PG No. 19 C_{3v} $3m$ (3m1 setting) [trigonal] (axial, internal axial quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(2,2)}[g](A_2)$

** symmetry

1

** expression

$$-\frac{\sqrt{5}G_u\left(x^2+y^2-2z^2\right)}{10}+\frac{\sqrt{15}G_v\left(x-y\right)\left(x+y\right)}{10}+\frac{\sqrt{15}G_{xy}xy}{5}+\frac{\sqrt{15}G_{xz}xz}{5}+\frac{\sqrt{15}G_{yz}yz}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(2,0)}[g](A_2)$

** symmetry

z

** expression

$$\frac{\sqrt{10}G_uz}{5}+\frac{\sqrt{30}G_{xz}x}{10}+\frac{\sqrt{30}G_{yz}y}{10}$$

$\vec{\mathbb{G}}_1^{(2,2)}[g](A_2)$

** symmetry

z

** expression

$$-\frac{3\sqrt{35}G_uz\left(3x^2+3y^2-2z^2\right)}{70}+\frac{\sqrt{105}G_{vz}\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}G_{xy}xyz}{7}-\frac{\sqrt{105}G_{xz}x\left(x^2+y^2-4z^2\right)}{35}-\frac{\sqrt{105}G_{yz}y\left(x^2+y^2-4z^2\right)}{35}$$

$\vec{\mathbb{G}}_{1,1}^{(2,0)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(2,0)}[g](E)$

** symmetry

$-y$

x

** expression

$$\frac{\sqrt{10}G_uy}{10}+\frac{\sqrt{30}G_vy}{10}-\frac{\sqrt{30}G_{xy}x}{10}-\frac{\sqrt{30}G_{yz}z}{10}$$

$$-\frac{\sqrt{10}G_ux}{10}+\frac{\sqrt{30}G_vx}{10}+\frac{\sqrt{30}G_{xy}y}{10}+\frac{\sqrt{30}G_{xz}z}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(2,2)}[g](E)$, $\vec{\mathbb{G}}_{1,2}^{(2,2)}[g](E)$

** symmetry

$-y$

x

** expression

$$\frac{3\sqrt{35}G_uy\left(x^2+y^2-4z^2\right)}{70}-\frac{\sqrt{105}G_vy\left(7x^2-3y^2+2z^2\right)}{70}+\frac{\sqrt{105}G_{xy}x\left(x^2-4y^2+z^2\right)}{35}-\frac{\sqrt{105}G_{xz}xyz}{7}+\frac{\sqrt{105}G_{yz}z\left(x^2-4y^2+z^2\right)}{35}$$

$$-\frac{3\sqrt{35}G_ux\left(x^2+y^2-4z^2\right)}{70}+\frac{\sqrt{105}G_vx\left(3x^2-7y^2-2z^2\right)}{70}+\frac{\sqrt{105}G_{xy}y\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}G_{xz}z\left(4x^2-y^2-z^2\right)}{35}+\frac{\sqrt{105}G_{yz}xyz}{7}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(2,-2)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

G_u

$\vec{\mathbb{G}}_2^{(2,0)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$-\frac{\sqrt{14}G_u\left(x^2+y^2-2z^2\right)}{14}-\frac{\sqrt{42}G_v\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{42}G_{xy}xy}{7}+\frac{\sqrt{42}G_{xz}xz}{14}+\frac{\sqrt{42}G_{yz}yz}{14}$$

$\vec{\mathbb{G}}_2^{(2,2)}[g](A_2)$
** symmetry

$$-\frac{x^2}{2}-\frac{y^2}{2}+z^2$$

** expression

$$\frac{\sqrt{14}G_u\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{56}-\frac{5\sqrt{42}G_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{168}$$
$$-\frac{5\sqrt{42}G_{xy}xy\left(x^2+y^2-6z^2\right)}{84}-\frac{5\sqrt{42}G_{xz}xz\left(3x^2+3y^2-4z^2\right)}{84}-\frac{5\sqrt{42}G_{yz}yz\left(3x^2+3y^2-4z^2\right)}{84}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$G_{yz}$$

$$-G_{xz}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-2)}[g](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(2,-2)}[g](E,2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$G_v$$

$$-G_{xy}$$

$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E,1)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{42}G_uyz}{14}-\frac{3\sqrt{14}G_vyz}{14}+\frac{3\sqrt{14}G_{xy}xz}{14}+\frac{3\sqrt{14}G_{xz}xy}{14}-\frac{\sqrt{14}G_{yz}\left(2x^2-y^2-z^2\right)}{14}$$

$$-\frac{\sqrt{42}G_uxz}{14}-\frac{3\sqrt{14}G_vxz}{14}-\frac{3\sqrt{14}G_{xy}yz}{14}-\frac{\sqrt{14}G_{xz}\left(x^2-2y^2+z^2\right)}{14}-\frac{3\sqrt{14}G_{yz}xy}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,0)}[g](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(2,0)}[g](E,2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{42}G_u\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{14}G_v\left(x^2+y^2-2z^2\right)}{14}+\frac{3\sqrt{14}G_{xz}xz}{14}-\frac{3\sqrt{14}G_{yz}yz}{14}$$

$$\frac{\sqrt{42}G_uxy}{7}-\frac{\sqrt{14}G_{xy}\left(x^2+y^2-2z^2\right)}{14}-\frac{3\sqrt{14}G_{xz}yz}{14}-\frac{3\sqrt{14}G_{yz}xz}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{5\sqrt{42}G_u yz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}G_v yz\left(9x^2-5y^2+2z^2\right)}{84}-\frac{5\sqrt{14}G_{xy}xz\left(x^2-6y^2+z^2\right)}{42}$$
$$-\frac{5\sqrt{14}G_{xz}xy\left(x^2+y^2-6z^2\right)}{42}+\frac{\sqrt{14}G_{yz}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{42}$$

$$\frac{5\sqrt{42}G_u xz\left(3x^2+3y^2-4z^2\right)}{84}-\frac{5\sqrt{14}G_v xz\left(5x^2-9y^2-2z^2\right)}{84}-\frac{5\sqrt{14}G_{xy}yz\left(6x^2-y^2-z^2\right)}{42}$$
$$+\frac{\sqrt{14}G_{xz}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{42}+\frac{5\sqrt{14}G_{yz}xy\left(x^2+y^2-6z^2\right)}{42}$$

$\vec{\mathbb{G}}_{2,1}^{(2,2)}[g](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(2,2)}[g](E,2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}G_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{168}+\frac{\sqrt{14}G_v\left(19x^4-102x^2y^2-12x^2z^2+19y^4-12y^2z^2+4z^4\right)}{168}$$
$$+\frac{5\sqrt{14}G_{xy}xy\,(x-y)\,(x+y)}{12}+\frac{5\sqrt{14}G_{xz}xz\left(5x^2-9y^2-2z^2\right)}{84}+\frac{5\sqrt{14}G_{yz}yz\left(9x^2-5y^2+2z^2\right)}{84}$$

$$\frac{5\sqrt{42}G_u xy\left(x^2+y^2-6z^2\right)}{84}-\frac{5\sqrt{14}G_v xy\,(x-y)\,(x+y)}{12}+\frac{\sqrt{14}G_{xy}\left(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4\right)}{42}$$
$$-\frac{5\sqrt{14}G_{xz}yz\left(6x^2-y^2-z^2\right)}{42}+\frac{5\sqrt{14}G_{yz}xz\left(x^2-6y^2+z^2\right)}{42}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(2,-2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{2}G_v x}{2}-\frac{\sqrt{2}G_{xy}y}{2}$$

$\vec{\mathbb{G}}_{3}^{(2,0)}[g](A_1)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_u x\left(x^2-3y^2\right)}{24}+\frac{\sqrt{2}G_v x\left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{2}G_{xy}y\left(x^2+y^2-4z^2\right)}{8}+\frac{5\sqrt{2}G_{xz}z\,(x-y)\,(x+y)}{8}-\frac{5\sqrt{2}G_{yz}xyz}{4}$$

$\vec{\mathbb{G}}_{3}^{(2,2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_u x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{264}+\frac{\sqrt{11}G_v x\left(11x^4-104x^2y^2-6x^2z^2+53y^4-6y^2z^2+4z^4\right)}{88}$$
$$+\frac{\sqrt{11}G_{xy}y\left(26x^4-53x^2y^2+3x^2z^2+5y^4+3y^2z^2-2z^4\right)}{44}+\frac{7\sqrt{11}G_{xz}z\left(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2\right)}{44}+\frac{7\sqrt{11}G_{yz}xyz\left(5x^2-7y^2+2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{3}^{(2,-2)}[g](A_2,1)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_u z}{5}-\frac{\sqrt{5}G_{xz}x}{5}-\frac{\sqrt{5}G_{yz}y}{5}$$

$\vec{\mathbb{G}}_3^{(2,-2)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{2}G_v y}{2}+\frac{\sqrt{2}G_{xy}x}{2}$$

$\vec{\mathbb{G}}_3^{(2,0)}[g](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_u z\left(3x^2+3y^2-2z^2\right)}{15}-\frac{\sqrt{5}G_v z\left(x-y\right)\left(x+y\right)}{2}-\sqrt{5}G_{xy}xyz-\frac{\sqrt{5}G_{xz}x\left(x^2+y^2-4z^2\right)}{20}-\frac{\sqrt{5}G_{yz}y\left(x^2+y^2-4z^2\right)}{20}$$

$\vec{\mathbb{G}}_3^{(2,0)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{5\sqrt{6}G_u y\left(3x^2-y^2\right)}{24}+\frac{\sqrt{2}G_v y\left(x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{2}G_{xy}x\left(x^2+y^2-4z^2\right)}{8}+\frac{5\sqrt{2}G_{xz}xyz}{4}+\frac{5\sqrt{2}G_{yz}z\left(x-y\right)\left(x+y\right)}{8}$$

$\vec{\mathbb{G}}_3^{(2,2)}[g](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{330}G_u z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{264}-\frac{7\sqrt{110}G_v z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}-\frac{7\sqrt{110}G_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$+\frac{\sqrt{110}G_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{110}G_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_3^{(2,2)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{33}G_u y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{264}+\frac{\sqrt{11}G_v y\left(53x^4-104x^2y^2-6x^2z^2+11y^4-6y^2z^2+4z^4\right)}{88}$$
$$-\frac{\sqrt{11}G_{xy}x\left(5x^4-53x^2y^2+3x^2z^2+26y^4+3y^2z^2-2z^4\right)}{44}+\frac{7\sqrt{11}G_{xz}xyz\left(7x^2-5y^2-2z^2\right)}{44}-\frac{7\sqrt{11}G_{yz}z\left(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,1)$
** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{10}G_u y}{5}-\frac{\sqrt{30}G_v y}{30}+\frac{\sqrt{30}G_{xy}x}{30}-\frac{2\sqrt{30}G_{yz}z}{15}$$

$$\frac{\sqrt{10}G_u x}{5} - \frac{\sqrt{30}G_v x}{30} - \frac{\sqrt{30}G_{xy} y}{30} + \frac{2\sqrt{30}G_{xz} z}{15}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-2)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,-2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}G_v z}{3} - \frac{\sqrt{3}G_{xz} x}{3} + \frac{\sqrt{3}G_{yz} y}{3}$$

$$\frac{\sqrt{3}G_{xy} z}{3} + \frac{\sqrt{3}G_{xz} y}{3} + \frac{\sqrt{3}G_{yz} x}{3}$$

$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,1)$

** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{10}G_u y\left(x^2+y^2-4z^2\right)}{40} + \frac{\sqrt{30}G_v y\left(9x^2-11y^2+24z^2\right)}{120} + \frac{\sqrt{30}G_{xy} x\left(x^2+21y^2-24z^2\right)}{120} - \frac{\sqrt{30}G_{xz} xyz}{4} + \frac{\sqrt{30}G_{yz} z\left(21x^2-9y^2-4z^2\right)}{120}$$

$$-\frac{3\sqrt{10}G_u x\left(x^2+y^2-4z^2\right)}{40} - \frac{\sqrt{30}G_v x\left(11x^2-9y^2-24z^2\right)}{120} - \frac{\sqrt{30}G_{xy} y\left(21x^2+y^2-24z^2\right)}{120} + \frac{\sqrt{30}G_{xz} z\left(9x^2-21y^2+4z^2\right)}{120} + \frac{\sqrt{30}G_{yz} xyz}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(2,0)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,0)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}G_v z\left(3x^2+3y^2-2z^2\right)}{6} - \frac{\sqrt{3}G_{xz} x\left(x^2-9y^2+6z^2\right)}{12} - \frac{\sqrt{3}G_{yz} y\left(9x^2-y^2-6z^2\right)}{12}$$

$$\frac{\sqrt{3}G_{xy} z\left(3x^2+3y^2-2z^2\right)}{6} + \frac{\sqrt{3}G_{xz} y\left(3x^2-2y^2+3z^2\right)}{6} - \frac{\sqrt{3}G_{yz} x\left(2x^2-3y^2-3z^2\right)}{6}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E,1)$

** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{55}G_u y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88} + \frac{\sqrt{165}G_v y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{264}$$
$$-\frac{\sqrt{165}G_{xy} x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{132} + \frac{7\sqrt{165}G_{xz} xyz\left(x^2+y^2-2z^2\right)}{44}$$
$$-\frac{\sqrt{165}G_{yz} z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{132}$$

$$\frac{3\sqrt{55}G_u x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88} - \frac{\sqrt{165}G_v x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{264}$$
$$-\frac{\sqrt{165}G_{xy} y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{132}$$
$$-\frac{\sqrt{165}G_{xz} z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{132} - \frac{7\sqrt{165}G_{yz} xyz\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{3,1}^{(2,2)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,2)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}z\,(x-y)\,(x+y)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{21\sqrt{22}G_u z\,(x-y)\,(x+y)\left(x^2+y^2-2z^2\right)}{88}-\frac{\sqrt{66}G_v z\left(39x^4-174x^2y^2-20x^2z^2+39y^4-20y^2z^2+4z^4\right)}{264}-\frac{21\sqrt{66}G_{xy}xyz\,(x-y)\,(x+y)}{44}$$
$$+\frac{\sqrt{66}G_{xz}x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{132}+\frac{\sqrt{66}G_{yz}y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{132}$$

$$-\frac{21\sqrt{22}G_u xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{21\sqrt{66}G_v xyz\,(x-y)\,(x+y)}{44}-\frac{\sqrt{66}G_{xy}z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{66}$$
$$-\frac{\sqrt{66}G_{xz}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{66}+\frac{\sqrt{66}G_{yz}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{66}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{3}G_v xz}{2}-\frac{\sqrt{3}G_{xy}yz}{2}+\frac{\sqrt{3}G_{xz}\,(x-y)\,(x+y)}{4}-\frac{\sqrt{3}G_{yz}xy}{2}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A_1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{22}G_u xz\left(x^2-3y^2\right)}{88}+\frac{3\sqrt{66}G_v xz\left(3x^2+3y^2-4z^2\right)}{88}-\frac{3\sqrt{66}G_{xy}yz\left(3x^2+3y^2-4z^2\right)}{88}$$
$$+\frac{\sqrt{66}G_{xz}\left(x^4-21x^2y^2+15x^2z^2+6y^4-15y^2z^2\right)}{88}+\frac{\sqrt{66}G_{yz}xy\left(19x^2-9y^2-30z^2\right)}{88}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A_1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{2002}G_u xz\left(x^2-3y^2\right)\left(3x^2+3y^2-8z^2\right)}{1144}+\frac{\sqrt{6006}G_v xz\left(19x^4-160x^2y^2-10x^2z^2+85y^4-10y^2z^2+4z^4\right)}{1144}$$
$$+\frac{\sqrt{6006}G_{xy}yz\left(40x^4-85x^2y^2+5x^2z^2+7y^4+5y^2z^2-2z^4\right)}{572}$$
$$-\frac{\sqrt{6006}G_{xz}\left(2x^6-7x^4y^2-23x^4z^2-8x^2y^4+90x^2y^2z^2+8x^2z^4+y^6-7y^4z^2-8y^2z^4\right)}{572}$$
$$-\frac{\sqrt{6006}G_{yz}xy\left(5x^4-2x^2y^2-44x^2z^2-7y^4+76y^2z^2-16z^4\right)}{572}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_2,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{3\sqrt{70}G_u\left(x^2+y^2-2z^2\right)}{70}+\frac{\sqrt{210}G_v\,(x-y)\,(x+y)}{140}+\frac{\sqrt{210}G_{xy}xy}{70}-\frac{2\sqrt{210}G_{xz}xz}{35}-\frac{2\sqrt{210}G_{yz}yz}{35}$$

$\vec{\mathbb{G}}_4^{(2,-2)}[g](A_2,2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{3}G_v yz}{2}+\frac{\sqrt{3}G_{xy}xz}{2}+\frac{\sqrt{3}G_{xz}xy}{2}+\frac{\sqrt{3}G_{yz}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A_2,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{\sqrt{385}G_u\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{308}+\frac{3\sqrt{1155}G_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{308}$$
$$+\frac{3\sqrt{1155}G_{xy}xy\left(x^2+y^2-6z^2\right)}{154}-\frac{\sqrt{1155}G_{xz}xz\left(3x^2+3y^2-4z^2\right)}{308}-\frac{\sqrt{1155}G_{yz}yz\left(3x^2+3y^2-4z^2\right)}{308}$$

$\vec{\mathbb{G}}_4^{(2,0)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{7\sqrt{22}G_u yz\left(3x^2-y^2\right)}{88}+\frac{3\sqrt{66}G_v yz\left(3x^2+3y^2-4z^2\right)}{88}+\frac{3\sqrt{66}G_{xy}xz\left(3x^2+3y^2-4z^2\right)}{88}$$
$$+\frac{\sqrt{66}G_{xz}xy\left(9x^2-19y^2+30z^2\right)}{88}-\frac{\sqrt{66}G_{yz}\left(6x^4-21x^2y^2-15x^2z^2+y^4+15y^2z^2\right)}{88}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A_2,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{3\sqrt{715}G_u\left(5x^6+15x^4y^2-90x^4z^2+15x^2y^4-180x^2y^2z^2+120x^2z^4+5y^6-90y^4z^2+120y^2z^4-16z^6\right)}{2288}$$
$$+\frac{7\sqrt{2145}G_v\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{2288}+\frac{7\sqrt{2145}G_{xy}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{7\sqrt{2145}G_{xz}xz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}+\frac{7\sqrt{2145}G_{yz}yz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}$$

$\vec{\mathbb{G}}_4^{(2,2)}[g](A_2,2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{9\sqrt{2002}G_u yz\left(3x^2-y^2\right)\left(3x^2+3y^2-8z^2\right)}{1144}+\frac{\sqrt{6006}G_v yz\left(85x^4-160x^2y^2-10x^2z^2+19y^4-10y^2z^2+4z^4\right)}{1144}$$
$$-\frac{\sqrt{6006}G_{xy}xz\left(7x^4-85x^2y^2+5x^2z^2+40y^4+5y^2z^2-2z^4\right)}{572}-\frac{\sqrt{6006}G_{xz}xy\left(7x^4+2x^2y^2-76x^2z^2-5y^4+44y^2z^2+16z^4\right)}{572}$$
$$+\frac{\sqrt{6006}G_{yz}\left(x^6-8x^4y^2-7x^4z^2-7x^2y^4+90x^2y^2z^2-8x^2z^4+2y^6-23y^4z^2+8y^2z^4\right)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{7}G_u yz}{7}+\frac{\sqrt{21}G_v yz}{14}-\frac{\sqrt{21}G_{xy}xz}{14}-\frac{\sqrt{21}G_{xz}xy}{14}-\frac{\sqrt{21}G_{yz}\left(x^2+3y^2-4z^2\right)}{28}$$

$$-\frac{3\sqrt{7}G_u xz}{7}+\frac{\sqrt{21}G_v xz}{14}+\frac{\sqrt{21}G_{xy}yz}{14}+\frac{\sqrt{21}G_{xz}\left(3x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{21}G_{yz}xy}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{6}G_v\,(x-y)\,(x+y)}{4}-\frac{\sqrt{6}G_{xy}xy}{2}$$

$$\frac{\sqrt{6}G_vxy}{2}+\frac{\sqrt{6}G_{xy}\,(x-y)\,(x+y)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-2)}[g](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(2,-2)}[g](E,3)$
** symmetry

$$-\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{14}G_u\,(x-y)\,(x+y)}{28}-\frac{\sqrt{42}G_v\left(x^2+y^2-2z^2\right)}{28}+\frac{\sqrt{42}G_{xz}xz}{7}-\frac{\sqrt{42}G_{yz}yz}{7}$$

$$-\frac{3\sqrt{14}G_uxy}{14}+\frac{\sqrt{42}G_{xy}\left(x^2+y^2-2z^2\right)}{28}-\frac{\sqrt{42}G_{xz}yz}{7}-\frac{\sqrt{42}G_{yz}xz}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{17\sqrt{154}G_uyz\left(3x^2+3y^2-4z^2\right)}{616}-\frac{\sqrt{462}G_vyz\left(33x^2-51y^2+40z^2\right)}{616}-\frac{\sqrt{462}G_{xy}xz\left(9x^2+93y^2-40z^2\right)}{616}$$
$$-\frac{9\sqrt{462}G_{xz}xy\left(x^2+y^2-6z^2\right)}{616}+\frac{\sqrt{462}G_{yz}\left(6x^4+3x^2y^2-39x^2z^2-3y^4+15y^2z^2+4z^4\right)}{616}$$

$$\frac{17\sqrt{154}G_uxz\left(3x^2+3y^2-4z^2\right)}{616}+\frac{\sqrt{462}G_vxz\left(51x^2-33y^2-40z^2\right)}{616}+\frac{\sqrt{462}G_{xy}yz\left(93x^2+9y^2-40z^2\right)}{616}$$
$$+\frac{\sqrt{462}G_{xz}\left(3x^4-3x^2y^2-15x^2z^2-6y^4+39y^2z^2-4z^4\right)}{616}+\frac{9\sqrt{462}G_{yz}xy\left(x^2+y^2-6z^2\right)}{616}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{7\sqrt{11}G_u\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{44}+\frac{\sqrt{33}G_v\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{44}$$
$$-\frac{\sqrt{33}G_{xy}xy\left(x^2+y^2-6z^2\right)}{22}+\frac{7\sqrt{33}G_{xz}xz\left(x^2-3y^2\right)}{44}-\frac{7\sqrt{33}G_{yz}yz\left(3x^2-y^2\right)}{44}$$

$$-\frac{7\sqrt{11}G_uxy\,(x-y)\,(x+y)}{11}+\frac{\sqrt{33}G_vxy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}G_{xy}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}G_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}G_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,0)}[g](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(2,0)}[g](E,3)$

** symmetry

$$-\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{77}G_u\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}G_v\left(2x^4-3x^2y^2-9x^2z^2+2y^4-9y^2z^2+3z^4\right)}{77}$$
$$-\frac{\sqrt{231}G_{xy}xy\,(x-y)\,(x+y)}{22}+\frac{\sqrt{231}G_{xz}xz\left(2x^2-33y^2+9z^2\right)}{154}+\frac{\sqrt{231}G_{yz}yz\left(33x^2-2y^2-9z^2\right)}{154}$$

$$\frac{2\sqrt{77}G_uxy\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{231}G_vxy\,(x-y)\,(x+y)}{22}+\frac{\sqrt{231}G_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}$$
$$-\frac{\sqrt{231}G_{xz}yz\left(39x^2-31y^2+18z^2\right)}{308}+\frac{\sqrt{231}G_{yz}xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,1)$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{21\sqrt{286}G_uyz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}-\frac{7\sqrt{858}G_vyz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{1144}$$
$$+\frac{7\sqrt{858}G_{xy}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}+\frac{7\sqrt{858}G_{xz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}$$
$$-\frac{\sqrt{858}G_{yz}\left(x^6-4x^4y^2-11x^4z^2-11x^2y^4+90x^2y^2z^2-4x^2z^4-6y^6+101y^4z^2-116y^2z^4+8z^6\right)}{572}$$

$$-\frac{21\sqrt{286}G_uxz\left(5x^4+10x^2y^2-20x^2z^2+5y^4-20y^2z^2+8z^4\right)}{1144}$$
$$+\frac{7\sqrt{858}G_vxz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{1144}+\frac{7\sqrt{858}G_{xy}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{572}$$
$$-\frac{\sqrt{858}G_{xz}\left(6x^6+11x^4y^2-101x^4z^2+4x^2y^4-90x^2y^2z^2+116x^2z^4-y^6+11y^4z^2+4y^2z^4-8z^6\right)}{572}$$
$$-\frac{7\sqrt{858}G_{yz}xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$-\frac{9\sqrt{1001}G_u\left(x^2+y^2-10z^2\right)\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{2288}+\frac{\sqrt{3003}G_v\,(x-y)\,(x+y)\left(17x^4-230x^2y^2-8x^2z^2+17y^4-8y^2z^2+8z^4\right)}{2288}$$
$$+\frac{\sqrt{3003}G_{xy}xy\left(49x^4-166x^2y^2+8x^2z^2+49y^4+8y^2z^2-8z^4\right)}{1144}$$
$$+\frac{3\sqrt{3003}G_{xz}xz\left(7x^4-58x^2y^2-4x^2z^2+23y^4+12y^2z^2\right)}{1144}+\frac{3\sqrt{3003}G_{yz}yz\left(23x^4-58x^2y^2+12x^2z^2+7y^4-4y^2z^2\right)}{1144}$$

$$-\frac{9\sqrt{1001}G_uxy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{3003}G_vxy\left(25x^4-82x^2y^2-4x^2z^2+25y^4-4y^2z^2+4z^4\right)}{572}$$
$$-\frac{\sqrt{3003}G_{xy}\,(x-y)\,(x+y)\left(2x^4-29x^2y^2+x^2z^2+2y^4+y^2z^2-z^4\right)}{286}$$
$$+\frac{3\sqrt{3003}G_{xz}yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{286}-\frac{3\sqrt{3003}G_{yz}xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{286}$$

$\vec{\mathbb{G}}_{4,1}^{(2,2)}[g](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(2,2)}[g](E,3)$

** symmetry

$$-\frac{\sqrt{5}\,(x-y)\,(x+y)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{143}G_u\,(x-y)\,(x+y)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$-\frac{\sqrt{429}G_v\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$
$$-\frac{21\sqrt{429}G_{xy}xy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}-\frac{7\sqrt{429}G_{xz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{572}$$
$$-\frac{7\sqrt{429}G_{yz}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{572}$$

$$-\frac{21\sqrt{143}G_uxy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}+\frac{21\sqrt{429}G_vxy\,(x-y)\,(x+y)\left(x^2+y^2-10z^2\right)}{572}$$
$$-\frac{\sqrt{429}G_{xy}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$
$$+\frac{7\sqrt{429}G_{xz}yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{286}-\frac{7\sqrt{429}G_{yz}xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{286}$$