PG No. 15 $D_{4\mathrm{h}}$ $4/mmm$ [tetragonal] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,0)}[q](A_{2g})$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_{x}y}{2} - \frac{\sqrt{2}Q_{y}x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E_{g})$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E_{g})$

** symmetry

$$x$$

$$-y$$

** expression

$$\frac{\sqrt{2}Q_{y}z}{2} - \frac{\sqrt{2}Q_{z}y}{2}$$

$$\frac{\sqrt{2}Q_{x}z}{2} - \frac{\sqrt{2}Q_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](A_{1u})$

** symmetry

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{6}Q_{x}yz}{2} - \frac{\sqrt{6}Q_{y}xz}{2}$$

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](B_{1u})$

** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{2}Q_{x}yz}{2} + \frac{\sqrt{2}Q_{y}xz}{2} - \sqrt{2}Q_{z}xy$$

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](B_{2u})$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{2}Q_{x}xz}{2} + \frac{\sqrt{2}Q_{y}yz}{2} + \frac{\sqrt{2}Q_{z}\left(x - y\right)\left(x + y\right)}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E_{u})$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E_{u})$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{x}\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}Q_{y}xy}{2} + \frac{\sqrt{2}Q_{z}xz}{2}$$

$$-\frac{\sqrt{2}Q_{x}xy}{2} + \frac{\sqrt{2}Q_{y}\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{2}Q_{z}yz}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_{2g})$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_xy\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_yx\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_{1g})$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_xx\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_yy\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_zz\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_{2g})$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_xy\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_yx\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_zxyz$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_g,1)$
** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

$$\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_yz\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{3}Q_zy\left(4x^2-y^2-z^2\right)}{4}$$

$$-\frac{\sqrt{3}Q_xz\left(x^2-4y^2+z^2\right)}{4}+\frac{\sqrt{3}Q_zx\left(x^2-4y^2+z^2\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_g,2)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\sqrt{5}Q_xxyz+\frac{\sqrt{5}Q_yz\left(2x^2+y^2-z^2\right)}{4}+\frac{\sqrt{5}Q_zy\left(2x^2-y^2+z^2\right)}{4}$$

$$\frac{\sqrt{5}Q_xz\left(x^2+2y^2-z^2\right)}{4}-\sqrt{5}Q_yxyz-\frac{\sqrt{5}Q_zx\left(x^2-2y^2-z^2\right)}{4}$$

* Harmonics for rank 4
$\vec{\mathbb{G}}_4^{(1,0)}[q](A_{1u},1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_xyz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}Q_yxz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}Q_zxy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_{1u},2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^{4}-12x^{2}y^{2}+6x^{2}z^{2}+y^{4}+6y^{2}z^{2}-2z^{4}\right)}{12}$$

** expression

$$-\frac{\sqrt{3}Q_{x}yz\left(9x^{2}+2y^{2}-5z^{2}\right)}{6}+\frac{\sqrt{3}Q_{y}xz\left(2x^{2}+9y^{2}-5z^{2}\right)}{6}+\frac{7\sqrt{3}Q_{z}xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A_{2u})$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_{x}xz\left(x^{2}-3y^{2}\right)}{4}+\frac{\sqrt{7}Q_{y}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}Q_{z}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](B_{1u})$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

** expression

$$-\frac{Q_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}+\frac{Q_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}-\frac{Q_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](B_{2u})$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{4}-\frac{Q_{y}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{4}-\frac{Q_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_{u},1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_{u},1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_{x}\left(y^{2}-2yz-z^{2}\right)\left(y^{2}+2yz-z^{2}\right)}{4}-\frac{\sqrt{7}Q_{y}xy\left(y^{2}-3z^{2}\right)}{4}+\frac{\sqrt{7}Q_{z}xz\left(3y^{2}-z^{2}\right)}{4}$$

$$-\frac{\sqrt{7}Q_{x}xy\left(x^{2}-3z^{2}\right)}{4}+\frac{\sqrt{7}Q_{y}\left(x^{2}-2xz-z^{2}\right)\left(x^{2}+2xz-z^{2}\right)}{4}+\frac{\sqrt{7}Q_{z}yz\left(3x^{2}-z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_{u},2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_{u},2)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x}\left(y-z\right)\left(y+z\right)\left(6x^{2}-y^{2}-z^{2}\right)}{4}-\frac{Q_{y}xy\left(6x^{2}-y^{2}-15z^{2}\right)}{4}+\frac{Q_{z}xz\left(6x^{2}-15y^{2}-z^{2}\right)}{4}$$

$$\frac{Q_{x}xy\left(x^{2}-6y^{2}+15z^{2}\right)}{4}-\frac{Q_{y}\left(x-z\right)\left(x+z\right)\left(x^{2}-6y^{2}+z^{2}\right)}{4}-\frac{Q_{z}yz\left(15x^{2}-6y^{2}+z^{2}\right)}{4}$$