PG No. 29 $T_{\rm h}$ $m\bar{3}$ [cubic] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1
  $\vec{\mathbb{Q}}_{1,1}^{(1,0)}[g](T_u)$, $\vec{\mathbb{Q}}_{1,2}^{(1,0)}[g](T_u)$, $\vec{\mathbb{Q}}_{1,3}^{(1,0)}[g](T_u)$
  ** symmetry

$$x$$

$$y$$

$$z$$

  ** expression

$$\frac{\sqrt{2}G_{y}z}{2} - \frac{\sqrt{2}G_{z}y}{2}$$

$$- \frac{\sqrt{2}G_{x}z}{2} + \frac{\sqrt{2}G_{z}x}{2}$$

$$\frac{\sqrt{2}G_{x}y}{2} - \frac{\sqrt{2}G_{y}x}{2}$$

* Harmonics for rank 2
  $\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E_g)$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E_g)$
  ** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

  ** expression

$$\frac{\sqrt{6}G_{x}yz}{2} - \frac{\sqrt{6}G_{y}xz}{2}$$

$$\frac{\sqrt{2}G_{x}yz}{2} + \frac{\sqrt{2}G_{y}xz}{2} - \sqrt{2}G_{z}xy$$

  $\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](T_g)$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](T_g)$, $\vec{\mathbb{Q}}_{2,3}^{(1,0)}[g](T_g)$
  ** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

  ** expression

$$\frac{\sqrt{2}G_{x} \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{2}G_{y}xy}{2} + \frac{\sqrt{2}G_{z}xz}{2}$$

$$\frac{\sqrt{2}G_{x}xy}{2} - \frac{\sqrt{2}G_{y} \left(x - z\right) \left(x + z\right)}{2} - \frac{\sqrt{2}G_{z}yz}{2}$$

$$- \frac{\sqrt{2}G_{x}xz}{2} + \frac{\sqrt{2}G_{y}yz}{2} + \frac{\sqrt{2}G_{z} \left(x - y\right) \left(x + y\right)}{2}$$

* Harmonics for rank 3
  $\vec{\mathbb{Q}}_{3}^{(1,0)}[g](A_u)$
  ** symmetry

$$\sqrt{15}xyz$$

  ** expression

$$\frac{\sqrt{5}G_{x}x \left(y - z\right) \left(y + z\right)}{2} - \frac{\sqrt{5}G_{y}y \left(x - z\right) \left(x + z\right)}{2} + \frac{\sqrt{5}G_{z}z \left(x - y\right) \left(x + y\right)}{2}$$

  $\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](T_u,1)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](T_u,1)$, $\vec{\mathbb{Q}}_{3,3}^{(1,0)}[g](T_u,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}G_y z\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{3}G_z y\left(4x^2-y^2-z^2\right)}{4}$$

$$\frac{\sqrt{3}G_x z\left(x^2-4y^2+z^2\right)}{4}-\frac{\sqrt{3}G_z x\left(x^2-4y^2+z^2\right)}{4}$$

$$-\frac{\sqrt{3}G_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}G_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](T_u,2)$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](T_u,2)$, $\vec{\mathbb{Q}}_{3,3}^{(1,0)}[g](T_u,2)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\sqrt{5}G_x xyz+\frac{\sqrt{5}G_y z\left(2x^2+y^2-z^2\right)}{4}+\frac{\sqrt{5}G_z y\left(2x^2-y^2+z^2\right)}{4}$$

$$\frac{\sqrt{5}G_x z\left(x^2+2y^2-z^2\right)}{4}-\sqrt{5}G_y xyz-\frac{\sqrt{5}G_z x\left(x^2-2y^2-z^2\right)}{4}$$

$$\frac{\sqrt{5}G_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}G_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}G_z xyz$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_{4}^{(1,0)}[g](A_g)$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}G_x yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}G_y xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}G_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E_g)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E_g)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{3}G_x yz\left(9x^2+2y^2-5z^2\right)}{6}+\frac{\sqrt{3}G_y xz\left(2x^2+9y^2-5z^2\right)}{6}+\frac{7\sqrt{3}G_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$$-\frac{G_x yz\left(3x^2-4y^2+3z^2\right)}{2}+\frac{G_y xz\left(4x^2-3y^2-3z^2\right)}{2}-\frac{G_z xy\left(x^2+y^2-6z^2\right)}{2}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](T_g,1)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](T_g,1)$, $\vec{\mathbb{Q}}_{4,3}^{(1,0)}[g](T_g,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y - z\right)\left(y + z\right)}{2}$$

$$- \frac{\sqrt{35}xz\left(x - z\right)\left(x + z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_{x}\left(y^{2} - 2yz - z^{2}\right)\left(y^{2} + 2yz - z^{2}\right)}{4} - \frac{\sqrt{7}G_{y}xy\left(y^{2} - 3z^{2}\right)}{4} + \frac{\sqrt{7}G_{z}xz\left(3y^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7}G_{x}xy\left(x^{2} - 3z^{2}\right)}{4} + \frac{\sqrt{7}G_{y}\left(x^{2} - 2xz - z^{2}\right)\left(x^{2} + 2xz - z^{2}\right)}{4} + \frac{\sqrt{7}G_{z}yz\left(3x^{2} - z^{2}\right)}{4}$$

$$- \frac{\sqrt{7}G_{x}xz\left(x^{2} - 3y^{2}\right)}{4} + \frac{\sqrt{7}G_{y}yz\left(3x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7}G_{z}\left(x^{2} - 2xy - y^{2}\right)\left(x^{2} + 2xy - y^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](T_g,2)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](T_g,2)$, $\vec{\mathbb{Q}}_{4,3}^{(1,0)}[g](T_g,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2} - y^{2} - z^{2}\right)}{2}$$

$$- \frac{\sqrt{5}xz\left(x^{2} - 6y^{2} + z^{2}\right)}{2}$$

$$- \frac{\sqrt{5}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{2}$$

** expression

$$\frac{G_{x}\left(y - z\right)\left(y + z\right)\left(6x^{2} - y^{2} - z^{2}\right)}{4} - \frac{G_{y}xy\left(6x^{2} - y^{2} - 15z^{2}\right)}{4} + \frac{G_{z}xz\left(6x^{2} - 15y^{2} - z^{2}\right)}{4}$$

$$- \frac{G_{x}xy\left(x^{2} - 6y^{2} + 15z^{2}\right)}{4} + \frac{G_{y}\left(x - z\right)\left(x + z\right)\left(x^{2} - 6y^{2} + z^{2}\right)}{4} + \frac{G_{z}yz\left(15x^{2} - 6y^{2} + z^{2}\right)}{4}$$

$$\frac{G_{x}xz\left(x^{2} + 15y^{2} - 6z^{2}\right)}{4} - \frac{G_{y}yz\left(15x^{2} + y^{2} - 6z^{2}\right)}{4} - \frac{G_{z}\left(x - y\right)\left(x + y\right)\left(x^{2} + y^{2} - 6z^{2}\right)}{4}$$