PG No. 31 $\quad T_{\rm d} \quad \bar{4}3m \quad$ [cubic] (polar, internal polar quadrupole)

* Harmonics for rank 0

$\vec{\mathbb{Q}}_{0}^{(2,2)}[q](A_1)$

** symmetry

$$1$$

** expression

$$-\frac{\sqrt{5}Q_{u}\left(x^{2}+y^{2}-2z^{2}\right)}{10}+\frac{\sqrt{15}Q_{v}\left(x-y\right)\left(x+y\right)}{10}+\frac{\sqrt{15}Q_{xy}xy}{5}+\frac{\sqrt{15}Q_{xz}xz}{5}+\frac{\sqrt{15}Q_{yz}yz}{5}$$

* Harmonics for rank 1

$\vec{\mathbb{Q}}_{1,1}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{1,2}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{1,3}^{(2,0)}[q](T_2)$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$-\frac{\sqrt{10}Q_{u}x}{10}+\frac{\sqrt{30}Q_{v}x}{10}+\frac{\sqrt{30}Q_{xy}y}{10}+\frac{\sqrt{30}Q_{xz}z}{10}$$

$$-\frac{\sqrt{10}Q_{u}y}{10}-\frac{\sqrt{30}Q_{v}y}{10}+\frac{\sqrt{30}Q_{xy}x}{10}+\frac{\sqrt{30}Q_{yz}z}{10}$$

$$\frac{\sqrt{10}Q_{u}z}{5}+\frac{\sqrt{30}Q_{xz}x}{10}+\frac{\sqrt{30}Q_{yz}y}{10}$$

$\vec{\mathbb{Q}}_{1,1}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{1,2}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{1,3}^{(2,2)}[q](T_2)$

** symmetry

$$x$$

$$y$$

$$z$$

** expression

$$-\frac{3\sqrt{35}Q_{u}x\left(x^{2}+y^{2}-4z^{2}\right)}{70}+\frac{\sqrt{105}Q_{v}x\left(3x^{2}-7y^{2}-2z^{2}\right)}{70}+\frac{\sqrt{105}Q_{xy}y\left(4x^{2}-y^{2}-z^{2}\right)}{35}+\frac{\sqrt{105}Q_{xz}z\left(4x^{2}-y^{2}-z^{2}\right)}{35}+\frac{\sqrt{105}Q_{yz}xyz}{7}$$

$$-\frac{3\sqrt{35}Q_{u}y\left(x^{2}+y^{2}-4z^{2}\right)}{70}+\frac{\sqrt{105}Q_{v}y\left(7x^{2}-3y^{2}+2z^{2}\right)}{70}-\frac{\sqrt{105}Q_{xy}x\left(x^{2}-4y^{2}+z^{2}\right)}{35}+\frac{\sqrt{105}Q_{xz}xyz}{7}-\frac{\sqrt{105}Q_{yz}z\left(x^{2}-4y^{2}+z^{2}\right)}{35}$$

$$-\frac{3\sqrt{35}Q_{u}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{70}+\frac{\sqrt{105}Q_{v}z\left(x-y\right)\left(x+y\right)}{14}+\frac{\sqrt{105}Q_{xy}xyz}{7}-\frac{\sqrt{105}Q_{xz}x\left(x^{2}+y^{2}-4z^{2}\right)}{35}-\frac{\sqrt{105}Q_{yz}y\left(x^{2}+y^{2}-4z^{2}\right)}{35}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_{2,1}^{(2,-2)}[q](E)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-2)}[q](E)$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$Q_{u}$$

$$Q_{v}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,0)}[q](E)$, $\vec{\mathbb{Q}}_{2,2}^{(2,0)}[q](E)$

** symmetry

$$-\frac{x^{2}}{2}-\frac{y^{2}}{2}+z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$- \frac{\sqrt{14} Q_{u} \left(x^{2} + y^{2} - 2 z^{2}\right)}{14} - \frac{\sqrt{42} Q_{v} \left(x - y\right) \left(x + y\right)}{14} - \frac{\sqrt{42} Q_{xy} x y}{7} + \frac{\sqrt{42} Q_{xz} x z}{14} + \frac{\sqrt{42} Q_{yz} y z}{14}$$

$$- \frac{\sqrt{42} Q_{u} \left(x - y\right) \left(x + y\right)}{14} + \frac{\sqrt{14} Q_{v} \left(x^{2} + y^{2} - 2 z^{2}\right)}{14} + \frac{3 \sqrt{14} Q_{xz} x z}{14} - \frac{3 \sqrt{14} Q_{yz} y z}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,2)}[q](E)$, $\vec{\mathbb{Q}}_{2,2}^{(2,2)}[q](E)$
** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{14} Q_{u} \left(3 x^{4} + 6 x^{2} y^{2} - 24 x^{2} z^{2} + 3 y^{4} - 24 y^{2} z^{2} + 8 z^{4}\right)}{56} - \frac{5 \sqrt{42} Q_{v} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{168} - \frac{5 \sqrt{42} Q_{xy} x y \left(x^{2} + y^{2} - 6 z^{2}\right)}{84} - \frac{5 \sqrt{42} Q_{xz} x z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{84} - \frac{5 \sqrt{42} Q_{yz} y z \left(3 x^{2} + 3 y^{2} - 4 z^{2}\right)}{84}$$

$$- \frac{5 \sqrt{42} Q_{u} \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 6 z^{2}\right)}{168} + \frac{\sqrt{14} Q_{v} \left(19 x^{4} - 102 x^{2} y^{2} - 12 x^{2} z^{2} + 19 y^{4} - 12 y^{2} z^{2} + 4 z^{4}\right)}{168} + \frac{5 \sqrt{14} Q_{xy} x y \left(x - y\right) \left(x + y\right)}{12} + \frac{5 \sqrt{14} Q_{xz} x z \left(5 x^{2} - 9 y^{2} - 2 z^{2}\right)}{84} + \frac{5 \sqrt{14} Q_{yz} y z \left(9 x^{2} - 5 y^{2} + 2 z^{2}\right)}{84}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,3}^{(2,-2)}[q](T_2)$
** symmetry

$$\sqrt{3} y z$$

$$\sqrt{3} x z$$

$$\sqrt{3} x y$$

** expression

$$Q_{yz}$$

$$Q_{xz}$$

$$Q_{xy}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,3}^{(2,0)}[q](T_2)$
** symmetry

$$\sqrt{3} y z$$

$$\sqrt{3} x z$$

$$\sqrt{3} x y$$

** expression

$$\frac{\sqrt{42} Q_{u} y z}{14} - \frac{3 \sqrt{14} Q_{v} y z}{14} + \frac{3 \sqrt{14} Q_{xy} x z}{14} + \frac{3 \sqrt{14} Q_{xz} x y}{14} - \frac{\sqrt{14} Q_{yz} \left(2 x^{2} - y^{2} - z^{2}\right)}{14}$$

$$\frac{\sqrt{42} Q_{u} x z}{14} + \frac{3 \sqrt{14} Q_{v} x z}{14} + \frac{3 \sqrt{14} Q_{xy} y z}{14} + \frac{\sqrt{14} Q_{xz} \left(x^{2} - 2 y^{2} + z^{2}\right)}{14} + \frac{3 \sqrt{14} Q_{yz} x y}{14}$$

$$- \frac{\sqrt{42} Q_{u} x y}{7} + \frac{\sqrt{14} Q_{xy} \left(x^{2} + y^{2} - 2 z^{2}\right)}{14} + \frac{3 \sqrt{14} Q_{xz} y z}{14} + \frac{3 \sqrt{14} Q_{yz} x z}{14}$$

$\vec{\mathbb{Q}}_{2,1}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,2}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{2,3}^{(2,2)}[q](T_2)$
** symmetry

$$\sqrt{3} y z$$

$$\sqrt{3}xz$$

$$\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}Q_u yz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}Q_v yz\left(9x^2-5y^2+2z^2\right)}{84}-\frac{5\sqrt{14}Q_{xy}xz\left(x^2-6y^2+z^2\right)}{42}-\frac{5\sqrt{14}Q_{xz}xy\left(x^2+y^2-6z^2\right)}{42}+\frac{\sqrt{14}Q_{yz}\left(x^4-3x^2y^2-3x^2z^2-4y^4+27y^2z^2-4z^4\right)}{42}$$

$$-\frac{5\sqrt{42}Q_u xz\left(3x^2+3y^2-4z^2\right)}{84}+\frac{5\sqrt{14}Q_v xz\left(5x^2-9y^2-2z^2\right)}{84}+\frac{5\sqrt{14}Q_{xy}yz\left(6x^2-y^2-z^2\right)}{42}-\frac{\sqrt{14}Q_{xz}\left(4x^4+3x^2y^2-27x^2z^2-y^4+3y^2z^2+4z^4\right)}{42}-\frac{5\sqrt{14}Q_{yz}xy\left(x^2+y^2-6z^2\right)}{42}$$

$$-\frac{5\sqrt{42}Q_u xy\left(x^2+y^2-6z^2\right)}{84}+\frac{5\sqrt{14}Q_v xy\left(x-y\right)\left(x+y\right)}{12}-\frac{\sqrt{14}Q_{xy}\left(4x^4-27x^2y^2+3x^2z^2+4y^4+3y^2z^2-z^4\right)}{42}+\frac{5\sqrt{14}Q_{xz}yz\left(6x^2-y^2-z^2\right)}{42}-\frac{5\sqrt{14}Q_{yz}xz\left(x^2-6y^2+z^2\right)}{42}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}_3^{(2,-2)}[q](A_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}Q_{xy}z}{3}+\frac{\sqrt{3}Q_{xz}y}{3}+\frac{\sqrt{3}Q_{yz}x}{3}$$

$\vec{\mathbb{Q}}_3^{(2,0)}[q](A_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{3}Q_{xy}z\left(3x^2+3y^2-2z^2\right)}{6}+\frac{\sqrt{3}Q_{xz}y\left(3x^2-2y^2+3z^2\right)}{6}-\frac{\sqrt{3}Q_{yz}x\left(2x^2-3y^2-3z^2\right)}{6}$$

$\vec{\mathbb{Q}}_3^{(2,2)}[q](A_1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$-\frac{21\sqrt{22}Q_u xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{21\sqrt{66}Q_v xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xy}z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{66}-\frac{\sqrt{66}Q_{xz}y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{66}+\frac{\sqrt{66}Q_{yz}x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{66}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-2)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-2)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,3}^{(2,-2)}[q](T_1)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{Q_u x}{2}-\frac{\sqrt{3}Q_v x}{6}+\frac{\sqrt{3}Q_{xy}y}{3}-\frac{\sqrt{3}Q_{xz}z}{3}$$

$$\frac{Q_u y}{2}-\frac{\sqrt{3}Q_v y}{6}-\frac{\sqrt{3}Q_{xy}x}{3}+\frac{\sqrt{3}Q_{yz}z}{3}$$

$$\frac{\sqrt{3}Q_{v}z}{3} + \frac{\sqrt{3}Q_{xz}x}{3} - \frac{\sqrt{3}Q_{yz}y}{3}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,0)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,0)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,3}^{(2,0)}[q](T_1)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{Q_{u}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{4} + \frac{\sqrt{3}Q_{v}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{12} + \frac{\sqrt{3}Q_{xy}y\left(6x^{2}+y^{2}-9z^{2}\right)}{12} - \frac{\sqrt{3}Q_{xz}z\left(6x^{2}-9y^{2}+z^{2}\right)}{12}$$

$$\frac{Q_{u}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{4} - \frac{\sqrt{3}Q_{v}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{12} - \frac{\sqrt{3}Q_{xy}x\left(x^{2}+6y^{2}-9z^{2}\right)}{12} - \frac{\sqrt{3}Q_{yz}z\left(9x^{2}-6y^{2}-z^{2}\right)}{12}$$

$$\frac{\sqrt{3}Q_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{6} + \frac{\sqrt{3}Q_{xz}x\left(x^{2}-9y^{2}+6z^{2}\right)}{12} + \frac{\sqrt{3}Q_{yz}y\left(9x^{2}-y^{2}-6z^{2}\right)}{12}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,2)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,2}^{(2,2)}[q](T_1)$, $\vec{\mathbb{Q}}_{3,3}^{(2,2)}[q](T_1)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{22}Q_{u}x\left(2x^{4}+11x^{2}y^{2}-31x^{2}z^{2}+9y^{4}-87y^{2}z^{2}+30z^{4}\right)}{88} - \frac{\sqrt{66}Q_{v}x\left(2x^{4}-73x^{2}y^{2}+53x^{2}z^{2}+51y^{4}-87y^{2}z^{2}-12z^{4}\right)}{264} - \frac{\sqrt{66}Q_{xy}y\left(12x^{4}-46x^{2}y^{2}+66x^{2}z^{2}+5y^{4}-4y^{2}z^{2}-9z^{4}\right)}{132} + \frac{\sqrt{66}Q_{xz}z\left(12x^{4}+66x^{2}y^{2}-46x^{2}z^{2}-9y^{4}-4y^{2}z^{2}+5z^{4}\right)}{132} + \frac{21\sqrt{66}Q_{yz}xyz\left(y-z\right)\left(y+z\right)}{44}$$

$$\frac{\sqrt{22}Q_{u}y\left(9x^{4}+11x^{2}y^{2}-87x^{2}z^{2}+2y^{4}-31y^{2}z^{2}+30z^{4}\right)}{88} - \frac{\sqrt{66}Q_{v}y\left(51x^{4}-73x^{2}y^{2}-87x^{2}z^{2}+2y^{4}+53y^{2}z^{2}-12z^{4}\right)}{264} + \frac{\sqrt{66}Q_{xy}x\left(5x^{4}-46x^{2}y^{2}-4x^{2}z^{2}+12y^{4}+66y^{2}z^{2}-9z^{4}\right)}{132} - \frac{21\sqrt{66}Q_{xz}xyz\left(x-z\right)\left(x+z\right)}{44} + \frac{\sqrt{66}Q_{yz}z\left(9x^{4}-66x^{2}y^{2}+4x^{2}z^{2}-12y^{4}+46y^{2}z^{2}-5z^{4}\right)}{132}$$

$$-\frac{21\sqrt{22}Q_{u}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88} + \frac{\sqrt{66}Q_{v}z\left(39x^{4}-174x^{2}y^{2}-20x^{2}z^{2}+39y^{4}-20y^{2}z^{2}+4z^{4}\right)}{264} + \frac{21\sqrt{66}Q_{xy}xyz\left(x-y\right)\left(x+y\right)}{44} - \frac{\sqrt{66}Q_{xz}x\left(5x^{4}-4x^{2}y^{2}-46x^{2}z^{2}-9y^{4}+66y^{2}z^{2}+12z^{4}\right)}{132} - \frac{\sqrt{66}Q_{yz}y\left(9x^{4}+4x^{2}y^{2}-66x^{2}z^{2}-5y^{4}+46y^{2}z^{2}-12z^{4}\right)}{132}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,3}^{(2,-2)}[q](T_2)$

** symmetry

$$\frac{x\left(2x^{2}-3y^{2}-3z^{2}\right)}{2}$$

$$-\frac{y\left(3x^{2}-2y^{2}+3z^{2}\right)}{2}$$

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{15} Q_{u} x}{10} + \frac{3 \sqrt{5} Q_{v} x}{10} - \frac{\sqrt{5} Q_{xy} y}{5} - \frac{\sqrt{5} Q_{xz} z}{5}$$

$$- \frac{\sqrt{15} Q_{u} y}{10} - \frac{3 \sqrt{5} Q_{v} y}{10} - \frac{\sqrt{5} Q_{xy} x}{5} - \frac{\sqrt{5} Q_{yz} z}{5}$$

$$\frac{\sqrt{15} Q_{u} z}{5} - \frac{\sqrt{5} Q_{xz} x}{5} - \frac{\sqrt{5} Q_{yz} y}{5}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,3}^{(2,0)}[q](T_2)$

** symmetry

$$\frac{x \left(2 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2}$$

$$- \frac{y \left(3 x^{2} - 2 y^{2} + 3 z^{2}\right)}{2}$$

$$- \frac{z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{15} Q_{u} x \left(4 x^{2} - 21 y^{2} + 9 z^{2}\right)}{60} + \frac{\sqrt{5} Q_{v} x \left(4 x^{2} - y^{2} - 11 z^{2}\right)}{20} + \frac{\sqrt{5} Q_{xy} y \left(4 x^{2} - y^{2} - z^{2}\right)}{20} + \frac{\sqrt{5} Q_{xz} z \left(4 x^{2} - y^{2} - z^{2}\right)}{20} - \sqrt{5} Q_{yz} x y z$$

$$\frac{\sqrt{15} Q_{u} y \left(21 x^{2} - 4 y^{2} - 9 z^{2}\right)}{60} + \frac{\sqrt{5} Q_{v} y \left(x^{2} - 4 y^{2} + 11 z^{2}\right)}{20} - \frac{\sqrt{5} Q_{xy} x \left(x^{2} - 4 y^{2} + z^{2}\right)}{20} - \sqrt{5} Q_{xz} x y z - \frac{\sqrt{5} Q_{yz} z \left(x^{2} - 4 y^{2} + z^{2}\right)}{20}$$

$$- \frac{\sqrt{15} Q_{u} z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{15} - \frac{\sqrt{5} Q_{v} z \left(x - y\right) \left(x + y\right)}{2} - \sqrt{5} Q_{xy} x y z - \frac{\sqrt{5} Q_{xz} x \left(x^{2} + y^{2} - 4 z^{2}\right)}{20} - \frac{\sqrt{5} Q_{yz} y \left(x^{2} + y^{2} - 4 z^{2}\right)}{20}$$

$\vec{\mathbb{Q}}_{3,1}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,2}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{3,3}^{(2,2)}[q](T_2)$

** symmetry

$$\frac{x \left(2 x^{2} - 3 y^{2} - 3 z^{2}\right)}{2}$$

$$- \frac{y \left(3 x^{2} - 2 y^{2} + 3 z^{2}\right)}{2}$$

$$- \frac{z \left(3 x^{2} + 3 y^{2} - 2 z^{2}\right)}{2}$$

** expression

$$- \frac{\sqrt{330} Q_{u} x \left(4 x^{4} + x^{2} y^{2} - 41 x^{2} z^{2} - 3 y^{4} + 15 y^{2} z^{2} + 18 z^{4}\right)}{264} + \frac{\sqrt{110} Q_{v} x \left(4 x^{4} - 27 x^{2} y^{2} - 13 x^{2} z^{2} + 11 y^{4} + 15 y^{2} z^{2} + 4 z^{4}\right)}{88} + \frac{\sqrt{110} Q_{xy} y \left(8 x^{4} - 12 x^{2} y^{2} - 12 x^{2} z^{2} + y^{4} + 2 y^{2} z^{2} + z^{4}\right)}{44} + \frac{\sqrt{110} Q_{xz} z \left(8 x^{4} - 12 x^{2} y^{2} - 12 x^{2} z^{2} + y^{4} + 2 y^{2} z^{2} + z^{4}\right)}{44} + \frac{7 \sqrt{110} Q_{yz} x y z \left(2 x^{2} - y^{2} - z^{2}\right)}{44}$$

$$\frac{\sqrt{330} Q_{u} y \left(3 x^{4} - x^{2} y^{2} - 15 x^{2} z^{2} - 4 y^{4} + 41 y^{2} z^{2} - 18 z^{4}\right)}{264} - \frac{\sqrt{110} Q_{v} y \left(11 x^{4} - 27 x^{2} y^{2} + 15 x^{2} z^{2} + 4 y^{4} - 13 y^{2} z^{2} + 4 z^{4}\right)}{88} + \frac{\sqrt{110} Q_{xy} x \left(x^{4} - 12 x^{2} y^{2} + 2 x^{2} z^{2} + 8 y^{4} - 12 y^{2} z^{2} + z^{4}\right)}{44} - \frac{7 \sqrt{110} Q_{xz} x y z \left(x^{2} - 2 y^{2} + z^{2}\right)}{44} + \frac{\sqrt{110} Q_{yz} z \left(x^{4} - 12 x^{2} y^{2} + 2 x^{2} z^{2} + 8 y^{4} - 12 y^{2} z^{2} + z^{4}\right)}{44}$$

$$\frac{\sqrt{330} Q_{u} z \left(15 x^{4} + 30 x^{2} y^{2} - 40 x^{2} z^{2} + 15 y^{4} - 40 y^{2} z^{2} + 8 z^{4}\right)}{264} - \frac{7 \sqrt{110} Q_{v} z \left(x - y\right) \left(x + y\right) \left(x^{2} + y^{2} - 2 z^{2}\right)}{88} - \frac{7 \sqrt{110} Q_{xy} x y z \left(x^{2} + y^{2} - 2 z^{2}\right)}{44} + \frac{\sqrt{110} Q_{xz} x \left(x^{4} + 2 x^{2} y^{2} - 12 x^{2} z^{2} + y^{4} - 12 y^{2} z^{2} + 8 z^{4}\right)}{44} + \frac{\sqrt{110} Q_{yz} y \left(x^{4} + 2 x^{2} y^{2} - 12 x^{2} z^{2} + y^{4} - 12 y^{2} z^{2} + 8 z^{4}\right)}{44}$$

* Harmonics for rank 4

$\vec{\mathbb{Q}}_{4}^{(2,-2)}[q](A_1)$

** symmetry

$$\frac{\sqrt{21} \left(x^{4} - 3 x^{2} y^{2} - 3 x^{2} z^{2} + y^{4} - 3 y^{2} z^{2} + z^{4}\right)}{6}$$

** expression

$$- \frac{\sqrt{30} Q_{u} \left(x^{2} + y^{2} - 2 z^{2}\right)}{20} + \frac{3 \sqrt{10} Q_{v} \left(x - y\right) \left(x + y\right)}{20} - \frac{\sqrt{10} Q_{xy} x y}{5} - \frac{\sqrt{10} Q_{xz} x z}{5} - \frac{\sqrt{10} Q_{yz} y z}{5}$$

$\vec{\mathbb{Q}}_{4}^{(2,0)}[q](A_1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{165}Q_u\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{66}+\frac{\sqrt{55}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{22}+\frac{\sqrt{55}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{22}+\frac{\sqrt{55}Q_{xz}xz\left(x^2-6y^2+z^2\right)}{22}-\frac{\sqrt{55}Q_{yz}yz\left(6x^2-y^2-z^2\right)}{22}$$

$\vec{\mathbb{Q}}_{4}^{(2,2)}[q](A_1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{15015}Q_u\left(x^6-15x^4z^2+15x^2z^4+y^6-15y^4z^2+15y^2z^4-2z^6\right)}{572}+\frac{3\sqrt{5005}Q_v\left(x-y\right)\left(x+y\right)\left(x^4-9x^2y^2-5x^2z^2+y^4-5y^2z^2+5z^4\right)}{572}+\frac{\sqrt{5005}Q_{xy}xy\left(7x^4-19x^2y^2-13x^2z^2+7y^4-13y^2z^2+13z^4\right)}{286}+\frac{\sqrt{5005}Q_{xz}xz\left(7x^4-13x^2y^2-19x^2z^2+13y^4-13y^2z^2+7z^4\right)}{286}+\frac{\sqrt{5005}Q_{yz}yz\left(13x^4-13x^2y^2-13x^2z^2+7y^4-19y^2z^2+7z^4\right)}{286}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](E)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](E)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{42}Q_u\left(x^2+y^2-2z^2\right)}{28}-\frac{3\sqrt{14}Q_v\left(x-y\right)\left(x+y\right)}{28}+\frac{2\sqrt{14}Q_{xy}xy}{7}-\frac{\sqrt{14}Q_{xz}xz}{7}-\frac{\sqrt{14}Q_{yz}yz}{7}$$

$$-\frac{3\sqrt{14}Q_u\left(x-y\right)\left(x+y\right)}{28}+\frac{\sqrt{42}Q_v\left(x^2+y^2-2z^2\right)}{28}-\frac{\sqrt{42}Q_{xz}xz}{7}+\frac{\sqrt{42}Q_{yz}yz}{7}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](E)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](E)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{231}Q_u\left(8x^4-33x^2y^2-15x^2z^2+8y^4-15y^2z^2+5z^4\right)}{231}+\frac{\sqrt{77}Q_v\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{77}Q_{xy}xy\left(x^2+y^2-6z^2\right)}{14}-\frac{\sqrt{77}Q_{xz}xz\left(16x^2-33y^2-5z^2\right)}{154}+\frac{\sqrt{77}Q_{yz}yz\left(33x^2-16y^2+5z^2\right)}{154}$$

$$\frac{\sqrt{77}Q_u\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{77}+\frac{\sqrt{231}Q_v\left(2x^4-3x^2y^2-9x^2z^2+2y^4-9y^2z^2+3z^4\right)}{77}+\frac{\sqrt{231}Q_{xy}xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{231}Q_{xz}xz\left(2x^2-33y^2+9z^2\right)}{154}-\frac{\sqrt{231}Q_{yz}yz\left(33x^2-2y^2-9z^2\right)}{154}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](E)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](E)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{429}Q_u\left(x^6+45x^4y^2-60x^4z^2+45x^2y^4-540x^2y^2z^2+150x^2z^4+y^6-60y^4z^2+150y^2z^4-20z^6\right)}{1144}$$
$$-\frac{21\sqrt{143}Q_v\left(x-y\right)\left(x+y\right)\left(x^4-20x^2y^2+6x^2z^2+y^4+6y^2z^2-6z^4\right)}{1144}-\frac{7\sqrt{143}Q_{xy}xy\left(x^4-4x^2y^2+2x^2z^2+y^4+2y^2z^2-2z^4\right)}{52}$$
$$+\frac{7\sqrt{143}Q_{xz}xz\left(x^4+56x^2y^2-22x^2z^2-11y^4-34y^2z^2+10z^4\right)}{572}-\frac{7\sqrt{143}Q_{yz}yz\left(11x^4-56x^2y^2+34x^2z^2-y^4+22y^2z^2-10z^4\right)}{572}$$

$$-\frac{21\sqrt{143}Q_u\left(x-y\right)\left(x+y\right)\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{1144}$$
$$+\frac{\sqrt{429}Q_v\left(13x^6-45x^4y^2-150x^4z^2-45x^2y^4+540x^2y^2z^2+60x^2z^4+13y^6-150y^4z^2+60y^2z^4-8z^6\right)}{1144}$$
$$+\frac{21\sqrt{429}Q_{xy}xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{7\sqrt{429}Q_{xz}xz\left(7x^4-4x^2y^2-22x^2z^2-11y^4+26y^2z^2+4z^4\right)}{572}$$
$$+\frac{7\sqrt{429}Q_{yz}yz\left(11x^4+4x^2y^2-26x^2z^2-7y^4+22y^2z^2-4z^4\right)}{572}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,3}^{(2,-2)}[q](T_1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{2}Q_uyz}{4}-\frac{\sqrt{6}Q_vyz}{4}+\frac{\sqrt{6}Q_{yz}\left(y-z\right)\left(y+z\right)}{4}$$

$$\frac{3\sqrt{2}Q_uxz}{4}-\frac{\sqrt{6}Q_vxz}{4}-\frac{\sqrt{6}Q_{xz}\left(x-z\right)\left(x+z\right)}{4}$$

$$\frac{\sqrt{6}Q_vxy}{2}+\frac{\sqrt{6}Q_{xy}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,3}^{(2,0)}[q](T_1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{11}Q_uyz\left(18x^2+11y^2-17z^2\right)}{44}+\frac{\sqrt{33}Q_vyz\left(6x^2-15y^2+13z^2\right)}{44}+\frac{7\sqrt{33}Q_{xy}xz\left(3y^2-z^2\right)}{44}$$
$$+\frac{7\sqrt{33}Q_{xz}xy\left(y^2-3z^2\right)}{44}-\frac{\sqrt{33}Q_{yz}\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{44}$$

$$-\frac{\sqrt{11}Q_uxz\left(11x^2+18y^2-17z^2\right)}{44}-\frac{\sqrt{33}Q_vxz\left(15x^2-6y^2-13z^2\right)}{44}-\frac{7\sqrt{33}Q_{xy}yz\left(3x^2-z^2\right)}{44}$$
$$-\frac{\sqrt{33}Q_{xz}\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{44}-\frac{7\sqrt{33}Q_{yz}xy\left(x^2-3z^2\right)}{44}$$

$$-\frac{7\sqrt{11}Q_uxy\left(x-y\right)\left(x+y\right)}{11}+\frac{\sqrt{33}Q_vxy\left(x^2+y^2-6z^2\right)}{22}$$
$$+\frac{\sqrt{33}Q_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{44}+\frac{7\sqrt{33}Q_{xz}yz\left(3x^2-y^2\right)}{44}+\frac{7\sqrt{33}Q_{yz}xz\left(x^2-3y^2\right)}{44}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](T_1)$, $\vec{\mathbb{Q}}_{4,3}^{(2,2)}[q](T_1)$

** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{3\sqrt{1001}Q_{u}yz\left(2x^{4}+13x^{2}y^{2}-17x^{2}z^{2}+11y^{4}-41y^{2}z^{2}+14z^{4}\right)}{572}-\frac{\sqrt{3003}Q_{v}yz\left(2x^{4}-47x^{2}y^{2}+43x^{2}z^{2}+17y^{4}-41y^{2}z^{2}+8z^{4}\right)}{572} -\frac{3\sqrt{3003}Q_{xy}xz\left(3x^{2}y^{2}-x^{2}z^{2}-8y^{4}+13y^{2}z^{2}-z^{4}\right)}{286}-\frac{3\sqrt{3003}Q_{xz}xy\left(x^{2}y^{2}-3x^{2}z^{2}+y^{4}-13y^{2}z^{2}+8z^{4}\right)}{286} +\frac{\sqrt{3003}Q_{yz}\left(y-z\right)\left(y+z\right)\left(x^{4}-x^{2}y^{2}-x^{2}z^{2}-2y^{4}+29y^{2}z^{2}-2z^{4}\right)}{286}$$

$$\frac{3\sqrt{1001}Q_{u}xz\left(11x^{4}+13x^{2}y^{2}-41x^{2}z^{2}+2y^{4}-17y^{2}z^{2}+14z^{4}\right)}{572} -\frac{\sqrt{3003}Q_{v}xz\left(17x^{4}-47x^{2}y^{2}-41x^{2}z^{2}+2y^{4}+43y^{2}z^{2}+8z^{4}\right)}{572}-\frac{3\sqrt{3003}Q_{xy}yz\left(8x^{4}-3x^{2}y^{2}-13x^{2}z^{2}+y^{2}z^{2}+z^{4}\right)}{286} +\frac{\sqrt{3003}Q_{xz}\left(x-z\right)\left(x+z\right)\left(2x^{4}+x^{2}y^{2}-29x^{2}z^{2}-y^{4}+y^{2}z^{2}+2z^{4}\right)}{286}+\frac{3\sqrt{3003}Q_{yz}xy\left(x^{4}+x^{2}y^{2}-13x^{2}z^{2}-3y^{2}z^{2}+8z^{4}\right)}{286}$$

$$-\frac{9\sqrt{1001}Q_{u}xy\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-10z^{2}\right)}{572}+\frac{\sqrt{3003}Q_{v}xy\left(25x^{4}-82x^{2}y^{2}-4x^{2}z^{2}+25y^{4}-4y^{2}z^{2}+4z^{4}\right)}{572} -\frac{\sqrt{3003}Q_{xy}\left(x-y\right)\left(x+y\right)\left(2x^{4}-29x^{2}y^{2}+x^{2}z^{2}+2y^{4}+y^{2}z^{2}-z^{4}\right)}{286} +\frac{3\sqrt{3003}Q_{xz}yz\left(8x^{4}-13x^{2}y^{2}-3x^{2}z^{2}+y^{4}+y^{2}z^{2}\right)}{286}-\frac{3\sqrt{3003}Q_{yz}xz\left(x^{4}-13x^{2}y^{2}+x^{2}z^{2}+8y^{4}-3y^{2}z^{2}\right)}{286}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,-2)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,3}^{(2,-2)}[q](T_2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{3\sqrt{14}Q_{u}yz}{28}+\frac{3\sqrt{42}Q_{v}yz}{28}+\frac{\sqrt{42}Q_{xy}xz}{7}+\frac{\sqrt{42}Q_{xz}xy}{7}+\frac{\sqrt{42}Q_{yz}\left(2x^{2}-y^{2}-z^{2}\right)}{28}$$

$$-\frac{3\sqrt{14}Q_{u}xz}{28}-\frac{3\sqrt{42}Q_{v}xz}{28}+\frac{\sqrt{42}Q_{xy}yz}{7}-\frac{\sqrt{42}Q_{xz}\left(x^{2}-2y^{2}+z^{2}\right)}{28}+\frac{\sqrt{42}Q_{yz}xy}{7}$$

$$\frac{3\sqrt{14}Q_{u}xy}{14}-\frac{\sqrt{42}Q_{xy}\left(x^{2}+y^{2}-2z^{2}\right)}{28}+\frac{\sqrt{42}Q_{xz}yz}{7}+\frac{\sqrt{42}Q_{yz}xz}{7}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,0)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,3}^{(2,0)}[q](T_2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^{2}-y^{2}-z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^{2}-6y^{2}+z^{2}\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{77}Q_{u}yz\left(24x^{2}-25y^{2}+17z^{2}\right)}{308}+\frac{\sqrt{231}Q_{v}yz\left(24x^{2}+3y^{2}-11z^{2}\right)}{308}+\frac{\sqrt{231}Q_{xy}xz\left(18x^{2}+39y^{2}-31z^{2}\right)}{308} +\frac{\sqrt{231}Q_{xz}xy\left(18x^{2}-31y^{2}+39z^{2}\right)}{308}-\frac{\sqrt{231}Q_{yz}\left(12x^{4}-36x^{2}y^{2}-36x^{2}z^{2}+y^{4}+30y^{2}z^{2}+z^{4}\right)}{308}$$

$$\frac{\sqrt{77}Q_u xz\left(25x^2-24y^2-17z^2\right)}{308}-\frac{\sqrt{231}Q_v xz\left(3x^2+24y^2-11z^2\right)}{308}+\frac{\sqrt{231}Q_{xy} yz\left(39x^2+18y^2-31z^2\right)}{308}-\frac{\sqrt{231}Q_{xz}\left(x^4-36x^2y^2+30x^2z^2+12y^4-36y^2z^2+z^4\right)}{308}-\frac{\sqrt{231}Q_{yz} xy\left(31x^2-18y^2-39z^2\right)}{308}$$

$$-\frac{2\sqrt{77}Q_u xy\left(x^2+y^2-6z^2\right)}{77}-\frac{\sqrt{231}Q_v xy\left(x-y\right)\left(x+y\right)}{22}-\frac{\sqrt{231}Q_{xy}\left(x^4+30x^2y^2-36x^2z^2+y^4-36y^2z^2+12z^4\right)}{308}+\frac{\sqrt{231}Q_{xz} yz\left(39x^2-31y^2+18z^2\right)}{308}-\frac{\sqrt{231}Q_{yz} xz\left(31x^2-39y^2-18z^2\right)}{308}$$

$\vec{\mathbb{Q}}_{4,1}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,2}^{(2,2)}[q](T_2)$, $\vec{\mathbb{Q}}_{4,3}^{(2,2)}[q](T_2)$

** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{21\sqrt{143}Q_u yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{572}+\frac{21\sqrt{429}Q_v yz\left(8x^4-13x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{572}-\frac{7\sqrt{429}Q_{xy} xz\left(2x^4-23x^2y^2+x^2z^2+8y^4+7y^2z^2-z^4\right)}{286}-\frac{7\sqrt{429}Q_{xz} xy\left(2x^4+x^2y^2-23x^2z^2-y^4+7y^2z^2+8z^4\right)}{286}+\frac{\sqrt{429}Q_{yz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}$$

$$\frac{21\sqrt{143}Q_u xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{572}-\frac{21\sqrt{429}Q_v xz\left(x^4-13x^2y^2+x^2z^2+8y^4-3y^2z^2\right)}{572}-\frac{7\sqrt{429}Q_{xy} yz\left(8x^4-23x^2y^2+7x^2z^2+2y^4+y^2z^2-z^4\right)}{286}+\frac{\sqrt{429}Q_{xz}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}+\frac{7\sqrt{429}Q_{yz} xy\left(x^4-x^2y^2-7x^2z^2-2y^4+23y^2z^2-8z^4\right)}{286}$$

$$\frac{21\sqrt{143}Q_u xy\left(x^4+2x^2y^2-16x^2z^2+y^4-16y^2z^2+16z^4\right)}{572}-\frac{21\sqrt{429}Q_v xy\left(x-y\right)\left(x+y\right)\left(x^2+y^2-10z^2\right)}{572}+\frac{\sqrt{429}Q_{xy}\left(2x^6-15x^4y^2-15x^4z^2-15x^2y^4+180x^2y^2z^2-15x^2z^4+2y^6-15y^4z^2-15y^2z^4+2z^6\right)}{286}-\frac{7\sqrt{429}Q_{xz} yz\left(8x^4+7x^2y^2-23x^2z^2-y^4+y^2z^2+2z^4\right)}{286}+\frac{7\sqrt{429}Q_{yz} xz\left(x^4-7x^2y^2-x^2z^2-8y^4+23y^2z^2-2z^4\right)}{286}$$