PG No. 19 $C_{3\mathrm{v}}$ $3m$ (3m1 setting) [trigonal] (axial, internal polar quadrupole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(2,1)}[q](A_2)$

** symmetry

$$z$$

** expression

$$\frac{\sqrt{30}Q_{v}xy}{5} - \frac{\sqrt{30}Q_{xy}\left(x-y\right)\left(x+y\right)}{10} + \frac{\sqrt{30}Q_{xz}yz}{10} - \frac{\sqrt{30}Q_{yz}xz}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(2,1)}[q](E)$, $\vec{\mathbb{G}}_{1,2}^{(2,1)}[q](E)$

** symmetry

$$-y$$

$$x$$

** expression

$$-\frac{3\sqrt{10}Q_{u}xz}{10} + \frac{\sqrt{30}Q_{v}xz}{10} + \frac{\sqrt{30}Q_{xy}yz}{10} - \frac{\sqrt{30}Q_{xz}\left(x-z\right)\left(x+z\right)}{10} - \frac{\sqrt{30}Q_{yz}xy}{10}$$

$$-\frac{3\sqrt{10}Q_{u}yz}{10} - \frac{\sqrt{30}Q_{v}yz}{10} + \frac{\sqrt{30}Q_{xy}xz}{10} - \frac{\sqrt{30}Q_{xz}xy}{10} - \frac{\sqrt{30}Q_{yz}\left(y-z\right)\left(y+z\right)}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(2,-1)}[q](A_2)$

** symmetry

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{\sqrt{2}Q_{xz}y}{2} - \frac{\sqrt{2}Q_{yz}x}{2}$$

$\vec{\mathbb{G}}_{2}^{(2,1)}[q](A_2)$

** symmetry

$$-\frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$\frac{5\sqrt{42}Q_{v}xyz}{14} - \frac{5\sqrt{42}Q_{xy}z\left(x-y\right)\left(x+y\right)}{28} - \frac{\sqrt{42}Q_{xz}y\left(x^{2}+y^{2}-4z^{2}\right)}{28} + \frac{\sqrt{42}Q_{yz}x\left(x^{2}+y^{2}-4z^{2}\right)}{28}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}Q_{u}x}{2} + \frac{\sqrt{6}Q_{v}x}{6} + \frac{\sqrt{6}Q_{xy}y}{6} - \frac{\sqrt{6}Q_{xz}z}{6}$$

$$\frac{\sqrt{2}Q_{u}y}{2} - \frac{\sqrt{6}Q_{v}y}{6} + \frac{\sqrt{6}Q_{xy}x}{6} - \frac{\sqrt{6}Q_{yz}z}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E,2)$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\frac{\sqrt{6}Q_{v}z}{3} - \frac{\sqrt{6}Q_{xz}x}{6} + \frac{\sqrt{6}Q_{yz}y}{6}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E,1)$, $\vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$-\frac{\sqrt{42}Q_{u}x\left(x^{2}+y^{2}-4z^{2}\right)}{28} - \frac{\sqrt{14}Q_{v}x\left(x^{2}-9y^{2}+6z^{2}\right)}{28} - \frac{\sqrt{14}Q_{xy}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14} + \frac{\sqrt{14}Q_{xz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$$-\frac{\sqrt{42}Q_{u}y\left(x^{2}+y^{2}-4z^{2}\right)}{28} - \frac{\sqrt{14}Q_{v}y\left(9x^{2}-y^{2}-6z^{2}\right)}{28} + \frac{\sqrt{14}Q_{xy}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14} + \frac{\sqrt{14}Q_{yz}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E,2)$, $\vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E,2)$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$-\frac{5\sqrt{42}Q_{u}xyz}{14} + \frac{\sqrt{14}Q_{xy}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{28} - \frac{\sqrt{14}Q_{xz}y\left(9x^{2}-y^{2}-6z^{2}\right)}{28} + \frac{\sqrt{14}Q_{yz}x\left(x^{2}-9y^{2}+6z^{2}\right)}{28}$$

$$-\frac{5\sqrt{42}Q_{u}z\left(x-y\right)\left(x+y\right)}{28} + \frac{\sqrt{14}Q_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{28} - \frac{\sqrt{14}Q_{xz}x\left(2x^{2}-3y^{2}-3z^{2}\right)}{14} - \frac{\sqrt{14}Q_{yz}y\left(3x^{2}-2y^{2}+3z^{2}\right)}{14}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{3}Q_{v}yz}{2} + \frac{\sqrt{3}Q_{xy}xz}{2} - \frac{\sqrt{3}Q_{xz}xy}{2} - \frac{\sqrt{3}Q_{yz}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(2,1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$-\frac{7\sqrt{2}Q_{u}yz\left(3x^{2}-y^{2}\right)}{8} + \frac{\sqrt{6}Q_{v}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{24} + \frac{\sqrt{6}Q_{xy}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{24}$$
$$- \frac{\sqrt{6}Q_{xz}xy\left(17x^{2}-11y^{2}-18z^{2}\right)}{24} + \frac{\sqrt{6}Q_{yz}\left(2x^{4}-21x^{2}y^{2}+9x^{2}z^{2}+5y^{4}-9y^{2}z^{2}\right)}{24}$$

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](A_2,1)$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{30}Q_{v}xy}{10} + \frac{\sqrt{30}Q_{xy}\left(x-y\right)\left(x+y\right)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$\vec{\mathbb{G}}_{3}^{(2,-1)}[q](A_2,2)$

** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{3}Q_v xz}{2}+\frac{\sqrt{3}Q_{xy}yz}{2}+\frac{\sqrt{3}Q_{xz}\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{3}Q_{yz}xy}{2}$$

$\vec{\mathbb{G}}_{3}^{(2,1)}[q](A_2,1)$
** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{15}Q_v xy\left(x^2+y^2-6z^2\right)}{6}+\frac{\sqrt{15}Q_{xy}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{12}-\frac{\sqrt{15}Q_{xz}yz\left(3x^2+3y^2-4z^2\right)}{12}+\frac{\sqrt{15}Q_{yz}xz\left(3x^2+3y^2-4z^2\right)}{12}$$

$\vec{\mathbb{G}}_{3}^{(2,1)}[q](A_2,2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{7\sqrt{2}Q_u xz\left(x^2-3y^2\right)}{8}-\frac{\sqrt{6}Q_v xz\left(3x^2+3y^2-4z^2\right)}{24}+\frac{\sqrt{6}Q_{xy}yz\left(3x^2+3y^2-4z^2\right)}{24}+\frac{\sqrt{6}Q_{xz}\left(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2\right)}{24}+\frac{\sqrt{6}Q_{yz}xy\left(11x^2-17y^2+18z^2\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E,1)$
** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$-\frac{\sqrt{15}Q_u xz}{5}-\frac{3\sqrt{5}Q_v xz}{10}-\frac{3\sqrt{5}Q_{xy}yz}{10}+\frac{\sqrt{5}Q_{xz}\left(x^2-5y^2+4z^2\right)}{20}+\frac{3\sqrt{5}Q_{yz}xy}{10}$$

$$-\frac{\sqrt{15}Q_u yz}{5}+\frac{3\sqrt{5}Q_v yz}{10}-\frac{3\sqrt{5}Q_{xy}xz}{10}+\frac{3\sqrt{5}Q_{xz}xy}{10}-\frac{\sqrt{5}Q_{yz}\left(5x^2-y^2-4z^2\right)}{20}$$

$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E,2)$
** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{6}Q_u xy}{2}+\frac{\sqrt{2}Q_{xy}\left(x^2+y^2-2z^2\right)}{4}$$

$$\frac{\sqrt{6}Q_u\left(x-y\right)\left(x+y\right)}{4}+\frac{\sqrt{2}Q_v\left(x^2+y^2-2z^2\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E,1)$
** symmetry

$$\frac{\sqrt{6}y\left(x^2+y^2-4z^2\right)}{4}$$

$$-\frac{\sqrt{6}x\left(x^2+y^2-4z^2\right)}{4}$$

** expression

$$\frac{\sqrt{30}Q_u xz\left(3x^2+3y^2-4z^2\right)}{24}+\frac{\sqrt{10}Q_v xz\left(x^2-27y^2+8z^2\right)}{24}+\frac{\sqrt{10}Q_{xy}yz\left(15x^2-13y^2+8z^2\right)}{24}+\frac{\sqrt{10}Q_{xz}\left(x^4+3x^2y^2-9x^2z^2+2y^4-15y^2z^2+4z^4\right)}{24}-\frac{\sqrt{10}Q_{yz}xy\left(x^2+y^2-6z^2\right)}{24}$$

$$-\frac{\sqrt{30}Q_u yz\left(3x^2+3y^2-4z^2\right)}{24}+\frac{\sqrt{10}Q_v yz\left(27x^2-y^2-8z^2\right)}{24}-\frac{\sqrt{10}Q_{xy}xz\left(13x^2-15y^2-8z^2\right)}{24}-\frac{\sqrt{10}Q_{xz}xy\left(x^2+y^2-6z^2\right)}{24}+\frac{\sqrt{10}Q_{yz}\left(2x^4+3x^2y^2-15x^2z^2+y^4-9y^2z^2+4z^4\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E,2)$

** symmetry

$$-\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

$$\sqrt{15}xyz$$

** expression

$$-\frac{\sqrt{3}Q_u xy\left(x^2+y^2-6z^2\right)}{3}-\frac{7Q_v xy\left(x-y\right)\left(x+y\right)}{6}+\frac{Q_{xy}\left(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4\right)}{12}+\frac{7Q_{xz}yz\left(3x^2+y^2-2z^2\right)}{12}+\frac{7Q_{yz}xz\left(x^2+3y^2-2z^2\right)}{12}$$

$$-\frac{\sqrt{3}Q_u\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{6}-\frac{Q_v\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{6}-\frac{7Q_{xy}xy\left(x-y\right)\left(x+y\right)}{6}+\frac{7Q_{xz}xz\left(x-z\right)\left(x+z\right)}{6}-\frac{7Q_{yz}yz\left(y-z\right)\left(y+z\right)}{6}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{2}Q_u y\left(3x^2-y^2\right)}{8}-\frac{\sqrt{6}Q_v y\left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{6}Q_{xy}x\left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{6}Q_{xz}xyz}{4}-\frac{\sqrt{6}Q_{yz}z\left(x-y\right)\left(x+y\right)}{8}$$

$\vec{\mathbb{G}}_{4}^{(2,1)}[q](A_1)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{77}Q_u y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{88}+\frac{\sqrt{231}Q_v y\left(7x^4-16x^2y^2+6x^2z^2+y^4+6y^2z^2-4z^4\right)}{88}-\frac{\sqrt{231}Q_{xy}x\left(x^4-7x^2y^2-3x^2z^2+4y^4-3y^2z^2+2z^4\right)}{44}-\frac{\sqrt{231}Q_{xz}xyz\left(11x^2-y^2-10z^2\right)}{44}-\frac{\sqrt{231}Q_{yz}z\left(x^4+9x^2y^2-5x^2z^2-4y^4+5y^2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_2,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{\sqrt{105}Q_v xyz}{7}+\frac{\sqrt{105}Q_{xy}z\left(x-y\right)\left(x+y\right)}{14}-\frac{\sqrt{105}Q_{xz}y\left(x^2+y^2-4z^2\right)}{28}+\frac{\sqrt{105}Q_{yz}x\left(x^2+y^2-4z^2\right)}{28}$$

$\vec{\mathbb{G}}_{4}^{(2,-1)}[q](A_2,2)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{2}Q_u x\left(x^2-3y^2\right)}{8}+\frac{\sqrt{6}Q_v x\left(x^2+y^2-4z^2\right)}{8}-\frac{\sqrt{6}Q_{xy}y\left(x^2+y^2-4z^2\right)}{8}+\frac{\sqrt{6}Q_{xz}z\left(x-y\right)\left(x+y\right)}{8}-\frac{\sqrt{6}Q_{yz}xyz}{4}$$

$\vec{\mathbb{G}}_{4}^{(2,1)}[q](A_2,1)$

** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$-\frac{7\sqrt{330}Q_{v}xyz\left(x^{2}+y^{2}-2z^{2}\right)}{44}+\frac{7\sqrt{330}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-2z^{2}\right)}{88}$$
$$+\frac{\sqrt{330}Q_{xz}y\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}-\frac{\sqrt{330}Q_{yz}x\left(x^{4}+2x^{2}y^{2}-12x^{2}z^{2}+y^{4}-12y^{2}z^{2}+8z^{4}\right)}{88}$$

$\vec{\mathbb{G}}_{4}^{(2,1)}[q](A_{2},2)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$-\frac{3\sqrt{77}Q_{u}x\left(x^{2}-3y^{2}\right)\left(x^{2}+y^{2}-8z^{2}\right)}{88}-\frac{\sqrt{231}Q_{v}x\left(x^{4}-16x^{2}y^{2}+6x^{2}z^{2}+7y^{4}+6y^{2}z^{2}-4z^{4}\right)}{88}$$
$$-\frac{\sqrt{231}Q_{xy}y\left(4x^{4}-7x^{2}y^{2}-3x^{2}z^{2}+y^{4}-3y^{2}z^{2}+2z^{4}\right)}{44}+\frac{\sqrt{231}Q_{xz}z\left(4x^{4}-9x^{2}y^{2}-5x^{2}z^{2}-y^{4}+5y^{2}z^{2}\right)}{44}+\frac{\sqrt{231}Q_{yz}xyz\left(x^{2}-11y^{2}+10z^{2}\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

** expression

$$-\frac{3\sqrt{14}Q_{u}x\left(x^{2}+y^{2}-4z^{2}\right)}{56}-\frac{\sqrt{42}Q_{v}x\left(x^{2}+5y^{2}-8z^{2}\right)}{56}+\frac{\sqrt{42}Q_{xy}y\left(x^{2}-3y^{2}+8z^{2}\right)}{56}-\frac{\sqrt{42}Q_{xz}z\left(x^{2}-13y^{2}+4z^{2}\right)}{56}-\frac{\sqrt{42}Q_{yz}xyz}{4}$$

$$-\frac{3\sqrt{14}Q_{u}y\left(x^{2}+y^{2}-4z^{2}\right)}{56}+\frac{\sqrt{42}Q_{v}y\left(5x^{2}+y^{2}-8z^{2}\right)}{56}-\frac{\sqrt{42}Q_{xy}x\left(3x^{2}-y^{2}-8z^{2}\right)}{56}-\frac{\sqrt{42}Q_{xz}xyz}{4}+\frac{\sqrt{42}Q_{yz}z\left(13x^{2}-y^{2}-4z^{2}\right)}{56}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^{2}-2xy-y^{2}\right)\left(x^{2}+2xy-y^{2}\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\sqrt{3}Q_{v}xyz+\frac{\sqrt{3}Q_{xy}z\left(x-y\right)\left(x+y\right)}{2}-\frac{\sqrt{3}Q_{xz}y\left(3x^{2}-y^{2}\right)}{4}-\frac{\sqrt{3}Q_{yz}x\left(x^{2}-3y^{2}\right)}{4}$$

$$-\frac{\sqrt{3}Q_{v}z\left(x-y\right)\left(x+y\right)}{2}+\sqrt{3}Q_{xy}xyz+\frac{\sqrt{3}Q_{xz}x\left(x^{2}-3y^{2}\right)}{4}-\frac{\sqrt{3}Q_{yz}y\left(3x^{2}-y^{2}\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^{2}+y^{2}-6z^{2}\right)}{2}$$

** expression

$$-\frac{6\sqrt{7}Q_{u}xyz}{7}-\frac{\sqrt{21}Q_{xy}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}+\frac{\sqrt{21}Q_{xz}y\left(2x^{2}-y^{2}+z^{2}\right)}{14}-\frac{\sqrt{21}Q_{yz}x\left(x^{2}-2y^{2}-z^{2}\right)}{14}$$

$$-\frac{3\sqrt{7}Q_{u}z\left(x-y\right)\left(x+y\right)}{7}-\frac{\sqrt{21}Q_{v}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}+\frac{\sqrt{21}Q_{xz}x\left(x^{2}-5y^{2}+2z^{2}\right)}{28}+\frac{\sqrt{21}Q_{yz}y\left(5x^{2}-y^{2}-2z^{2}\right)}{28}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$\frac{3\sqrt{11}Q_u x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}+\frac{\sqrt{33}Q_v x\left(x^4-12x^2y^2+2x^2z^2-13y^4+114y^2z^2-20z^4\right)}{88}+\frac{\sqrt{33}Q_{xy}y\left(4x^4+x^2y^2-27x^2z^2-3y^4+29y^2z^2-10z^4\right)}{44}-\frac{\sqrt{33}Q_{xz}z\left(4x^4+15x^2y^2-13x^2z^2+11y^4-27y^2z^2+4z^4\right)}{44}+\frac{7\sqrt{33}Q_{yz}xyz\left(x^2+y^2-2z^2\right)}{44}$$

$$\frac{3\sqrt{11}Q_u y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}+\frac{\sqrt{33}Q_v y\left(13x^4+12x^2y^2-114x^2z^2-y^4-2y^2z^2+20z^4\right)}{88}-\frac{\sqrt{33}Q_{xy}x\left(3x^4-x^2y^2-29x^2z^2-4y^4+27y^2z^2+10z^4\right)}{44}+\frac{7\sqrt{33}Q_{xz}xyz\left(x^2+y^2-2z^2\right)}{44}-\frac{\sqrt{33}Q_{yz}z\left(11x^4+15x^2y^2-27x^2z^2+4y^4-13y^2z^2+4z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{9\sqrt{154}Q_u xyz\left(x-y\right)\left(x+y\right)}{22}+\frac{\sqrt{462}Q_v xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{462}Q_{xy}z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}-\frac{\sqrt{462}Q_{xz}y\left(9x^4-14x^2y^2-12x^2z^2+y^4+4y^2z^2\right)}{88}+\frac{\sqrt{462}Q_{yz}x\left(x^4-14x^2y^2+4x^2z^2+9y^4-12y^2z^2\right)}{88}$$

$$\frac{9\sqrt{154}Q_u z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}-\frac{\sqrt{462}Q_v z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{462}Q_{xy}xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{462}Q_{xz}x\left(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2\right)}{44}+\frac{\sqrt{462}Q_{yz}y\left(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E,3)$, $\vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E,3)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{21\sqrt{22}Q_u xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{21\sqrt{66}Q_v xyz\left(x-y\right)\left(x+y\right)}{44}-\frac{\sqrt{66}Q_{xy}z\left(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4\right)}{44}+\frac{\sqrt{66}Q_{xz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}+\frac{\sqrt{66}Q_{yz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$$\frac{21\sqrt{22}Q_u z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{88}+\frac{\sqrt{66}Q_v z\left(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4\right)}{88}+\frac{21\sqrt{66}Q_{xy}xyz\left(x-y\right)\left(x+y\right)}{44}+\frac{\sqrt{66}Q_{xz}x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}-\frac{\sqrt{66}Q_{yz}y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$