# PG No. 32 $O_{\rm h}$ $m\bar{3}m$ [ cubic ] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_{0}^{(1,1)}[g](A_{1u})$

** symmetry

$$1$$

** expression

$$\frac{\sqrt{3}G_{x}x}{3} + \frac{\sqrt{3}G_{y}y}{3} + \frac{\sqrt{3}G_{z}z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,3}^{(1,-1)}[g](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$G_{x}$$

$$G_{y}$$

$$G_{z}$$

$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,2}^{(1,1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{1,3}^{(1,1)}[g](T_{1g})$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$\frac{\sqrt{10}G_{x} \left(2x^{2} - y^{2} - z^{2}\right)}{10} + \frac{3\sqrt{10}G_{y}xy}{10} + \frac{3\sqrt{10}G_{z}xz}{10}$$

$$\frac{3\sqrt{10}G_{x}xy}{10} - \frac{\sqrt{10}G_{y} \left(x^{2} - 2y^{2} + z^{2}\right)}{10} + \frac{3\sqrt{10}G_{z}yz}{10}$$

$$\frac{3\sqrt{10}G_{x}xz}{10} + \frac{3\sqrt{10}G_{y}yz}{10} - \frac{\sqrt{10}G_{z} \left(x^{2} + y^{2} - 2z^{2}\right)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_{u})$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_{u})$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$- \frac{\sqrt{6}G_{x}x}{6} - \frac{\sqrt{6}G_{y}y}{6} + \frac{\sqrt{6}G_{z}z}{3}$$

$$\frac{\sqrt{2}G_{x}x}{2} - \frac{\sqrt{2}G_{y}y}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_{u})$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_{u})$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3} \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$-\frac{\sqrt{21}G_x x\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_y y\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$$\frac{\sqrt{7}G_x x\left(3x^2-7y^2-2z^2\right)}{14}+\frac{\sqrt{7}G_y y\left(7x^2-3y^2+2z^2\right)}{14}+\frac{5\sqrt{7}G_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{2,3}^{(1,-1)}[g](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_y z}{2}+\frac{\sqrt{2}G_z y}{2}$$

$$\frac{\sqrt{2}G_x z}{2}+\frac{\sqrt{2}G_z x}{2}$$

$$\frac{\sqrt{2}G_x y}{2}+\frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{2,3}^{(1,1)}[g](T_{2u})$

** symmetry

$$\sqrt{3}yz$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7}-\frac{\sqrt{7}G_y z\left(x^2-4y^2+z^2\right)}{7}-\frac{\sqrt{7}G_z y\left(x^2+y^2-4z^2\right)}{7}$$

$$\frac{\sqrt{7}G_x z\left(4x^2-y^2-z^2\right)}{7}+\frac{5\sqrt{7}G_y xyz}{7}-\frac{\sqrt{7}G_z x\left(x^2+y^2-4z^2\right)}{7}$$

$$\frac{\sqrt{7}G_x y\left(4x^2-y^2-z^2\right)}{7}-\frac{\sqrt{7}G_y x\left(x^2-4y^2+z^2\right)}{7}+\frac{5\sqrt{7}G_z xyz}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,-1)}[g](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_x yz+G_y xz+G_z xy$$

$\vec{\mathbb{G}}_{3}^{(1,1)}[g](A_{2g})$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}G_x yz\left(6x^2-y^2-z^2\right)}{6}-\frac{\sqrt{15}G_y xz\left(x^2-6y^2+z^2\right)}{6}-\frac{\sqrt{15}G_z xy\left(x^2+y^2-6z^2\right)}{6}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(1,-1)}[g](T_{1g})$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(2x^2-y^2-z^2\right)}{10}-\frac{\sqrt{15}G_yxy}{5}-\frac{\sqrt{15}G_zxz}{5}$$

$$-\frac{\sqrt{15}G_xxy}{5}-\frac{\sqrt{15}G_y\left(x^2-2y^2+z^2\right)}{10}-\frac{\sqrt{15}G_zyz}{5}$$

$$-\frac{\sqrt{15}G_xxz}{5}-\frac{\sqrt{15}G_yyz}{5}-\frac{\sqrt{15}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](T_{1g})$, $\vec{\mathbb{G}}_{3,3}^{(1,1)}[g](T_{1g})$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$\frac{G_x\left(8x^4-24x^2y^2-24x^2z^2+3y^4+6y^2z^2+3z^4\right)}{12}+\frac{5G_yxy\left(4x^2-3y^2-3z^2\right)}{12}+\frac{5G_zxz\left(4x^2-3y^2-3z^2\right)}{12}$$

$$-\frac{5G_xxy\left(3x^2-4y^2+3z^2\right)}{12}+\frac{G_y\left(3x^4-24x^2y^2+6x^2z^2+8y^4-24y^2z^2+3z^4\right)}{12}-\frac{5G_zyz\left(3x^2-4y^2+3z^2\right)}{12}$$

$$-\frac{5G_xxz\left(3x^2+3y^2-4z^2\right)}{12}-\frac{5G_yyz\left(3x^2+3y^2-4z^2\right)}{12}+\frac{G_z\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(1,-1)}[g](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{G_x\left(y-z\right)\left(y+z\right)}{2}+G_yxy-G_zxz$$

$$-G_xxy-\frac{G_y\left(x-z\right)\left(x+z\right)}{2}+G_zyz$$

$$G_xxz-G_yyz+\frac{G_z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](T_{2g})$, $\vec{\mathbb{G}}_{3,3}^{(1,1)}[g](T_{2g})$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{12}-\frac{\sqrt{15}G_yxy\left(2x^2-5y^2+9z^2\right)}{12}+\frac{\sqrt{15}G_zxz\left(2x^2+9y^2-5z^2\right)}{12}$$

$$-\frac{\sqrt{15}G_xxy\left(5x^2-2y^2-9z^2\right)}{12}+\frac{\sqrt{15}G_y\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{12}-\frac{\sqrt{15}G_zyz\left(9x^2+2y^2-5z^2\right)}{12}$$

$$\frac{\sqrt{15}G_xxz\left(5x^2-9y^2-2z^2\right)}{12}+\frac{\sqrt{15}G_yyz\left(9x^2-5y^2+2z^2\right)}{12}-\frac{\sqrt{15}G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}G_xx\left(2x^2-3y^2-3z^2\right)}{6}-\frac{\sqrt{3}G_yy\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}G_zz\left(3x^2+3y^2-2z^2\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_{1u})$

** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}G_xx\left(x^4-5x^2y^2-5x^2z^2+3y^4-3y^2z^2+3z^4\right)}{66}+\frac{\sqrt{1155}G_yy\left(3x^4-5x^2y^2-3x^2z^2+y^4-5y^2z^2+3z^4\right)}{66}$$
$$+\frac{\sqrt{1155}G_zz\left(3x^4-3x^2y^2-5x^2z^2+3y^4-5y^2z^2+z^4\right)}{66}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{105}G_xx\left(x^2-6y^2+3z^2\right)}{42}+\frac{\sqrt{105}G_yy\left(6x^2-y^2-3z^2\right)}{42}-\frac{\sqrt{105}G_zz\left(3x^2+3y^2-2z^2\right)}{42}$$

$$\frac{\sqrt{35}G_xx\left(x^2-3z^2\right)}{14}-\frac{\sqrt{35}G_yy\left(y^2-3z^2\right)}{14}-\frac{3\sqrt{35}G_zz\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{33}G_xx\left(5x^4-88x^2y^2+38x^2z^2+33y^4+66y^2z^2-30z^4\right)}{132}-\frac{\sqrt{33}G_yy\left(33x^4-88x^2y^2+66x^2z^2+5y^4+38y^2z^2-30z^4\right)}{132}$$
$$+\frac{\sqrt{33}G_zz\left(3x^4+132x^2y^2-50x^2z^2+3y^4-50y^2z^2+10z^4\right)}{132}$$

$$\frac{\sqrt{11}G_xx\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$+\frac{\sqrt{11}G_yy\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}+\frac{21\sqrt{11}G_zz\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(1,-1)}[g](T_{1u})$

** symmetry

$$\frac{\sqrt{35} y z \left(y - z\right) \left(y + z\right)}{2}$$

$$- \frac{\sqrt{35} x z \left(x - z\right) \left(x + z\right)}{2}$$

$$\frac{\sqrt{35} x y \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{5} G_{y} z \left(3 y^{2} - z^{2}\right)}{4} + \frac{\sqrt{5} G_{z} y \left(y^{2} - 3 z^{2}\right)}{4}$$

$$- \frac{\sqrt{5} G_{x} z \left(3 x^{2} - z^{2}\right)}{4} - \frac{\sqrt{5} G_{z} x \left(x^{2} - 3 z^{2}\right)}{4}$$

$$\frac{\sqrt{5} G_{x} y \left(3 x^{2} - y^{2}\right)}{4} + \frac{\sqrt{5} G_{y} x \left(x^{2} - 3 y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](T_{1u})$, $\vec{\mathbb{G}}_{4,3}^{(1,1)}[g](T_{1u})$

** symmetry

$$\frac{\sqrt{35} y z \left(y - z\right) \left(y + z\right)}{2}$$

$$- \frac{\sqrt{35} x z \left(x - z\right) \left(x + z\right)}{2}$$

$$\frac{\sqrt{35} x y \left(x - y\right) \left(x + y\right)}{2}$$

** expression

$$\frac{9 \sqrt{77} G_{x} x y z \left(y - z\right) \left(y + z\right)}{22} - \frac{\sqrt{77} G_{y} z \left(3 x^{2} y^{2} - x^{2} z^{2} - 6 y^{4} + 11 y^{2} z^{2} - z^{4}\right)}{22} - \frac{\sqrt{77} G_{z} y \left(x^{2} y^{2} - 3 x^{2} z^{2} + y^{4} - 11 y^{2} z^{2} + 6 z^{4}\right)}{22}$$

$$- \frac{\sqrt{77} G_{x} z \left(6 x^{4} - 3 x^{2} y^{2} - 11 x^{2} z^{2} + y^{2} z^{2} + z^{4}\right)}{22} - \frac{9 \sqrt{77} G_{y} x y z \left(x - z\right) \left(x + z\right)}{22} + \frac{\sqrt{77} G_{z} x \left(x^{4} + x^{2} y^{2} - 11 x^{2} z^{2} - 3 y^{2} z^{2} + 6 z^{4}\right)}{22}$$

$$\frac{\sqrt{77} G_{x} y \left(6 x^{4} - 11 x^{2} y^{2} - 3 x^{2} z^{2} + y^{4} + y^{2} z^{2}\right)}{22} - \frac{\sqrt{77} G_{y} x \left(x^{4} - 11 x^{2} y^{2} + x^{2} z^{2} + 6 y^{4} - 3 y^{2} z^{2}\right)}{22} + \frac{9 \sqrt{77} G_{z} x y z \left(x - y\right) \left(x + y\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(1,-1)}[g](T_{2u})$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x z \left(x^{2} - 6 y^{2} + z^{2}\right)}{2}$$

$$- \frac{\sqrt{5} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{2}$$

** expression

$$\frac{3 \sqrt{35} G_{x} x y z}{7} + \frac{\sqrt{35} G_{y} z \left(6 x^{2} - 3 y^{2} - z^{2}\right)}{28} + \frac{\sqrt{35} G_{z} y \left(6 x^{2} - y^{2} - 3 z^{2}\right)}{28}$$

$$- \frac{\sqrt{35} G_{x} z \left(3 x^{2} - 6 y^{2} + z^{2}\right)}{28} + \frac{3 \sqrt{35} G_{y} x y z}{7} - \frac{\sqrt{35} G_{z} x \left(x^{2} - 6 y^{2} + 3 z^{2}\right)}{28}$$

$$- \frac{\sqrt{35} G_{x} y \left(3 x^{2} + y^{2} - 6 z^{2}\right)}{28} - \frac{\sqrt{35} G_{y} x \left(x^{2} + 3 y^{2} - 6 z^{2}\right)}{28} + \frac{3 \sqrt{35} G_{z} x y z}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](T_{2u})$, $\vec{\mathbb{G}}_{4,3}^{(1,1)}[g](T_{2u})$

** symmetry

$$\frac{\sqrt{5} y z \left(6 x^{2} - y^{2} - z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}G_xxyz\left(2x^2-y^2-z^2\right)}{22}-\frac{\sqrt{11}G_yz\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{\sqrt{11}G_zy\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}$$

$$-\frac{\sqrt{11}G_xz\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{21\sqrt{11}G_yxyz\left(x^2-2y^2+z^2\right)}{22}+\frac{\sqrt{11}G_zx\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}$$

$$-\frac{\sqrt{11}G_xy\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}G_yx\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}G_zxyz\left(x^2+y^2-2z^2\right)}{22}$$