PG No. 13 $C_{4\mathrm{v}}$ $4mm$ [tetragonal] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(1,1)}[g](A_2)$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_2)$

** symmetry

z

** expression

G_z

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_2)$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x xz}{10} + \frac{3\sqrt{10}G_y yz}{10} - \frac{\sqrt{10}G_z \left(x^2 + y^2 - 2z^2\right)}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](E),\ \vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](E)$

** symmetry

$-y$

x

** expression

$-G_y$

G_x

$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](E),\ \vec{\mathbb{G}}_{1,2}^{(1,1)}[g](E)$

** symmetry

$-y$

x

** expression

$$-\frac{3\sqrt{10}G_x xy}{10} + \frac{\sqrt{10}G_y \left(x^2 - 2y^2 + z^2\right)}{10} - \frac{3\sqrt{10}G_z yz}{10}$$

$$\frac{\sqrt{10}G_x \left(2x^2 - y^2 - z^2\right)}{10} + \frac{3\sqrt{10}G_y xy}{10} + \frac{3\sqrt{10}G_z xz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_2)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_x x\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_y y\left(x^2+y^2-4z^2\right)}{14}-\frac{\sqrt{21}G_z z\left(3x^2+3y^2-2z^2\right)}{14}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_1)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_x y}{2}+\frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_1)$
** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_x y\left(4x^2-y^2-z^2\right)}{7}-\frac{\sqrt{7}G_y x\left(x^2-4y^2+z^2\right)}{7}+\frac{5\sqrt{7}G_z xyz}{7}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{2}G_x x}{2}-\frac{\sqrt{2}G_y y}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_2)$
** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_x x\left(3x^2-7y^2-2z^2\right)}{14}+\frac{\sqrt{7}G_y y\left(7x^2-3y^2+2z^2\right)}{14}+\frac{5\sqrt{7}G_z z\left(x-y\right)\left(x+y\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E),\ \vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_y z}{2}+\frac{\sqrt{2}G_z y}{2}$$

$$-\frac{\sqrt{2}G_x z}{2}-\frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E),\ \vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E)$
** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7}-\frac{\sqrt{7}G_y z\left(x^2-4y^2+z^2\right)}{7}-\frac{\sqrt{7}G_z y\left(x^2+y^2-4z^2\right)}{7}$$

$$-\frac{\sqrt{7}G_x z\left(4x^2-y^2-z^2\right)}{7}-\frac{5\sqrt{7}G_y xyz}{7}+\frac{\sqrt{7}G_z x\left(x^2+y^2-4z^2\right)}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_2)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xz}{5} - \frac{\sqrt{15}G_y yz}{5} - \frac{\sqrt{15}G_z\left(x^2 + y^2 - 2z^2\right)}{10}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_2)$

** symmetry

$$-\frac{z\left(3x^2 + 3y^2 - 2z^2\right)}{2}$$

** expression

$$-\frac{5G_x xz\left(3x^2 + 3y^2 - 4z^2\right)}{12} - \frac{5G_y yz\left(3x^2 + 3y^2 - 4z^2\right)}{12} + \frac{G_z\left(3x^4 + 6x^2y^2 - 24x^2z^2 + 3y^4 - 24y^2z^2 + 8z^4\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_x yz + G_y xz + G_z xy$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}G_x yz\left(6x^2 - y^2 - z^2\right)}{6} - \frac{\sqrt{15}G_y xz\left(x^2 - 6y^2 + z^2\right)}{6} - \frac{\sqrt{15}G_z xy\left(x^2 + y^2 - 6z^2\right)}{6}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_2)$

** symmetry

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$G_x xz - G_y yz + \frac{G_z\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_2)$

** symmetry

$$\frac{\sqrt{15}z\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x xz\left(5x^2 - 9y^2 - 2z^2\right)}{12} + \frac{\sqrt{15}G_y yz\left(9x^2 - 5y^2 + 2z^2\right)}{12} - \frac{\sqrt{15}G_z\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E,1),\ \vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E,1)$

** symmetry

$$\frac{y\left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

$$\frac{x\left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x xy}{5} + \frac{\sqrt{15}G_y\left(x^2 - 2y^2 + z^2\right)}{10} + \frac{\sqrt{15}G_z yz}{5}$$

$$\frac{\sqrt{15}G_x\left(2x^2 - y^2 - z^2\right)}{10} - \frac{\sqrt{15}G_y xy}{5} - \frac{\sqrt{15}G_z xz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$-G_x xy - \frac{G_y\,(x-z)\,(x+z)}{2} + G_z yz$$

$$\frac{G_x\,(y-z)\,(y+z)}{2} + G_y xy - G_z xz$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E,1)$

** symmetry

$$\frac{y\,(3x^2 - 2y^2 + 3z^2)}{2}$$

$$\frac{x\,(2x^2 - 3y^2 - 3z^2)}{2}$$

** expression

$$\frac{5G_x xy\,(3x^2 - 4y^2 + 3z^2)}{12} - \frac{G_y\,(3x^4 - 24x^2y^2 + 6x^2z^2 + 8y^4 - 24y^2z^2 + 3z^4)}{12} + \frac{5G_z yz\,(3x^2 - 4y^2 + 3z^2)}{12}$$

$$\frac{G_x\,(8x^4 - 24x^2y^2 - 24x^2z^2 + 3y^4 + 6y^2z^2 + 3z^4)}{12} + \frac{5G_y xy\,(4x^2 - 3y^2 - 3z^2)}{12} + \frac{5G_z xz\,(4x^2 - 3y^2 - 3z^2)}{12}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E,2)$

** symmetry

$$-\frac{\sqrt{15}y\,(x-z)\,(x+z)}{2}$$

$$\frac{\sqrt{15}x\,(y-z)\,(y+z)}{2}$$

** expression

$$-\frac{\sqrt{15}G_x xy\,(5x^2 - 2y^2 - 9z^2)}{12} + \frac{\sqrt{15}G_y\,(x-z)\,(x+z)\,(x^2 - 6y^2 + z^2)}{12} - \frac{\sqrt{15}G_z yz\,(9x^2 + 2y^2 - 5z^2)}{12}$$

$$\frac{\sqrt{15}G_x\,(y-z)\,(y+z)\,(6x^2 - y^2 - z^2)}{12} - \frac{\sqrt{15}G_y xy\,(2x^2 - 5y^2 + 9z^2)}{12} + \frac{\sqrt{15}G_z xz\,(2x^2 + 9y^2 - 5z^2)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_1)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y\,(3x^2 - y^2)}{4} + \frac{\sqrt{5}G_y x\,(x^2 - 3y^2)}{4}$$

$\vec{\mathbb{G}}_{4}^{(1,1)}[g](A_1)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{77}G_x y\,(6x^4 - 11x^2y^2 - 3x^2z^2 + y^4 + y^2z^2)}{22} - \frac{\sqrt{77}G_y x\,(x^4 - 11x^2y^2 + x^2z^2 + 6y^4 - 3y^2z^2)}{22} + \frac{9\sqrt{77}G_z xyz\,(x-y)\,(x+y)}{22}$$

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_2,1)$

** symmetry

$$\frac{\sqrt{21}\,(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4)}{6}$$

$$\frac{\sqrt{3}G_x x\left(2x^2 - 3y^2 - 3z^2\right)}{6} - \frac{\sqrt{3}G_y y\left(3x^2 - 2y^2 + 3z^2\right)}{6} - \frac{\sqrt{3}G_z z\left(3x^2 + 3y^2 - 2z^2\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_2, 2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}G_x x\left(x^2 - 6y^2 + 3z^2\right)}{42} + \frac{\sqrt{105}G_y y\left(6x^2 - y^2 - 3z^2\right)}{42} - \frac{\sqrt{105}G_z z\left(3x^2 + 3y^2 - 2z^2\right)}{42}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2, 1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2 y^2 - 3x^2 z^2 + y^4 - 3y^2 z^2 + z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}G_x x\left(x^4 - 5x^2 y^2 - 5x^2 z^2 + 3y^4 - 3y^2 z^2 + 3z^4\right)}{66} + \frac{\sqrt{1155}G_y y\left(3x^4 - 5x^2 y^2 - 3x^2 z^2 + y^4 - 5y^2 z^2 + 3z^4\right)}{66}$$
$$+ \frac{\sqrt{1155}G_z z\left(3x^4 - 3x^2 y^2 - 5x^2 z^2 + 3y^4 - 5y^2 z^2 + z^4\right)}{66}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_2, 2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2 y^2 + 6x^2 z^2 + y^4 + 6y^2 z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}G_x x\left(5x^4 - 88x^2 y^2 + 38x^2 z^2 + 33y^4 + 66y^2 z^2 - 30z^4\right)}{132} - \frac{\sqrt{33}G_y y\left(33x^4 - 88x^2 y^2 + 66x^2 z^2 + 5y^4 + 38y^2 z^2 - 30z^4\right)}{132}$$
$$+ \frac{\sqrt{33}G_z z\left(3x^4 + 132x^2 y^2 - 50x^2 z^2 + 3y^4 - 50y^2 z^2 + 10z^4\right)}{132}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_1)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x y\left(3x^2 + y^2 - 6z^2\right)}{28} - \frac{\sqrt{35}G_y x\left(x^2 + 3y^2 - 6z^2\right)}{28} + \frac{3\sqrt{35}G_z xyz}{7}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_1)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2 + y^2 - 6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x y\left(6x^4 + 5x^2 y^2 - 51x^2 z^2 - y^4 + 5y^2 z^2 + 6z^4\right)}{22} + \frac{\sqrt{11}G_y x\left(x^4 - 5x^2 y^2 - 5x^2 z^2 - 6y^4 + 51y^2 z^2 - 6z^4\right)}{22} - \frac{21\sqrt{11}G_z xyz\left(x^2 + y^2 - 2z^2\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_2)$
** symmetry

$$\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}G_x x\left(x^2 - 3z^2\right)}{14} - \frac{\sqrt{35}G_y y\left(y^2 - 3z^2\right)}{14} - \frac{3\sqrt{35}G_z z\left(x - y\right)\left(x + y\right)}{14}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_2)$
** symmetry

$$\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{11}G_x x\left(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4\right)}{44}$$

$$+ \frac{\sqrt{11}G_y y\left(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4\right)}{44} + \frac{21\sqrt{11}G_z z\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E,1),\ \vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y - z\right)\left(y + z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_y z\left(3y^2 - z^2\right)}{4} + \frac{\sqrt{5}G_z y\left(y^2 - 3z^2\right)}{4}$$

$$-\frac{\sqrt{5}G_x z\left(3x^2 - z^2\right)}{4} - \frac{\sqrt{5}G_z x\left(x^2 - 3z^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E,2),\ \vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2 - y^2 - z^2\right)}{2}$$

$$\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$\frac{3\sqrt{35}G_x xyz}{7} + \frac{\sqrt{35}G_y z\left(6x^2 - 3y^2 - z^2\right)}{28} + \frac{\sqrt{35}G_z y\left(6x^2 - y^2 - 3z^2\right)}{28}$$

$$\frac{\sqrt{35}G_x z\left(3x^2 - 6y^2 + z^2\right)}{28} - \frac{3\sqrt{35}G_y xyz}{7} + \frac{\sqrt{35}G_z x\left(x^2 - 6y^2 + 3z^2\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E,1),\ \vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E,1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y - z\right)\left(y + z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$\frac{9\sqrt{77}G_x xyz\left(y - z\right)\left(y + z\right)}{22} - \frac{\sqrt{77}G_y z\left(3x^2y^2 - x^2z^2 - 6y^4 + 11y^2z^2 - z^4\right)}{22} - \frac{\sqrt{77}G_z y\left(x^2y^2 - 3x^2z^2 + y^4 - 11y^2z^2 + 6z^4\right)}{22}$$

$$-\frac{\sqrt{77}G_x z\left(6x^4 - 3x^2y^2 - 11x^2z^2 + y^2z^2 + z^4\right)}{22} - \frac{9\sqrt{77}G_y xyz\left(x - z\right)\left(x + z\right)}{22} + \frac{\sqrt{77}G_z x\left(x^4 + x^2y^2 - 11x^2z^2 - 3y^2z^2 + 6z^4\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E,2),\ \vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E,2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2 - y^2 - z^2\right)}{2}$$

$$\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}G_x xyz\left(2x^2 - y^2 - z^2\right)}{22} - \frac{\sqrt{11}G_y z\left(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4\right)}{22} - \frac{\sqrt{11}G_z y\left(6x^4 + 5x^2y^2 - 51x^2z^2 - y^4 + 5y^2z^2 + 6z^4\right)}{22}$$

$$\frac{\sqrt{11}G_x z\left(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4\right)}{22} + \frac{21\sqrt{11}G_y xyz\left(x^2 - 2y^2 + z^2\right)}{22} - \frac{\sqrt{11}G_z x\left(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4\right)}{22}$$