PG No. 20 $D_{3\mathrm{d}}$ $\bar{3}m$ (-3m1 setting) [trigonal] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,0)}[q](A_{2g})$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_{x}y}{2} - \frac{\sqrt{2}Q_{y}x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E_{g})$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E_{g})$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}Q_{y}z}{2} - \frac{\sqrt{2}Q_{z}y}{2}$$

$$-\frac{\sqrt{2}Q_{x}z}{2} + \frac{\sqrt{2}Q_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](A_{1u})$

** symmetry

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{6}Q_{x}yz}{2} - \frac{\sqrt{6}Q_{y}xz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E_{u},1)$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E_{u},1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{x}\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}Q_{y}xy}{2} + \frac{\sqrt{2}Q_{z}xz}{2}$$

$$-\frac{\sqrt{2}Q_{x}xy}{2} + \frac{\sqrt{2}Q_{y}\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{2}Q_{z}yz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E_{u},2)$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E_{u},2)$

** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{x}yz}{2} + \frac{\sqrt{2}Q_{y}xz}{2} - \sqrt{2}Q_{z}xy$$

$$\frac{\sqrt{2}Q_{x}xz}{2} - \frac{\sqrt{2}Q_{y}yz}{2} - \frac{\sqrt{2}Q_{z}\left(x - y\right)\left(x + y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{1g})$

** symmetry

$$\frac{\sqrt{10}x\left(x^{2} - 3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{30}Q_{x}xyz}{4}+\frac{\sqrt{30}Q_{y}z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}Q_{z}y\left(3x^{2}-y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{2g},1)$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_{x}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{3}Q_{y}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{2g},2)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_{x}z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}Q_{y}xyz}{4}+\frac{\sqrt{30}Q_{z}x\left(x^{2}-3y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_{g},1)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_{g},1)$

** symmetry

$$-\frac{\sqrt{6}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

** expression

$$\frac{5\sqrt{2}Q_{x}xyz}{4}-\frac{\sqrt{2}Q_{y}z\left(11x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{\sqrt{2}Q_{z}y\left(x^{2}+y^{2}-4z^{2}\right)}{8}$$

$$\frac{\sqrt{2}Q_{x}z\left(x^{2}+11y^{2}-4z^{2}\right)}{8}-\frac{5\sqrt{2}Q_{y}xyz}{4}-\frac{\sqrt{2}Q_{z}x\left(x^{2}+y^{2}-4z^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_{g},2)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_{g},2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_{x}x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_{y}y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_{z}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{5}Q_{x}y\left(x^{2}-y^{2}+2z^{2}\right)}{4}-\frac{\sqrt{5}Q_{y}x\left(x^{2}-y^{2}-2z^{2}\right)}{4}-\sqrt{5}Q_{z}xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A_{1u},1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$-\frac{\sqrt{5}Q_{x}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{5}Q_{y}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A_{1u},2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x\left(3x^2y^2-3x^2z^2-y^4+3y^2z^2\right)}{8}-\frac{\sqrt{14}Q_yxy\left(3x^2-y^2-6z^2\right)}{8}+\frac{3\sqrt{14}Q_zxz\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_{2u})$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_xxy\left(x^2-3y^2+6z^2\right)}{8}-\frac{\sqrt{14}Q_y\left(x^4-3x^2y^2-3x^2z^2+3y^2z^2\right)}{8}-\frac{3\sqrt{14}Q_zyz\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_u,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_x\left(3x^2y^2-3x^2z^2+3y^4-21y^2z^2+4z^4\right)}{8}+\frac{3\sqrt{2}Q_yxy\left(x^2+y^2-6z^2\right)}{8}-\frac{\sqrt{2}Q_zxz\left(3x^2+3y^2-4z^2\right)}{8}$$

$$\frac{3\sqrt{2}Q_xxy\left(x^2+y^2-6z^2\right)}{8}-\frac{\sqrt{2}Q_y\left(3x^4+3x^2y^2-21x^2z^2-3y^2z^2+4z^4\right)}{8}-\frac{\sqrt{2}Q_zyz\left(3x^2+3y^2-4z^2\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_u,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_xyz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}Q_yxz\left(x^2-3y^2\right)}{4}-\sqrt{7}Q_zxy\left(x-y\right)\left(x+y\right)$$

$$-\frac{\sqrt{7}Q_xxz\left(x^2-3y^2\right)}{4}+\frac{\sqrt{7}Q_yyz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}Q_z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_u,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{Q_xyz\left(3x^2-4y^2+3z^2\right)}{2}-\frac{Q_yxz\left(4x^2-3y^2-3z^2\right)}{2}+\frac{Q_zxy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{Q_xxz\left(x^2+15y^2-6z^2\right)}{4}+\frac{Q_yyz\left(15x^2+y^2-6z^2\right)}{4}+\frac{Q_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$