PG No. 22 C_{3h} $\bar{6}$ [hexagonal] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,0)}[q](A')$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_{x}y}{2}-\frac{\sqrt{2}Q_{y}x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E'')$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E'')$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}Q_{y}z}{2}-\frac{\sqrt{2}Q_{z}y}{2}$$

$$-\frac{\sqrt{2}Q_{x}z}{2}+\frac{\sqrt{2}Q_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](A'')$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

** expression

$$\frac{\sqrt{6}Q_{x}yz}{2}-\frac{\sqrt{6}Q_{y}xz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E')$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E')$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{x}\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{2}Q_{y}xy}{2}+\frac{\sqrt{2}Q_{z}xz}{2}$$

$$-\frac{\sqrt{2}Q_{x}xy}{2}+\frac{\sqrt{2}Q_{y}\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{2}Q_{z}yz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E'')$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E'')$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{x}yz}{2}+\frac{\sqrt{2}Q_{y}xz}{2}-\sqrt{2}Q_{z}xy$$

$$\frac{\sqrt{2}Q_{x}xz}{2}-\frac{\sqrt{2}Q_{y}yz}{2}-\frac{\sqrt{2}Q_{z}\left(x-y\right)\left(x+y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A')$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A'',1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_x z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}Q_y xyz}{4}+\frac{\sqrt{30}Q_z x\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](A'',2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{30}Q_x xyz}{4}+\frac{\sqrt{30}Q_y z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}Q_z y\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E')$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E')$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_z z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{5}Q_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_z xyz$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E'')$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E'')$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{5\sqrt{2}Q_x xyz}{4}-\frac{\sqrt{2}Q_y z\left(11x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{2}Q_z y\left(x^2+y^2-4z^2\right)}{8}$$

$$\frac{\sqrt{2}Q_x z\left(x^2+11y^2-4z^2\right)}{8}-\frac{5\sqrt{2}Q_y xyz}{4}-\frac{\sqrt{2}Q_z x\left(x^2+y^2-4z^2\right)}{8}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A',1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x xy\left(x^2-3y^2+6z^2\right)}{8}-\frac{\sqrt{14}Q_y\left(x^4-3x^2y^2-3x^2z^2+3y^2z^2\right)}{8}-\frac{3\sqrt{14}Q_z yz\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A',2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_{x}\left(3x^{2}y^{2} - 3x^{2}z^{2} - y^{4} + 3y^{2}z^{2}\right)}{8} - \frac{\sqrt{14}Q_{y}xy\left(3x^{2} - y^{2} - 6z^{2}\right)}{8} + \frac{3\sqrt{14}Q_{z}xz\left(x^{2} - 3y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{4}^{(1,0)}[q](A'')$

** symmetry

$$\frac{3x^{4}}{8} + \frac{3x^{2}y^{2}}{4} - 3x^{2}z^{2} + \frac{3y^{4}}{8} - 3y^{2}z^{2} + z^{4}$$

** expression

$$-\frac{\sqrt{5}Q_{x}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{4} + \frac{\sqrt{5}Q_{y}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E')$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E')$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_{x}\left(3x^{2}y^{2} - 3x^{2}z^{2} + 3y^{4} - 21y^{2}z^{2} + 4z^{4}\right)}{8} + \frac{3\sqrt{2}Q_{y}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{8} - \frac{\sqrt{2}Q_{z}xz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{8}$$

$$\frac{3\sqrt{2}Q_{x}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{8} - \frac{\sqrt{2}Q_{y}\left(3x^{4} + 3x^{2}y^{2} - 21x^{2}z^{2} - 3y^{2}z^{2} + 4z^{4}\right)}{8} - \frac{\sqrt{2}Q_{z}yz\left(3x^{2} + 3y^{2} - 4z^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E'',1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E'',1)$

** symmetry

$$\frac{\sqrt{35}\left(x^{2} - 2xy - y^{2}\right)\left(x^{2} + 2xy - y^{2}\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x - y\right)\left(x + y\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_{x}yz\left(3x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7}Q_{y}xz\left(x^{2} - 3y^{2}\right)}{4} - \sqrt{7}Q_{z}xy\left(x - y\right)\left(x + y\right)$$

$$-\frac{\sqrt{7}Q_{x}xz\left(x^{2} - 3y^{2}\right)}{4} + \frac{\sqrt{7}Q_{y}yz\left(3x^{2} - y^{2}\right)}{4} + \frac{\sqrt{7}Q_{z}\left(x^{2} - 2xy - y^{2}\right)\left(x^{2} + 2xy - y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E'',2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E'',2)$

** symmetry

$$-\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^{2} + y^{2} - 6z^{2}\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{2}$$

** expression

$$\frac{Q_{x}yz\left(3x^{2} - 4y^{2} + 3z^{2}\right)}{2} - \frac{Q_{y}xz\left(4x^{2} - 3y^{2} - 3z^{2}\right)}{2} + \frac{Q_{z}xy\left(x^{2} + y^{2} - 6z^{2}\right)}{2}$$

$$-\frac{Q_{x}xz\left(x^{2} + 15y^{2} - 6z^{2}\right)}{4} + \frac{Q_{y}yz\left(15x^{2} + y^{2} - 6z^{2}\right)}{4} + \frac{Q_{z}\left(x - y\right)\left(x + y\right)\left(x^{2} + y^{2} - 6z^{2}\right)}{4}$$