PG No. 27 $D_{6\mathrm{h}}$ $6/mmm$ [hexagonal] (polar, internal axial dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{Q}}_{1}^{(1,0)}[g](A_{2u})$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{2}G_{x}y}{2} - \frac{\sqrt{2}G_{y}x}{2}$$

$\vec{\mathbb{Q}}_{1,1}^{(1,0)}[g](E_{1u})$, $\vec{\mathbb{Q}}_{1,2}^{(1,0)}[g](E_{1u})$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{2}G_{y}z}{2} - \frac{\sqrt{2}G_{z}y}{2}$$

$$- \frac{\sqrt{2}G_{x}z}{2} + \frac{\sqrt{2}G_{z}x}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{Q}}_{2}^{(1,0)}[g](A_{1g})$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{6}G_{x}yz}{2} - \frac{\sqrt{6}G_{y}xz}{2}$$

$\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E_{1g})$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E_{1g})$

** symmetry

$$\sqrt{3}yz$$

$$- \sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_{x}\left(y - z\right)\left(y + z\right)}{2} - \frac{\sqrt{2}G_{y}xy}{2} + \frac{\sqrt{2}G_{z}xz}{2}$$

$$- \frac{\sqrt{2}G_{x}xy}{2} + \frac{\sqrt{2}G_{y}\left(x - z\right)\left(x + z\right)}{2} + \frac{\sqrt{2}G_{z}yz}{2}$$

$\vec{\mathbb{Q}}_{2,1}^{(1,0)}[g](E_{2g})$, $\vec{\mathbb{Q}}_{2,2}^{(1,0)}[g](E_{2g})$

** symmetry

$$\frac{\sqrt{3}\left(x - y\right)\left(x + y\right)}{2}$$

$$- \sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_{x}yz}{2} + \frac{\sqrt{2}G_{y}xz}{2} - \sqrt{2}G_{z}xy$$

$$\frac{\sqrt{2}G_{x}xz}{2} - \frac{\sqrt{2}G_{y}yz}{2} - \frac{\sqrt{2}G_{z}\left(x - y\right)\left(x + y\right)}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{Q}}_{3}^{(1,0)}[g](A_{2u})$

** symmetry

$$- \frac{z\left(3x^{2} + 3y^{2} - 2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{3}G_{x}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{3}G_{y}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{3}^{(1,0)}[g](B_{1u})$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{30}G_{x}z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{30}G_{y}xyz}{4}+\frac{\sqrt{30}G_{z}x\left(x^{2}-3y^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{3}^{(1,0)}[g](B_{2u})$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{30}G_{x}xyz}{4}+\frac{\sqrt{30}G_{y}z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{30}G_{z}y\left(3x^{2}-y^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E_{1u})$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E_{1u})$
** symmetry

$$-\frac{\sqrt{6}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

** expression

$$\frac{5\sqrt{2}G_{x}xyz}{4}-\frac{\sqrt{2}G_{y}z\left(11x^{2}+y^{2}-4z^{2}\right)}{8}+\frac{\sqrt{2}G_{z}y\left(x^{2}+y^{2}-4z^{2}\right)}{8}$$

$$\frac{\sqrt{2}G_{x}z\left(x^{2}+11y^{2}-4z^{2}\right)}{8}-\frac{5\sqrt{2}G_{y}xyz}{4}-\frac{\sqrt{2}G_{z}x\left(x^{2}+y^{2}-4z^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{3,1}^{(1,0)}[g](E_{2u})$, $\vec{\mathbb{Q}}_{3,2}^{(1,0)}[g](E_{2u})$
** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_{x}x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}G_{y}y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}G_{z}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\frac{\sqrt{5}G_{x}y\left(x^{2}-y^{2}+2z^{2}\right)}{4}-\frac{\sqrt{5}G_{y}x\left(x^{2}-y^{2}-2z^{2}\right)}{4}-\sqrt{5}G_{z}xyz$$

* Harmonics for rank 4
$\vec{\mathbb{Q}}_{4}^{(1,0)}[g](A_{1g})$
** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$-\frac{\sqrt{5}G_{x}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}+\frac{\sqrt{5}G_{y}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{4}^{(1,0)}[g](B_{1g})$
** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{14}G_{x}xy\left(x^{2}-3y^{2}+6z^{2}\right)}{8}-\frac{\sqrt{14}G_{y}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}+3y^{2}z^{2}\right)}{8}-\frac{3\sqrt{14}G_{z}yz\left(3x^{2}-y^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{4}^{(1,0)}[g](B_{2g})$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{14}G_{x}\left(3x^{2}y^{2}-3x^{2}z^{2}-y^{4}+3y^{2}z^{2}\right)}{8}-\frac{\sqrt{14}G_{y}xy\left(3x^{2}-y^{2}-6z^{2}\right)}{8}+\frac{3\sqrt{14}G_{z}xz\left(x^{2}-3y^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E_{1g})$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E_{1g})$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{2}G_{x}\left(3x^{2}y^{2}-3x^{2}z^{2}+3y^{4}-21y^{2}z^{2}+4z^{4}\right)}{8}+\frac{3\sqrt{2}G_{y}xy\left(x^{2}+y^{2}-6z^{2}\right)}{8}-\frac{\sqrt{2}G_{z}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{8}$$

$$\frac{3\sqrt{2}G_{x}xy\left(x^{2}+y^{2}-6z^{2}\right)}{8}-\frac{\sqrt{2}G_{y}\left(3x^{4}+3x^{2}y^{2}-21x^{2}z^{2}-3y^{2}z^{2}+4z^{4}\right)}{8}-\frac{\sqrt{2}G_{z}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{8}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E_{2g},1)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E_{2g},1)$
** symmetry

$$\frac{\sqrt{35}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{7}G_{x}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}G_{y}xz\left(x^{2}-3y^{2}\right)}{4}-\sqrt{7}G_{z}xy\left(x-y\right)\left(x+y\right)$$

$$-\frac{\sqrt{7}G_{x}xz\left(x^{2}-3y^{2}\right)}{4}+\frac{\sqrt{7}G_{y}yz\left(3x^{2}-y^{2}\right)}{4}+\frac{\sqrt{7}G_{z}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{4}$$

$\vec{\mathbb{Q}}_{4,1}^{(1,0)}[g](E_{2g},2)$, $\vec{\mathbb{Q}}_{4,2}^{(1,0)}[g](E_{2g},2)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$\frac{G_{x}yz\left(3x^{2}-4y^{2}+3z^{2}\right)}{2}-\frac{G_{y}xz\left(4x^{2}-3y^{2}-3z^{2}\right)}{2}+\frac{G_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

$$-\frac{G_{x}xz\left(x^{2}+15y^{2}-6z^{2}\right)}{4}+\frac{G_{y}yz\left(15x^{2}+y^{2}-6z^{2}\right)}{4}+\frac{G_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$