

* symmetry operation

Table 1: Symmetry operations for 3d polar vector.

| No. | tag | matrix (polar) | det | TR |
|-----|---------------------|--|-----|----|
| 1 | $\{1 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 2 | $\{2_{100} 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | 1 |
| 3 | $\{2_{010} 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | 1 |
| 4 | $\{2_{001} 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | 1 |
| 5 | $\{3_{111}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 6 | $\{3_{111}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 7 | $\{3_{1-1-1}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 8 | $\{3_{1-1-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 9 | $\{3_{-11-1}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 10 | $\{3_{-11-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 11 | $\{3_{-1-11}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 12 | $\{3_{-1-11}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | 1 |
| 13 | $\{-1 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|----------------------|--|-----|----|
| 14 | $\{m_{100} 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | 1 |
| 15 | $\{m_{010} 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | 1 |
| 16 | $\{m_{001} 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | 1 |
| 17 | $\{-3_{111}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 18 | $\{-3_{111}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 19 | $\{-3_{1-1-1}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 20 | $\{-3_{1-1-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 21 | $\{-3_{-11-1}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 22 | $\{-3_{-11-1}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 23 | $\{-3_{-1-11}^- 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 24 | $\{-3_{-1-11}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | 1 |
| 25 | $\{4_{100}'^+ 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 26 | $\{4_{100}'^- 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 27 | $\{4_{010}'^+ 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|--------------------|--|-----|----|
| 28 | $\{4_{010}^- 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 29 | $\{4_{001}^+ 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | -1 |
| 30 | $\{4_{001}^- 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | 1 | -1 |
| 31 | $\{2_{110}' 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 32 | $\{2_{1-10}' 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | 1 | -1 |
| 33 | $\{2_{011}' 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 34 | $\{2_{01-1}' 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 35 | $\{2_{101}' 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 36 | $\{2_{-101}' 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | 1 | -1 |
| 37 | $\{-4_{100}^+ 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 38 | $\{-4_{100}^- 0\}$ | $\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 39 | $\{-4_{010}^+ 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 40 | $\{-4_{010}^- 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 41 | $\{-4_{001}^+ 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | -1 |

continued ...

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| No. | tag | matrix (polar) | det | TR |
|-----|-------------------|---|-----|----|
| 42 | $\{-4_{001}' 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ | -1 | -1 |
| 43 | $\{m_{110}' 0\}$ | $\begin{bmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 44 | $\{m_{1-10}' 0\}$ | $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ | -1 | -1 |
| 45 | $\{m_{011}' 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 46 | $\{m_{01-1}' 0\}$ | $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 47 | $\{m_{101}' 0\}$ | $\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |
| 48 | $\{m_{-101}' 0\}$ | $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ | -1 | -1 |