

Table 1 Harmonics for rank 0.

No.	multipole	definition
1	$Q_0(A_{1g})$	$C_0$

Table 2 Harmonics for rank 1.

No.	multipole	definition
2	$Q_1(A_{2u})$	$C_0$
3	$Q_{1,1}(E_{1u})$	$C_1$
4	$Q_{1,2}(E_{1u})$	$S_1$

Table 3 Harmonics for rank 2.

No.	multipole	definition
5	$Q_2(A_{1g})$	$C_0$
6	$Q_{2,1}(E_{1g})$	$S_1$
7	$Q_{2,2}(E_{1g})$	$-C_1$
8	$Q_{2,1}(E_{2g})$	$C_2$
9	$Q_{2,2}(E_{2g})$	$-S_2$

Table 4 Harmonics for rank 3.

No.	multipole	definition
10	$Q_3(A_{2u})$	$C_0$
11	$Q_3(B_{1u})$	$S_3$
12	$Q_3(B_{2u})$	$C_3$
13	$Q_{3,1}(E_{1u})$	$C_1$
14	$Q_{3,2}(E_{1u})$	$S_1$
15	$Q_{3,1}(E_{2u})$	$S_2$
16	$Q_{3,2}(E_{2u})$	$C_2$

Table 5 Harmonics for rank 4.

No.	multipole	definition
17	$Q_4(A_{1g})$	$C_0$
18	$Q_4(B_{1g})$	$C_3$
19	$Q_4(B_{2g})$	$S_3$
20	$Q_{4,1}(E_{1g})$	$S_1$
21	$Q_{4,2}(E_{1g})$	$-C_1$
22	$Q_{4,1}(E_{2g}, 1)$	$C_4$
23	$Q_{4,2}(E_{2g}, 1)$	$S_4$
24	$Q_{4,1}(E_{2g}, 2)$	$C_2$
25	$Q_{4,2}(E_{2g}, 2)$	$-S_2$

Table 6 Harmonics for rank 5.

No.	multipole	definition
26	$\mathbb{Q}_5(A_{2u})$	$C_0$
27	$\mathbb{Q}_5(B_{1u})$	$S_3$
28	$\mathbb{Q}_5(B_{2u})$	$C_3$
29	$\mathbb{Q}_{5,1}(E_{1u}, 1)$	$C_5$
30	$\mathbb{Q}_{5,2}(E_{1u}, 1)$	$-S_5$
31	$\mathbb{Q}_{5,1}(E_{1u}, 2)$	$C_1$
32	$\mathbb{Q}_{5,2}(E_{1u}, 2)$	$S_1$
33	$\mathbb{Q}_{5,1}(E_{2u}, 1)$	$-S_4$
34	$\mathbb{Q}_{5,2}(E_{2u}, 1)$	$C_4$
35	$\mathbb{Q}_{5,1}(E_{2u}, 2)$	$S_2$
36	$\mathbb{Q}_{5,2}(E_{2u}, 2)$	$C_2$

Table 7 Harmonics for rank 6.

No.	multipole	definition
37	$\mathbb{Q}_6(A_{1g}, 1)$	$C_0$
38	$\mathbb{Q}_6(A_{1g}, 2)$	$C_6$
39	$\mathbb{Q}_6(A_{2g})$	$S_6$
40	$\mathbb{Q}_6(B_{1g})$	$C_3$
41	$\mathbb{Q}_6(B_{2g})$	$S_3$
42	$\mathbb{Q}_{6,1}(E_{1g}, 1)$	$S_5$
43	$\mathbb{Q}_{6,2}(E_{1g}, 1)$	$C_5$
44	$\mathbb{Q}_{6,1}(E_{1g}, 2)$	$S_1$
45	$\mathbb{Q}_{6,2}(E_{1g}, 2)$	$-C_1$
46	$\mathbb{Q}_{6,1}(E_{2g}, 1)$	$C_4$
47	$\mathbb{Q}_{6,2}(E_{2g}, 1)$	$S_4$
48	$\mathbb{Q}_{6,1}(E_{2g}, 2)$	$C_2$
49	$\mathbb{Q}_{6,2}(E_{2g}, 2)$	$-S_2$

Table 8 Harmonics for rank 7.

No.	multipole	definition
50	$\mathbb{Q}_7(A_{1u})$	$S_6$
51	$\mathbb{Q}_7(A_{2u}, 1)$	$C_0$
52	$\mathbb{Q}_7(A_{2u}, 2)$	$C_6$
53	$\mathbb{Q}_7(B_{1u})$	$S_3$
54	$\mathbb{Q}_7(B_{2u})$	$C_3$
55	$\mathbb{Q}_{7,1}(E_{1u}, 1)$	$C_7$
56	$\mathbb{Q}_{7,2}(E_{1u}, 1)$	$S_7$
57	$\mathbb{Q}_{7,1}(E_{1u}, 2)$	$C_5$
58	$\mathbb{Q}_{7,2}(E_{1u}, 2)$	$-S_5$
59	$\mathbb{Q}_{7,1}(E_{1u}, 3)$	$C_1$
60	$\mathbb{Q}_{7,2}(E_{1u}, 3)$	$S_1$
61	$\mathbb{Q}_{7,1}(E_{2u}, 1)$	$-S_4$
62	$\mathbb{Q}_{7,2}(E_{2u}, 1)$	$C_4$
63	$\mathbb{Q}_{7,1}(E_{2u}, 2)$	$S_2$
64	$\mathbb{Q}_{7,2}(E_{2u}, 2)$	$C_2$

Table 9 Harmonics for rank 8.

No.	multipole	definition
65	$\mathbb{Q}_8(A_{1g}, 1)$	$C_0$
66	$\mathbb{Q}_8(A_{1g}, 2)$	$C_6$
67	$\mathbb{Q}_8(A_{2g})$	$S_6$
68	$\mathbb{Q}_8(B_{1g})$	$C_3$
69	$\mathbb{Q}_8(B_{2g})$	$S_3$
70	$\mathbb{Q}_{8,1}(E_{1g}, 1)$	$S_7$
71	$\mathbb{Q}_{8,2}(E_{1g}, 1)$	$-C_7$
72	$\mathbb{Q}_{8,1}(E_{1g}, 2)$	$S_5$
73	$\mathbb{Q}_{8,2}(E_{1g}, 2)$	$C_5$
74	$\mathbb{Q}_{8,1}(E_{1g}, 3)$	$S_1$
75	$\mathbb{Q}_{8,2}(E_{1g}, 3)$	$-C_1$
76	$\mathbb{Q}_{8,1}(E_{2g}, 1)$	$C_8$
77	$\mathbb{Q}_{8,2}(E_{2g}, 1)$	$-S_8$
78	$\mathbb{Q}_{8,1}(E_{2g}, 2)$	$C_4$
79	$\mathbb{Q}_{8,2}(E_{2g}, 2)$	$S_4$
80	$\mathbb{Q}_{8,1}(E_{2g}, 3)$	$C_2$
81	$\mathbb{Q}_{8,2}(E_{2g}, 3)$	$-S_2$

Table 10 Harmonics for rank 9.

No.	multipole	definition
82	$\mathbb{Q}_9(A_{1u})$	$S_6$
83	$\mathbb{Q}_9(A_{2u}, 1)$	$C_0$
84	$\mathbb{Q}_9(A_{2u}, 2)$	$C_6$
85	$\mathbb{Q}_9(B_{1u}, 1)$	$S_9$
86	$\mathbb{Q}_9(B_{1u}, 2)$	$S_3$
87	$\mathbb{Q}_9(B_{2u}, 1)$	$C_9$
88	$\mathbb{Q}_9(B_{2u}, 2)$	$C_3$
89	$\mathbb{Q}_{9,1}(E_{1u}, 1)$	$C_7$
90	$\mathbb{Q}_{9,2}(E_{1u}, 1)$	$S_7$
91	$\mathbb{Q}_{9,1}(E_{1u}, 2)$	$C_5$
92	$\mathbb{Q}_{9,2}(E_{1u}, 2)$	$-S_5$
93	$\mathbb{Q}_{9,1}(E_{1u}, 3)$	$C_1$
94	$\mathbb{Q}_{9,2}(E_{1u}, 3)$	$S_1$
95	$\mathbb{Q}_{9,1}(E_{2u}, 1)$	$S_8$
96	$\mathbb{Q}_{9,2}(E_{2u}, 1)$	$C_8$
97	$\mathbb{Q}_{9,1}(E_{2u}, 2)$	$-S_4$
98	$\mathbb{Q}_{9,2}(E_{2u}, 2)$	$C_4$
99	$\mathbb{Q}_{9,1}(E_{2u}, 3)$	$S_2$
100	$\mathbb{Q}_{9,2}(E_{2u}, 3)$	$C_2$

Table 11 Harmonics for rank 10.

No.	multipole	definition
101	$\mathbb{Q}_{10}(A_{1g}, 1)$	$C_0$
102	$\mathbb{Q}_{10}(A_{1g}, 2)$	$C_6$
103	$\mathbb{Q}_{10}(A_{2g})$	$S_6$
104	$\mathbb{Q}_{10}(B_{1g}, 1)$	$C_9$
105	$\mathbb{Q}_{10}(B_{1g}, 2)$	$C_3$
106	$\mathbb{Q}_{10}(B_{2g}, 1)$	$S_9$
107	$\mathbb{Q}_{10}(B_{2g}, 2)$	$S_3$
108	$\mathbb{Q}_{10,1}(E_{1g}, 1)$	$S_7$
109	$\mathbb{Q}_{10,2}(E_{1g}, 1)$	$-C_7$
110	$\mathbb{Q}_{10,1}(E_{1g}, 2)$	$S_5$
111	$\mathbb{Q}_{10,2}(E_{1g}, 2)$	$C_5$
112	$\mathbb{Q}_{10,1}(E_{1g}, 3)$	$S_1$
113	$\mathbb{Q}_{10,2}(E_{1g}, 3)$	$-C_1$
114	$\mathbb{Q}_{10,1}(E_{2g}, 1)$	$C_{10}$
115	$\mathbb{Q}_{10,2}(E_{2g}, 1)$	$S_{10}$
116	$\mathbb{Q}_{10,1}(E_{2g}, 2)$	$C_8$
117	$\mathbb{Q}_{10,2}(E_{2g}, 2)$	$-S_8$
118	$\mathbb{Q}_{10,1}(E_{2g}, 3)$	$C_4$
119	$\mathbb{Q}_{10,2}(E_{2g}, 3)$	$S_4$
120	$\mathbb{Q}_{10,1}(E_{2g}, 4)$	$C_2$
121	$\mathbb{Q}_{10,2}(E_{2g}, 4)$	$-S_2$

Table 12 Harmonics for rank 11.

No.	multipole	definition
122	$\mathbb{Q}_{11}(A_{1u})$	$S_6$
123	$\mathbb{Q}_{11}(A_{2u}, 1)$	$C_0$
124	$\mathbb{Q}_{11}(A_{2u}, 2)$	$C_6$
125	$\mathbb{Q}_{11}(B_{1u}, 1)$	$S_9$
126	$\mathbb{Q}_{11}(B_{1u}, 2)$	$S_3$
127	$\mathbb{Q}_{11}(B_{2u}, 1)$	$C_9$
128	$\mathbb{Q}_{11}(B_{2u}, 2)$	$C_3$
129	$\mathbb{Q}_{11,1}(E_{1u}, 1)$	$C_{11}$
130	$\mathbb{Q}_{11,2}(E_{1u}, 1)$	$-S_{11}$
131	$\mathbb{Q}_{11,1}(E_{1u}, 2)$	$C_7$
132	$\mathbb{Q}_{11,2}(E_{1u}, 2)$	$S_7$
133	$\mathbb{Q}_{11,1}(E_{1u}, 3)$	$C_5$
134	$\mathbb{Q}_{11,2}(E_{1u}, 3)$	$-S_5$
135	$\mathbb{Q}_{11,1}(E_{1u}, 4)$	$C_1$
136	$\mathbb{Q}_{11,2}(E_{1u}, 4)$	$S_1$
137	$\mathbb{Q}_{11,1}(E_{2u}, 1)$	$-S_{10}$
138	$\mathbb{Q}_{11,2}(E_{2u}, 1)$	$C_{10}$
139	$\mathbb{Q}_{11,1}(E_{2u}, 2)$	$S_8$
140	$\mathbb{Q}_{11,2}(E_{2u}, 2)$	$C_8$
141	$\mathbb{Q}_{11,1}(E_{2u}, 3)$	$-S_4$
142	$\mathbb{Q}_{11,2}(E_{2u}, 3)$	$C_4$
143	$\mathbb{Q}_{11,1}(E_{2u}, 4)$	$S_2$
144	$\mathbb{Q}_{11,2}(E_{2u}, 4)$	$C_2$