PG No. 23 $C_{6\mathrm{h}}$ $6/m$ [hexagonal] (axial, internal polar dipole)

* Harmonics for rank 0

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,0)}[q](A_g)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}Q_{x}y}{2} - \frac{\sqrt{2}Q_{y}x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E_{1g})$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E_{1g})$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}Q_{y}z}{2} - \frac{\sqrt{2}Q_{z}y}{2}$$

$$- \frac{\sqrt{2}Q_{x}z}{2} + \frac{\sqrt{2}Q_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,0)}[q](A_u)$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{6}Q_{x}yz}{2} - \frac{\sqrt{6}Q_{y}xz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E_{1u})$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E_{1u})$

** symmetry

$$\sqrt{3}yz$$

$$- \sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{x}\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}Q_{y}xy}{2} + \frac{\sqrt{2}Q_{z}xz}{2}$$

$$- \frac{\sqrt{2}Q_{x}xy}{2} + \frac{\sqrt{2}Q_{y}\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{2}Q_{z}yz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E_{2u})$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E_{2u})$

** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$- \sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}Q_{x}yz}{2} + \frac{\sqrt{2}Q_{y}xz}{2} - \sqrt{2}Q_{z}xy$$

$$\frac{\sqrt{2}Q_{x}xz}{2} - \frac{\sqrt{2}Q_{y}yz}{2} - \frac{\sqrt{2}Q_{z}\left(x - y\right)\left(x + y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,0)}[q](A_g)$

** symmetry

$$- \frac{z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,1)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$-\frac{\sqrt{30}Q_x z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}Q_y xyz}{4}+\frac{\sqrt{30}Q_z x\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B_g,2)$

** symmetry

$$\frac{\sqrt{10}x\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{30}Q_x xyz}{4}+\frac{\sqrt{30}Q_y z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}Q_z y\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_{1g})$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_{1g})$

** symmetry

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{5\sqrt{2}Q_x xyz}{4}-\frac{\sqrt{2}Q_y z\left(11x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{2}Q_z y\left(x^2+y^2-4z^2\right)}{8}$$

$$\frac{\sqrt{2}Q_x z\left(x^2+11y^2-4z^2\right)}{8}-\frac{5\sqrt{2}Q_y xyz}{4}-\frac{\sqrt{2}Q_z x\left(x^2+y^2-4z^2\right)}{8}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E_{2g})$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E_{2g})$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_z z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{5}Q_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_z xyz$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,0)}[q](A_u)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{\sqrt{5}Q_x yz\left(3x^2+3y^2-4z^2\right)}{4}+\frac{\sqrt{5}Q_y xz\left(3x^2+3y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u,1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x xy\left(x^2-3y^2+6z^2\right)}{8}-\frac{\sqrt{14}Q_y\left(x^4-3x^2y^2-3x^2z^2+3y^2z^2\right)}{8}-\frac{3\sqrt{14}Q_z yz\left(3x^2-y^2\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B_u,2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{\sqrt{14}Q_x\left(3x^2y^2-3x^2z^2-y^4+3y^2z^2\right)}{8}-\frac{\sqrt{14}Q_y xy\left(3x^2-y^2-6z^2\right)}{8}+\frac{3\sqrt{14}Q_z xz\left(x^2-3y^2\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_{1u})$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_{1u})$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{2}Q_x\left(3x^2y^2-3x^2z^2+3y^4-21y^2z^2+4z^4\right)}{8}+\frac{3\sqrt{2}Q_y xy\left(x^2+y^2-6z^2\right)}{8}-\frac{\sqrt{2}Q_z xz\left(3x^2+3y^2-4z^2\right)}{8}$$

$$\frac{3\sqrt{2}Q_x xy\left(x^2+y^2-6z^2\right)}{8}-\frac{\sqrt{2}Q_y\left(3x^4+3x^2y^2-21x^2z^2-3y^2z^2+4z^4\right)}{8}-\frac{\sqrt{2}Q_z yz\left(3x^2+3y^2-4z^2\right)}{8}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_{2u},1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_{2u},1)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x yz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}Q_y xz\left(x^2-3y^2\right)}{4}-\sqrt{7}Q_z xy\left(x-y\right)\left(x+y\right)$$

$$-\frac{\sqrt{7}Q_x xz\left(x^2-3y^2\right)}{4}+\frac{\sqrt{7}Q_y yz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}Q_z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E_{2u},2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E_{2u},2)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{Q_x yz\left(3x^2-4y^2+3z^2\right)}{2}-\frac{Q_y xz\left(4x^2-3y^2-3z^2\right)}{2}+\frac{Q_z xy\left(x^2+y^2-6z^2\right)}{2}$$

$$-\frac{Q_x xz\left(x^2+15y^2-6z^2\right)}{4}+\frac{Q_y yz\left(15x^2+y^2-6z^2\right)}{4}+\frac{Q_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$