PG No. 5 C_{2h} $2/m$ (b-axis setting)   [monoclinic] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_0^{(1,1)}[g](A_u)$

** symmetry

1

** expression

$$\frac{\sqrt{3}G_x x}{3} + \frac{\sqrt{3}G_y y}{3} + \frac{\sqrt{3}G_z z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,-1)}[g](A_g)$

** symmetry

y

** expression

G_y

$\vec{\mathbb{G}}_1^{(1,1)}[g](A_g)$

** symmetry

y

** expression

$$\frac{3\sqrt{10}G_x xy}{10} - \frac{\sqrt{10}G_y \left(x^2 - 2y^2 + z^2\right)}{10} + \frac{3\sqrt{10}G_z yz}{10}$$

$\vec{\mathbb{G}}_1^{(1,-1)}[g](B_g, 1)$

** symmetry

x

** expression

G_x

$\vec{\mathbb{G}}_1^{(1,-1)}[g](B_g, 2)$

** symmetry

z

** expression

G_z

$\vec{\mathbb{G}}_1^{(1,1)}[g](B_g, 1)$

** symmetry

x

** expression

$$\frac{\sqrt{10}G_x \left(2x^2 - y^2 - z^2\right)}{10} + \frac{3\sqrt{10}G_y xy}{10} + \frac{3\sqrt{10}G_z xz}{10}$$

$\vec{\mathbb{G}}_1^{(1,1)}[g](B_g, 2)$

** symmetry

z

** expression

$$\frac{3\sqrt{10}G_x xz}{10} + \frac{3\sqrt{10}G_y yz}{10} - \frac{\sqrt{10}G_z \left(x^2 + y^2 - 2z^2\right)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{6}G_x x}{6} - \frac{\sqrt{6}G_y y}{6} + \frac{\sqrt{6}G_z z}{3}$$

1

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}G_x x}{2} - \frac{\sqrt{2}G_y y}{2}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](A_u, 3)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_x z}{2} + \frac{\sqrt{2}G_z x}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 1)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$-\frac{\sqrt{21}G_x x\,(x^2+y^2-4z^2)}{14} - \frac{\sqrt{21}G_y y\,(x^2+y^2-4z^2)}{14} - \frac{\sqrt{21}G_z z\,(3x^2+3y^2-2z^2)}{14}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 2)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{7}G_x x\,(3x^2-7y^2-2z^2)}{14} + \frac{\sqrt{7}G_y y\,(7x^2-3y^2+2z^2)}{14} + \frac{5\sqrt{7}G_z z\,(x-y)\,(x+y)}{14}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](A_u, 3)$

** symmetry

$$\sqrt{3}xz$$

** expression

$$\frac{\sqrt{7}G_x z\,(4x^2-y^2-z^2)}{7} + \frac{5\sqrt{7}G_y xyz}{7} - \frac{\sqrt{7}G_z x\,(x^2+y^2-4z^2)}{7}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_u, 1)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}G_y z}{2} + \frac{\sqrt{2}G_z y}{2}$$

$\vec{\mathbb{G}}_2^{(1,-1)}[g](B_u, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_x y}{2} + \frac{\sqrt{2}G_y x}{2}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_u, 1)$

** symmetry

$$\sqrt{3}yz$$

** expression

$$\frac{5\sqrt{7}G_x xyz}{7} - \frac{\sqrt{7}G_y z\,(x^2-4y^2+z^2)}{7} - \frac{\sqrt{7}G_z y\,(x^2+y^2-4z^2)}{7}$$

$\vec{\mathbb{G}}_2^{(1,1)}[g](B_u, 2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_xy\left(4x^2 - y^2 - z^2\right)}{7} - \frac{\sqrt{7}G_yx\left(x^2 - 4y^2 + z^2\right)}{7} + \frac{5\sqrt{7}G_zxyz}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$G_xyz + G_yxz + G_zxy$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 2)$

** symmetry

$$-\frac{y\left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxy}{5} - \frac{\sqrt{15}G_y\left(x^2 - 2y^2 + z^2\right)}{10} - \frac{\sqrt{15}G_zyz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](A_g, 3)$

** symmetry

$$-\frac{\sqrt{15}y\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$-G_xxy - \frac{G_y\left(x - z\right)\left(x + z\right)}{2} + G_zyz$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 1)$

** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{15}G_xyz\left(6x^2 - y^2 - z^2\right)}{6} - \frac{\sqrt{15}G_yxz\left(x^2 - 6y^2 + z^2\right)}{6} - \frac{\sqrt{15}G_zxy\left(x^2 + y^2 - 6z^2\right)}{6}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 2)$

** symmetry

$$-\frac{y\left(3x^2 - 2y^2 + 3z^2\right)}{2}$$

** expression

$$-\frac{5G_xxy\left(3x^2 - 4y^2 + 3z^2\right)}{12} + \frac{G_y\left(3x^4 - 24x^2y^2 + 6x^2z^2 + 8y^4 - 24y^2z^2 + 3z^4\right)}{12} - \frac{5G_zyz\left(3x^2 - 4y^2 + 3z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](A_g, 3)$

** symmetry

$$-\frac{\sqrt{15}y\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxy\left(5x^2 - 2y^2 - 9z^2\right)}{12} + \frac{\sqrt{15}G_y\left(x - z\right)\left(x + z\right)\left(x^2 - 6y^2 + z^2\right)}{12} - \frac{\sqrt{15}G_zyz\left(9x^2 + 2y^2 - 5z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_g, 1)$

** symmetry

$$\frac{x\left(2x^2 - 3y^2 - 3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(2x^2-y^2-z^2\right)}{10}-\frac{\sqrt{15}G_yxy}{5}-\frac{\sqrt{15}G_zxz}{5}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_g,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_xxz}{5}-\frac{\sqrt{15}G_yyz}{5}-\frac{\sqrt{15}G_z\left(x^2+y^2-2z^2\right)}{10}$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_g,3)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{G_x\left(y-z\right)\left(y+z\right)}{2}+G_yxy-G_zxz$$

$\vec{\mathbb{G}}_3^{(1,-1)}[g](B_g,4)$

** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$G_xxz-G_yyz+\frac{G_z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_g,1)$

** symmetry

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{G_x\left(8x^4-24x^2y^2-24x^2z^2+3y^4+6y^2z^2+3z^4\right)}{12}+\frac{5G_yxy\left(4x^2-3y^2-3z^2\right)}{12}+\frac{5G_zxz\left(4x^2-3y^2-3z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_g,2)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{5G_xxz\left(3x^2+3y^2-4z^2\right)}{12}-\frac{5G_yyz\left(3x^2+3y^2-4z^2\right)}{12}+\frac{G_z\left(3x^4+6x^2y^2-24x^2z^2+3y^4-24y^2z^2+8z^4\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_g,3)$

** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_x\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{12}-\frac{\sqrt{15}G_yxy\left(2x^2-5y^2+9z^2\right)}{12}+\frac{\sqrt{15}G_zxz\left(2x^2+9y^2-5z^2\right)}{12}$$

$\vec{\mathbb{G}}_3^{(1,1)}[g](B_g,4)$

** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_xxz\left(5x^2-9y^2-2z^2\right)}{12}+\frac{\sqrt{15}G_yyz\left(9x^2-5y^2+2z^2\right)}{12}-\frac{\sqrt{15}G_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$\frac{\sqrt{3}G_x x \left(2x^2 - 3y^2 - 3z^2\right)}{6} - \frac{\sqrt{3}G_y y \left(3x^2 - 2y^2 + 3z^2\right)}{6} - \frac{\sqrt{3}G_z z \left(3x^2 + 3y^2 - 2z^2\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{105}G_x x \left(x^2 - 6y^2 + 3z^2\right)}{42} + \frac{\sqrt{105}G_y y \left(6x^2 - y^2 - 3z^2\right)}{42} - \frac{\sqrt{105}G_z z \left(3x^2 + 3y^2 - 2z^2\right)}{42}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{5}\left(x - y\right)\left(x + y\right)\left(x^2 + y^2 - 6z^2\right)}{4}$$

** expression

$$\frac{\sqrt{35}G_x x \left(x^2 - 3z^2\right)}{14} - \frac{\sqrt{35}G_y y \left(y^2 - 3z^2\right)}{14} - \frac{3\sqrt{35}G_z z \left(x - y\right)\left(x + y\right)}{14}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,4)$

** symmetry

$$-\frac{\sqrt{35}xz\left(x - z\right)\left(x + z\right)}{2}$$

** expression

$$-\frac{\sqrt{5}G_x z \left(3x^2 - z^2\right)}{4} - \frac{\sqrt{5}G_z x \left(x^2 - 3z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](A_u,5)$

** symmetry

$$-\frac{\sqrt{5}xz\left(x^2 - 6y^2 + z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x z \left(3x^2 - 6y^2 + z^2\right)}{28} + \frac{3\sqrt{35}G_y xyz}{7} - \frac{\sqrt{35}G_z x \left(x^2 - 6y^2 + 3z^2\right)}{28}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,1)$

** symmetry

$$\frac{\sqrt{21}\left(x^4 - 3x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + z^4\right)}{6}$$

** expression

$$\frac{\sqrt{1155}G_x x \left(x^4 - 5x^2y^2 - 5x^2z^2 + 3y^4 - 3y^2z^2 + 3z^4\right)}{66} + \frac{\sqrt{1155}G_y y \left(3x^4 - 5x^2y^2 - 3x^2z^2 + y^4 - 5y^2z^2 + 3z^4\right)}{66}$$
$$+ \frac{\sqrt{1155}G_z z \left(3x^4 - 3x^2y^2 - 5x^2z^2 + 3y^4 - 5y^2z^2 + z^4\right)}{66}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,2)$

** symmetry

$$-\frac{\sqrt{15}\left(x^4 - 12x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{33}G_x x \left(5x^4 - 88x^2y^2 + 38x^2z^2 + 33y^4 + 66y^2z^2 - 30z^4\right)}{132} - \frac{\sqrt{33}G_y y \left(33x^4 - 88x^2y^2 + 66x^2z^2 + 5y^4 + 38y^2z^2 - 30z^4\right)}{132}$$
$$+ \frac{\sqrt{33}G_z z \left(3x^4 + 132x^2y^2 - 50x^2z^2 + 3y^4 - 50y^2z^2 + 10z^4\right)}{132}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u, 3)$

** symmetry

$$\frac{\sqrt{5}\,(x-y)\,(x+y)\,(x^2+y^2-6z^2)}{4}$$

** expression

$$\frac{\sqrt{11}G_x x\,(5x^4 - 4x^2y^2 - 46x^2z^2 - 9y^4 + 66y^2z^2 + 12z^4)}{44}$$
$$+ \frac{\sqrt{11}G_y y\,(9x^4 + 4x^2y^2 - 66x^2z^2 - 5y^4 + 46y^2z^2 - 12z^4)}{44} + \frac{21\sqrt{11}G_z z\,(x-y)\,(x+y)\,(x^2+y^2-2z^2)}{44}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u, 4)$

** symmetry

$$-\frac{\sqrt{35}xz\,(x-z)\,(x+z)}{2}$$

** expression

$$-\frac{\sqrt{77}G_x z\,(6x^4 - 3x^2y^2 - 11x^2z^2 + y^2z^2 + z^4)}{22} - \frac{9\sqrt{77}G_y xyz\,(x-z)\,(x+z)}{22} + \frac{\sqrt{77}G_z x\,(x^4 + x^2y^2 - 11x^2z^2 - 3y^2z^2 + 6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u, 5)$

** symmetry

$$-\frac{\sqrt{5}xz\,(x^2 - 6y^2 + z^2)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x z\,(6x^4 - 51x^2y^2 + 5x^2z^2 + 6y^4 + 5y^2z^2 - z^4)}{22} - \frac{21\sqrt{11}G_y xyz\,(x^2 - 2y^2 + z^2)}{22} + \frac{\sqrt{11}G_z x\,(x^4 - 5x^2y^2 - 5x^2z^2 - 6y^4 + 51y^2z^2 - 6z^4)}{22}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_u, 1)$

** symmetry

$$\frac{\sqrt{35}yz\,(y-z)\,(y+z)}{2}$$

** expression

$$\frac{\sqrt{5}G_y z\,(3y^2 - z^2)}{4} + \frac{\sqrt{5}G_z y\,(y^2 - 3z^2)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_u, 2)$

** symmetry

$$\frac{\sqrt{35}xy\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{5}G_x y\,(3x^2 - y^2)}{4} + \frac{\sqrt{5}G_y x\,(x^2 - 3y^2)}{4}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_u, 3)$

** symmetry

$$\frac{\sqrt{5}yz\,(6x^2 - y^2 - z^2)}{2}$$

** expression

$$\frac{3\sqrt{35}G_x xyz}{7} + \frac{\sqrt{35}G_y z\,(6x^2 - 3y^2 - z^2)}{28} + \frac{\sqrt{35}G_z y\,(6x^2 - y^2 - 3z^2)}{28}$$

$\vec{\mathbb{G}}_4^{(1,-1)}[g](B_u, 4)$

** symmetry

$$-\frac{\sqrt{5}xy\,(x^2 + y^2 - 6z^2)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x y\,(3x^2 + y^2 - 6z^2)}{28} - \frac{\sqrt{35}G_y x\,(x^2 + 3y^2 - 6z^2)}{28} + \frac{3\sqrt{35}G_z xyz}{7}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_u, 1)$

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

** expression

$$\frac{9\sqrt{77}G_x xyz\left(y-z\right)\left(y+z\right)}{22}-\frac{\sqrt{77}G_y z\left(3x^2y^2-x^2z^2-6y^4+11y^2z^2-z^4\right)}{22}-\frac{\sqrt{77}G_z y\left(x^2y^2-3x^2z^2+y^4-11y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_u,2)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77}G_x y\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}-\frac{\sqrt{77}G_y x\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}+\frac{9\sqrt{77}G_z xyz\left(x-y\right)\left(x+y\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_u,3)$
** symmetry

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{11}G_x xyz\left(2x^2-y^2-z^2\right)}{22}-\frac{\sqrt{11}G_y z\left(6x^4-51x^2y^2+5x^2z^2+6y^4+5y^2z^2-z^4\right)}{22}-\frac{\sqrt{11}G_z y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](B_u,4)$
** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}+\frac{\sqrt{11}G_y x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}-\frac{21\sqrt{11}G_z xyz\left(x^2+y^2-2z^2\right)}{22}$$