q}_9(A_u,3)$	$-\frac{\sqrt{4290}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)\left(3x^2+3y^2-14z^2\right)}{64}$
85	$\mathbb{Q}_9(A_u,4)$	$\frac{\sqrt{24310}y\left(3x^2-y^2\right)\left(3x^6-27x^4y^2+33x^2y^4-y^6\right)}{256}$
86	$\mathbb{Q}_9(A_u,5)$	$-\frac{\sqrt{2310}y\left(3x^2-y^2\right)\left(x^6+3x^4y^2-36x^4z^2+3x^2y^4-72x^2y^2z^2+120x^2z^4+y^6-36y^4z^2+120y^2z^4-64z^6\right)}{128}$
87	$\mathbb{Q}_9(A_u,6)$	$\frac{\sqrt{24310}x\left(x^2-3y^2\right)\left(x^6-33x^4y^2+27x^2y^4-3y^6\right)}{256}$
88	$\mathbb{Q}_9(A_u,7)$	$-\frac{\sqrt{2310}x\left(x^2-3y^2\right)\left(x^6+3x^4y^2-36x^4z^2+3x^2y^4-72x^2y^2z^2+120x^2z^4+y^6-36y^4z^2+120y^2z^4-64z^6\right)}{128}$
89	$\mathbb{Q}_{9,1}(E_u,1)$	$-\frac{3\sqrt{1430}x\left(x^2+y^2-16z^2\right)\left(x^6-21x^4y^2+35x^2y^4-7y^6\right)}{256}$
90	$\mathbb{Q}_{9,2}(E_u,1)$	$-\frac{3\sqrt{1430}y\left(x^2+y^2-16z^2\right)\left(7x^6-35x^4y^2+21x^2y^4-y^6\right)}{256}$
91	$\mathbb{Q}_{9,1}(E_u,2)$	$\frac{3\sqrt{286}x\left(x^4-10x^2y^2+5y^4\right)\left(x^4+2x^2y^2-28x^2z^2+y^4-28y^2z^2+56z^4\right)}{128}$
92	$\mathbb{Q}_{9,2}(E_u,2)$	$-\frac{3\sqrt{286}y\left(5x^4-10x^2y^2+y^4\right)\left(x^4+2x^2y^2-28x^2z^2+y^4-28y^2z^2+56z^4\right)}{128}$
93	$\mathbb{Q}_{9,1}(E_u,3)$	$\frac{3\sqrt{5}x\left(7x^8+28x^6y^2-280x^6z^2+42x^4y^4-840x^4y^2z^2+1120x^4z^4+28x^2y^6-840x^2y^4z^2+2240x^2y^2z^4-896x^2z^6+7y^8-280y^6z^2+1120y^4z^4-896y^2z^6+128z^8\right)}{128}$
94	$\mathbb{Q}_{9,2}(E_u,3)$	$\frac{3\sqrt{5}y\left(7x^8+28x^6y^2-280x^6z^2+42x^4y^4-840x^4y^2z^2+1120x^4z^4+28x^2y^6-840x^2y^4z^2+2240x^2y^2z^4-896x^2z^6+7y^8-280y^6z^2+1120y^4z^4-896y^2z^6+128z^8\right)}{128}$
95	$\mathbb{Q}_{9,1}(E_u,4)$	$\frac{3\sqrt{12155}xyz(x-y)(x+y)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{16}$
96	$\mathbb{Q}_{9,2}(E_u,4)$	$\frac{3\sqrt{12155}z\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{128}$
97	$\mathbb{Q}_{9,1}(E_u,5)$	$-\frac{3\sqrt{5005}xyz(x-y)(x+y)\left(x^4+2x^2y^2-8x^2z^2+y^4-8y^2z^2+8z^4\right)}{16}$
98	$\mathbb{Q}_{9,2}(E_u,5)$	$\frac{3\sqrt{5005}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(x^4+2x^2y^2-8x^2z^2+y^4-8y^2z^2+8z^4\right)}{64}$
99	$\mathbb{Q}_{9,1}(E_u,6)$	$-\frac{3\sqrt{110}xyz\left(7x^6+21x^4y^2-70x^4z^2+21x^2y^4-140x^2y^2z^2+112x^2z^4+7y^6-70y^4z^2+112y^2z^4-32z^6\right)}{32}$
100	$\mathbb{Q}_{9,2}(E_u,6)$	$-\frac{3\sqrt{110}z(x-y)(x+y)\left(7x^6+21x^4y^2-70x^4z^2+21x^2y^4-140x^2y^2z^2+112x^2z^4+7y^6-70y^4z^2+112y^2z^4-32z^6\right)}{64}$

Table 11 Harmonics for rank 10.

No.	multipole	expression
101	$\mathbb{Q}_{10}(A_g,1)$	$-\frac{63x^{10}}{256}-\frac{315x^8y^2}{256}+\frac{1575x^8z^2}{128}-\frac{315x^6y^4}{128}+\frac{1575x^6y^2z^2}{32}-\frac{525x^6z^4}{8}-\frac{315x^4y^6}{128}+\frac{4725x^4y^4z^2}{64}-\frac{1575x^4y^2z^4}{8}+\frac{315x^4z^6}{4}-\frac{315x^2y^8}{256}+\frac{1575x^2y^6z^2}{32}-\frac{1575x^2y^4z^4}{8}+\frac{315x^2y^2z^6}{2}-\frac{45x^2z^8}{2}-\frac{63y^{10}}{256}+\frac{1575y^8z^2}{128}-\frac{525y^6z^4}{8}+\frac{315y^4z^6}{4}-\frac{45y^2z^8}{2}+z^{10}$
102	$\mathbb{Q}_{10}(A_g,2)$	$\frac{\sqrt{4290}(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)\left(3x^4+6x^2y^2-96x^2z^2+3y^4-96y^2z^2+224z^4\right)}{512}$
103	$\mathbb{Q}_{10}(A_g,3)$	$\frac{\sqrt{4290}xy\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(3x^4+6x^2y^2-96x^2z^2+3y^4-96y^2z^2+224z^4\right)}{256}$
104	$\mathbb{Q}_{10}(A_g,4)$	$\frac{\sqrt{461890}xz\left(x^2-3y^2\right)\left(x^6-33x^4y^2+27x^2y^4-3y^6\right)}{256}$
105	$\mathbb{Q}_{10}(A_g,5)$	$-\frac{\sqrt{4290}xz\left(x^2-3y^2\right)\left(7x^6+21x^4y^2-84x^4z^2+21x^2y^4-168x^2y^2z^2+168x^2z^4+7y^6-84y^4z^2+168y^2z^4-64z^6\right)}{128}$
106	$\mathbb{Q}_{10}(A_g,6)$	$\frac{\sqrt{461890}yz\left(3x^2-y^2\right)\left(3x^6-27x^4y^2+33x^2y^4-y^6\right)}{256}$
107	$\mathbb{Q}_{10}(A_g,7)$	$-\frac{\sqrt{4290}yz\left(3x^2-y^2\right)\left(7x^6+21x^4y^2-84x^4z^2+21x^2y^4-168x^2y^2z^2+168x^2z^4+7y^6-84y^4z^2+168y^2z^4-64z^6\right)}{128}$
108	$\mathbb{Q}_{10,1}(E_g,1)$	$-\frac{\sqrt{72930}yz\left(3x^2+3y^2-16z^2\right)\left(7x^6-35x^4y^2+21x^2y^4-y^6\right)}{256}$
109	$\mathbb{Q}_{10,2}(E_g,1)$	$\frac{\sqrt{72930}xz\left(3x^2+3y^2-16z^2\right)\left(x^6-21x^4y^2+35x^2y^4-7y^6\right)}{256}$
110	$\mathbb{Q}_{10,1}(E_g,2)$	$\frac{\sqrt{858}yz\left(5x^4-10x^2y^2+y^4\right)\left(15x^4+30x^2y^2-140x^2z^2+15y^4-140y^2z^2+168z^4\right)}{128}$
111	$\mathbb{Q}_{10,2}(E_g,2)$	$\frac{\sqrt{858}xz\left(x^4-10x^2y^2+5y^4\right)\left(15x^4+30x^2y^2-140x^2z^2+15y^4-140y^2z^2+168z^4\right)}{128}$
112	$\mathbb{Q}_{10,1}(E_g,3)$	$\frac{\sqrt{55}yz\left(63x^8+252x^6y^2-840x^6z^2+378x^4y^4-2520x^4y^2z^2+2016x^4z^4+252x^2y^6-2520x^2y^4z^2+4032x^2y^2z^4-1152x^2z^6+63y^8-840y^6z^2+2016y^4z^4-1152y^2z^6+128z^8\right)}{128}$
113	$\mathbb{Q}_{10,2}(E_g,3)$	$-\frac{\sqrt{55}xz\left(63x^8+252x^6y^2-840x^6z^2+378x^4y^4-2520x^4y^2z^2+2016x^4z^4+252x^2y^6-2520x^2y^4z^2+4032x^2y^2z^4-1152x^2z^6+63y^8-840y^6z^2+2016y^4z^4-1152y^2z^6+128z^8\right)}{128}$
114	$\mathbb{Q}_{10,1}(E_g,4)$	$\frac{\sqrt{92378}(x-y)(x+y)\left(x^4-4x^3y-14x^2y^2-4xy^3+y^4\right)\left(x^4+4x^3y-14x^2y^2+4xy^3+y^4\right)}{512}$
115	$\mathbb{Q}_{10,2}(E_g,4)$	$\frac{\sqrt{92378}xy\left(x^4-10x^2y^2+5y^4\right)\left(5x^4-10x^2y^2+y^4\right)}{256}$
116	$\mathbb{Q}_{10,1}(E_g,5)$	$-\frac{\sqrt{12155}\left(x^2+y^2-18z^2\right)\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{256}$
117	$\mathbb{Q}_{10,2}(E_g,5)$	$\frac{\sqrt{12155}xy(x-y)(x+y)\left(x^2+y^2-18z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{32}$
118	$\mathbb{Q}_{10,1}(E_g,6)$	$-\frac{\sqrt{2145}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(x^6+3x^4y^2-42x^4z^2+3x^2y^4-84x^2y^2z^2+168x^2z^4+y^6-42y^4z^2+168y^2z^4-112z^6\right)}{128}$
119	$\mathbb{Q}_{10,2}(E_g,6)$	$-\frac{\sqrt{2145}xy(x-y)(x+y)\left(x^6+3x^4y^2-42x^4z^2+3x^2y^4-84x^2y^2z^2+168x^2z^4+y^6-42y^4z^2+168y^2z^4-112z^6\right)}{32}$
120	$\mathbb{Q}_{10,1}(E_g,7)$	$\frac{\sqrt{165}(x-y)(x+y)\left(7x^8+28x^6y^2-336x^6z^2+42x^4y^4-1008x^4y^2z^2+1680x^4z^4+28x^2y^6-1008x^2y^4z^2+3360x^2y^2z^4-1792x^2z^6+7y^8-336y^6z^2+1680y^4z^4-1792y^2z^6+384z^8\right)}{256}$
121	$\mathbb{Q}_{10,2}(E_g,7)$	$-\frac{\sqrt{165}xy\left(7x^8+28x^6y^2-336x^6z^2+42x^4y^4-1008x^4y^2z^2+1680x^4z^4+28x^2y^6-1008x^2y^4z^2+3360x^2y^2z^4-1792x^2z^6+7y^8-336y^6z^2+1680y^4z^4-1792y^2z^6+384z^8\right)}{128}$

Table 12 Harmonics for rank 11.

No.	multipole	expression
122	$\mathbb{Q}_{11}(A_u,1)$	$\frac{\sqrt{14586}xyz\left(x^2-3y^2\right)\left(3x^2-y^2\right)\left(15x^4+30x^2y^2-160x^2z^2+15y^4-160y^2z^2+224z^4\right)}{256}$
123	$\mathbb{Q}_{11}(A_u,2)$	$-\frac{z\left(693x^{10}+3465x^8y^2-11550x^8z^2+6930x^6y^4-46200x^6y^2z^2+36960x^6z^4+6930x^4y^6-69300x^4y^4z^2+110880x^4y^2z^4-31680x^4z^6+3465x^2y^8-46200x^2y^6z^2+110880x^2y^4z^4-63360x^2y^2z^6+7040x^2z^8+693y^{10}-11550y^8z^2+36960y^6z^4-31680y^4z^6+7040y^2z^8-256z^{10}\right)}{256}$
124	$\mathbb{Q}_{11}(A_u,3)$	$\frac{\sqrt{14586}z(x-y)(x+y)\left(x^2-4xy+y^2\right)\left(x^2+4xy+y^2\right)\left(15x^4+30x^2y^2-160x^2z^2+15y^4-160y^2z^2+224z^4\right)}{512}$
125	$\mathbb{Q}_{11}(A_u,4)$	$-\frac{\sqrt{46189}y\left(3x^2-y^2\right)\left(x^2+y^2-20z^2\right)\left(3x^6-27x^4y^2+33x^2y^4-y^6\right)}{512}$
126	$\mathbb{Q}_{11}(A_u,5)$	$\frac{\sqrt{30030}y\left(3x^2-y^2\right)\left(x^8+4x^6y^2-56x^6z^2+6x^4y^4-168x^4y^2z^2+336x^4z^4+4x^2y^6-168x^2y^4z^2+672x^2y^2z^4-448x^2z^6+y^8-56y^6z^2+336y^4z^4-448y^2z^6+128z^8\right)}{512}$
127	$\mathbb{Q}_{11}(A_u,6)$	$-\frac{\sqrt{46189}x\left(x^2-3y^2\right)\left(x^2+y^2-20z^2\right)\left(x^6-33x^4y^2+27x^2y^4-3y^6\right)}{512}$
128	$\mathbb{Q}_{11}(A_u,7)$	$\frac{\sqrt{30030}x\left(x^2-3y^2\right)\left(x^8+4x^6y^2-56x^6z^2+6x^4y^4-168x^4y^2z^2+336x^4z^4+4x^2y^6-168x^2y^4z^2+672x^2y^2z^4-448x^2z^6+y^8-56y^6z^2+336y^4z^4-448y^2z^6+128z^8\right)}{512}$
129	$\mathbb{Q}_{11,1}(E_u,1)$	$\frac{\sqrt{88179}x\left(x^{10}-55x^8y^2+330x^6y^4-462x^4y^6+165x^2y^8-11y^{10}\right)}{512}$
130	$\mathbb{Q}_{11,2}(E_u,1)$	$-\frac{\sqrt{88179}y\left(11x^{10}-165x^8y^2+462x^6y^4-330x^4y^6+55x^2y^8-y^{10}\right)}{512}$
131	$\mathbb{Q}_{11,1}(E_u,2)$	$\frac{\sqrt{36465}x\left(x^6-21x^4y^2+35x^2y^4-7y^6\right)\left(x^4+2x^2y^2-36x^2z^2+y^4-36y^2z^2+96z^4\right)}{512}$
132	$\mathbb{Q}_{11,2}(E_u,2)$	$\frac{\sqrt{36465}y\left(7x^6-35x^4y^2+21x^2y^4-y^6\right)\left(x^4+2x^2y^2-36x^2z^2+y^4-36y^2z^2+96z^4\right)}{512}$
133	$\mathbb{Q}_{11,1}(E_u,3)$	$-\frac{3\sqrt{143}x\left(x^4-10x^2y^2+5y^4\right)\left(5x^6+15x^4y^2-240x^4z^2+15x^2y^4-480x^2y^2z^2+1120x^2z^4+5y^6-240y^4z^2+1120y^2z^4-896z^6\right)}{512}$
134	$\mathbb{Q}_{11,2}(E_u,3)$	$\frac{3\sqrt{143}y\left(5x^4-10x^2y^2+y^4\right)\left(5x^6+15x^4y^2-240x^4z^2+15x^2y^4-480x^2y^2z^2+1120x^2z^4+5y^6-240y^4z^2+1120y^2z^4-896z^6\right)}{512}$
135	$\mathbb{Q}_{11,1}(E_u,4)$	$-\frac{\sqrt{66}x\left(21x^{10}+105x^8y^2-1260x^8z^2+210x^6y^4-5040x^6y^2z^2+8400x^6z^4+210x^4y^6-7560x^4y^4z^2+25200x^4y^2z^4-13440x^4z^6+105x^2y^8-5040x^2y^6z^2+25200x^2y^4z^4-26880x^2y^2z^6+5760x^2z^8+21y^{10}-1260y^8z^2+8400y^6z^4-13440y^4z^6+5760y^2z^8-512z^{10}\right)}{512}$
136	$\mathbb{Q}_{11,2}(E_u,4)$	$-\frac{\sqrt{66}y\left(21x^{10}+105x^8y^2-1260x^8z^2+210x^6y^4-5040x^6y^2z^2+8400x^6z^4+210x^4y^6-7560x^4y^4z^2+25200x^4y^2z^4-13440x^4z^6+105x^2y^8-5040x^2y^6z^2+25200x^2y^4z^4-26880x^2y^2z^6+5760x^2z^8+21y^{10}-1260y^8z^2+8400y^6z^4-13440y^4z^6+5760y^2z^8-512z^{10}\right)}{512}$
137	$\mathbb{Q}_{11,1}(E_u,5)$	$-\frac{\sqrt{1939938}xyz\left(x^4-10x^2y^2+5y^4\right)\left(5x^4-10x^2y^2+y^4\right)}{256}$
138	$\mathbb{Q}_{11,2}(E_u,5)$	$\frac{\sqrt{1939938}z(x-y)(x+y)\left(x^4-4x^3y-14x^2y^2-4xy^3+y^4\right)\left(x^4+4x^3y-14x^2y^2+4xy^3+y^4\right)}{512}$
139	$\mathbb{Q}_{11,1}(E_u,6)$	$-\frac{\sqrt{692835}xyz(x-y)(x+y)\left(x^2+y^2-6z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{32}$
140	$\mathbb{Q}_{11,2}(E_u,6)$	$-\frac{\sqrt{692835}z\left(x^2+y^2-6z^2\right)\left(x^4-4x^3y-6x^2y^2+4xy^3+y^4\right)\left(x^4+4x^3y-6x^2y^2-4xy^3+y^4\right)}{256}$
141	$\mathbb{Q}_{11,1}(E_u,7)$	$\frac{3\sqrt{1001}xyz(x-y)(x+y)\left(5x^6+15x^4y^2-70x^4z^2+15x^2y^4-140x^2y^2z^2+168x^2z^4+5y^6-70y^4z^2+168y^2z^4-80z^6\right)}{32}$
142	$\mathbb{Q}_{11,2}(E_u,7)$	$-\frac{3\sqrt{1001}z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)\left(5x^6+15x^4y^2-70x^4z^2+15x^2y^4-140x^2y^2z^2+168x^2z^4+5y^6-70y^4z^2+168y^2z^4-80z^6\right)}{128}$
143	$\mathbb{Q}_{11,1}(E_u,8)$	$\frac{\sqrt{2145}xyz\left(21x^8+84x^6y^2-336x^6z^2+126x^4y^4-1008x^4y^2z^2+1008x^4z^4+84x^2y^6-1008x^2y^4z^2+2016x^2y^2z^4-768x^2z^6+21y^8-336y^6z^2+1008y^4z^4-768y^2z^6+128z^8\right)}{128}$
144	$\mathbb{Q}_{11,2}(E_u,8)$	$\frac{\sqrt{2145}z(x-y)(x+y)\left(21x^8+84x^6y^2-336x^6z^2+126x^4y^4-1008x^4y^2z^2+1008x^4z^4+84x^2y^6-1008x^2y^4z^2+2016x^2y^2z^4-768x^2z^6+21y^8-336y^6z^2+1008y^4z^4-768y^2z^6+128z^8\right)}{256}$