PG No. 17 $C_{3\mathrm{i}}$ $\bar{3}$ [trigonal] (axial, internal axial dipole)

* Harmonics for rank 0

$\vec{\mathbb{G}}_{0}^{(1,1)}[g](A_u)$

** symmetry

$$1$$

** expression

$$\frac{\sqrt{3}G_{x}x}{3} + \frac{\sqrt{3}G_{y}y}{3} + \frac{\sqrt{3}G_{z}z}{3}$$

* Harmonics for rank 1

$\vec{\mathbb{G}}_{1}^{(1,-1)}[g](A_g)$

** symmetry

$$z$$

** expression

$$G_{z}$$

$\vec{\mathbb{G}}_{1}^{(1,1)}[g](A_g)$

** symmetry

$$z$$

** expression

$$\frac{3\sqrt{10}G_{x}xz}{10} + \frac{3\sqrt{10}G_{y}yz}{10} - \frac{\sqrt{10}G_{z}\left(x^{2} + y^{2} - 2z^{2}\right)}{10}$$

$\vec{\mathbb{G}}_{1,1}^{(1,-1)}[g](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(1,-1)}[g](E_g)$

** symmetry

$$x$$

$$y$$

** expression

$$G_{x}$$

$$G_{y}$$

$\vec{\mathbb{G}}_{1,1}^{(1,1)}[g](E_g)$, $\vec{\mathbb{G}}_{1,2}^{(1,1)}[g](E_g)$

** symmetry

$$x$$

$$y$$

** expression

$$\frac{\sqrt{10}G_{x}\left(2x^{2} - y^{2} - z^{2}\right)}{10} + \frac{3\sqrt{10}G_{y}xy}{10} + \frac{3\sqrt{10}G_{z}xz}{10}$$

$$\frac{3\sqrt{10}G_{x}xy}{10} - \frac{\sqrt{10}G_{y}\left(x^{2} - 2y^{2} + z^{2}\right)}{10} + \frac{3\sqrt{10}G_{z}yz}{10}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_{2}^{(1,-1)}[g](A_u)$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$- \frac{\sqrt{6}G_{x}x}{6} - \frac{\sqrt{6}G_{y}y}{6} + \frac{\sqrt{6}G_{z}z}{3}$$

$\vec{\mathbb{G}}_{2}^{(1,1)}[g](A_u)$

** symmetry

$$- \frac{x^{2}}{2} - \frac{y^{2}}{2} + z^{2}$$

** expression

$$-\frac{\sqrt{21}G_{x}x\left(x^{2}+y^{2}-4z^{2}\right)}{14}-\frac{\sqrt{21}G_{y}y\left(x^{2}+y^{2}-4z^{2}\right)}{14}-\frac{\sqrt{21}G_{z}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{14}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_u,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{\sqrt{2}G_{y}z}{2}+\frac{\sqrt{2}G_{z}y}{2}$$

$$-\frac{\sqrt{2}G_{x}z}{2}-\frac{\sqrt{2}G_{z}x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,-1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{2,2}^{(1,-1)}[g](E_u,2)$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{2}G_{x}x}{2}-\frac{\sqrt{2}G_{y}y}{2}$$

$$-\frac{\sqrt{2}G_{x}y}{2}-\frac{\sqrt{2}G_{y}x}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_u,1)$

** symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

** expression

$$\frac{5\sqrt{7}G_{x}xyz}{7}-\frac{\sqrt{7}G_{y}z\left(x^{2}-4y^{2}+z^{2}\right)}{7}-\frac{\sqrt{7}G_{z}y\left(x^{2}+y^{2}-4z^{2}\right)}{7}$$

$$-\frac{\sqrt{7}G_{x}z\left(4x^{2}-y^{2}-z^{2}\right)}{7}-\frac{5\sqrt{7}G_{y}xyz}{7}+\frac{\sqrt{7}G_{z}x\left(x^{2}+y^{2}-4z^{2}\right)}{7}$$

$\vec{\mathbb{G}}_{2,1}^{(1,1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{2,2}^{(1,1)}[g](E_u,2)$

** symmetry

$$\frac{\sqrt{3}\left(x-y\right)\left(x+y\right)}{2}$$

$$-\sqrt{3}xy$$

** expression

$$\frac{\sqrt{7}G_{x}x\left(3x^{2}-7y^{2}-2z^{2}\right)}{14}+\frac{\sqrt{7}G_{y}y\left(7x^{2}-3y^{2}+2z^{2}\right)}{14}+\frac{5\sqrt{7}G_{z}z\left(x-y\right)\left(x+y\right)}{14}$$

$$-\frac{\sqrt{7}G_{x}y\left(4x^{2}-y^{2}-z^{2}\right)}{7}+\frac{\sqrt{7}G_{y}x\left(x^{2}-4y^{2}+z^{2}\right)}{7}-\frac{5\sqrt{7}G_{z}xyz}{7}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_{3}^{(1,-1)}[g](A_g,1)$

** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{\sqrt{15}G_{x}xz}{5}-\frac{\sqrt{15}G_{y}yz}{5}-\frac{\sqrt{15}G_{z}\left(x^{2}+y^{2}-2z^{2}\right)}{10}$$

$\vec{\mathbb{G}}_{3}^{(1,-1)}[g](A_{g},2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{6}G_{x}xy}{2}+\frac{\sqrt{6}G_{y}\left(x-y\right)\left(x+y\right)}{4}$$

$\vec{\mathbb{G}}_{3}^{(1,-1)}[g](A_{g},3)$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{6}G_{x}\left(x-y\right)\left(x+y\right)}{4}-\frac{\sqrt{6}G_{y}xy}{2}$$

$\vec{\mathbb{G}}_{3}^{(1,1)}[g](A_{g},1)$
** symmetry

$$-\frac{z\left(3x^{2}+3y^{2}-2z^{2}\right)}{2}$$

** expression

$$-\frac{5G_{x}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}-\frac{5G_{y}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{12}+\frac{G_{z}\left(3x^{4}+6x^{2}y^{2}-24x^{2}z^{2}+3y^{4}-24y^{2}z^{2}+8z^{4}\right)}{12}$$

$\vec{\mathbb{G}}_{3}^{(1,1)}[g](A_{g},2)$
** symmetry

$$\frac{\sqrt{10}y\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{10}G_{x}xy\left(15x^{2}-13y^{2}-6z^{2}\right)}{24}-\frac{\sqrt{10}G_{y}\left(3x^{4}-21x^{2}y^{2}+3x^{2}z^{2}+4y^{4}-3y^{2}z^{2}\right)}{24}+\frac{7\sqrt{10}G_{z}yz\left(3x^{2}-y^{2}\right)}{24}$$

$\vec{\mathbb{G}}_{3}^{(1,1)}[g](A_{g},3)$
** symmetry

$$\frac{\sqrt{10}x\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{\sqrt{10}G_{x}\left(4x^{4}-21x^{2}y^{2}-3x^{2}z^{2}+3y^{4}+3y^{2}z^{2}\right)}{24}+\frac{\sqrt{10}G_{y}xy\left(13x^{2}-15y^{2}+6z^{2}\right)}{24}+\frac{7\sqrt{10}G_{z}xz\left(x^{2}-3y^{2}\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_{g},1)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_{g},1)$
** symmetry

$$-\frac{\sqrt{6}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{10}G_{x}\left(3x^{2}+y^{2}-4z^{2}\right)}{20}-\frac{\sqrt{10}G_{y}xy}{10}+\frac{2\sqrt{10}G_{z}xz}{5}$$

$$-\frac{\sqrt{10}G_{x}xy}{10}-\frac{\sqrt{10}G_{y}\left(x^{2}+3y^{2}-4z^{2}\right)}{20}+\frac{2\sqrt{10}G_{z}yz}{5}$$

$\vec{\mathbb{G}}_{3,1}^{(1,-1)}[g](E_{g},2)$, $\vec{\mathbb{G}}_{3,2}^{(1,-1)}[g](E_{g},2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$G_{x}yz+G_{y}xz+G_{z}xy$$

$$G_{x}xz-G_{y}yz+\frac{G_{z}\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_g,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_g,1)$

** symmetry

$$-\frac{\sqrt{6}x\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

$$-\frac{\sqrt{6}y\left(x^{2}+y^{2}-4z^{2}\right)}{4}$$

** expression

$$-\frac{\sqrt{6}G_{x}\left(4x^{4}+3x^{2}y^{2}-27x^{2}z^{2}-y^{4}+3y^{2}z^{2}+4z^{4}\right)}{24}-\frac{5\sqrt{6}G_{y}xy\left(x^{2}+y^{2}-6z^{2}\right)}{24}-\frac{5\sqrt{6}G_{z}xz\left(3x^{2}+3y^{2}-4z^{2}\right)}{24}$$

$$-\frac{5\sqrt{6}G_{x}xy\left(x^{2}+y^{2}-6z^{2}\right)}{24}+\frac{\sqrt{6}G_{y}\left(x^{4}-3x^{2}y^{2}-3x^{2}z^{2}-4y^{4}+27y^{2}z^{2}-4z^{4}\right)}{24}-\frac{5\sqrt{6}G_{z}yz\left(3x^{2}+3y^{2}-4z^{2}\right)}{24}$$

$\vec{\mathbb{G}}_{3,1}^{(1,1)}[g](E_g,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,1)}[g](E_g,2)$

** symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{15}G_{x}yz\left(6x^{2}-y^{2}-z^{2}\right)}{6}-\frac{\sqrt{15}G_{y}xz\left(x^{2}-6y^{2}+z^{2}\right)}{6}-\frac{\sqrt{15}G_{z}xy\left(x^{2}+y^{2}-6z^{2}\right)}{6}$$

$$\frac{\sqrt{15}G_{x}xz\left(5x^{2}-9y^{2}-2z^{2}\right)}{12}+\frac{\sqrt{15}G_{y}yz\left(9x^{2}-5y^{2}+2z^{2}\right)}{12}-\frac{\sqrt{15}G_{z}\left(x-y\right)\left(x+y\right)\left(x^{2}+y^{2}-6z^{2}\right)}{12}$$

* Harmonics for rank 4

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_u,1)$

** symmetry

$$\frac{3x^{4}}{8}+\frac{3x^{2}y^{2}}{4}-3x^{2}z^{2}+\frac{3y^{4}}{8}-3y^{2}z^{2}+z^{4}$$

** expression

$$\frac{3\sqrt{7}G_{x}x\left(x^{2}+y^{2}-4z^{2}\right)}{28}+\frac{3\sqrt{7}G_{y}y\left(x^{2}+y^{2}-4z^{2}\right)}{28}-\frac{\sqrt{7}G_{z}z\left(3x^{2}+3y^{2}-2z^{2}\right)}{7}$$

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_u,2)$

** symmetry

$$\frac{\sqrt{70}xz\left(x^{2}-3y^{2}\right)}{4}$$

** expression

$$\frac{3\sqrt{10}G_{x}z\left(x-y\right)\left(x+y\right)}{8}-\frac{3\sqrt{10}G_{y}xyz}{4}+\frac{\sqrt{10}G_{z}x\left(x^{2}-3y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_{4}^{(1,-1)}[g](A_u,3)$

** symmetry

$$\frac{\sqrt{70}yz\left(3x^{2}-y^{2}\right)}{4}$$

** expression

$$\frac{3\sqrt{10}G_{x}xyz}{4}+\frac{3\sqrt{10}G_{y}z\left(x-y\right)\left(x+y\right)}{8}+\frac{\sqrt{10}G_{z}y\left(3x^{2}-y^{2}\right)}{8}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,1)$
** symmetry

$$\frac{3x^4}{8}+\frac{3x^2y^2}{4}-3x^2z^2+\frac{3y^4}{8}-3y^2z^2+z^4$$

** expression

$$\frac{3\sqrt{55}G_x x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}+\frac{3\sqrt{55}G_y y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{88}+\frac{\sqrt{55}G_z z\left(15x^4+30x^2y^2-40x^2z^2+15y^4-40y^2z^2+8z^4\right)}{88}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,2)$
** symmetry

$$\frac{\sqrt{70}xz\left(x^2-3y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{154}G_x z\left(2x^4-9x^2y^2-x^2z^2+y^4+y^2z^2\right)}{44}+\frac{3\sqrt{154}G_y xyz\left(5x^2-7y^2+2z^2\right)}{44}-\frac{\sqrt{154}G_z x\left(x^2-3y^2\right)\left(x^2+y^2-8z^2\right)}{44}$$

$\vec{\mathbb{G}}_4^{(1,1)}[g](A_u,3)$
** symmetry

$$\frac{\sqrt{70}yz\left(3x^2-y^2\right)}{4}$$

** expression

$$\frac{3\sqrt{154}G_x xyz\left(7x^2-5y^2-2z^2\right)}{44}-\frac{3\sqrt{154}G_y z\left(x^4-9x^2y^2+x^2z^2+2y^4-y^2z^2\right)}{44}-\frac{\sqrt{154}G_z y\left(3x^2-y^2\right)\left(x^2+y^2-8z^2\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u,1)$
** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{3\sqrt{70}G_x xyz}{28}-\frac{\sqrt{70}G_y z\left(3x^2+9y^2-4z^2\right)}{56}-\frac{3\sqrt{70}G_z y\left(x^2+y^2-4z^2\right)}{56}$$

$$\frac{\sqrt{70}G_x z\left(9x^2+3y^2-4z^2\right)}{56}+\frac{3\sqrt{70}G_y xyz}{28}+\frac{3\sqrt{70}G_z x\left(x^2+y^2-4z^2\right)}{56}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u,2)$
** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}G_x x\left(x^2-3y^2\right)}{4}-\frac{\sqrt{5}G_y y\left(3x^2-y^2\right)}{4}$$

$$\frac{\sqrt{5}G_x y\left(3x^2-y^2\right)}{4}+\frac{\sqrt{5}G_y x\left(x^2-3y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,-1)}[g](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,-1)}[g](E_u,3)$
** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{35}G_x x\left(x^2-3z^2\right)}{14}+\frac{\sqrt{35}G_y y\left(y^2-3z^2\right)}{14}+\frac{3\sqrt{35}G_z z\left(x-y\right)\left(x+y\right)}{14}$$

$$\frac{\sqrt{35}G_x y\left(3x^2+y^2-6z^2\right)}{28}+\frac{\sqrt{35}G_y x\left(x^2+3y^2-6z^2\right)}{28}-\frac{3\sqrt{35}G_z xyz}{7}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u,1)$

** symmetry

$$-\frac{\sqrt{10}yz\left(3x^2+3y^2-4z^2\right)}{4}$$

$$\frac{\sqrt{10}xz\left(3x^2+3y^2-4z^2\right)}{4}$$

** expression

$$-\frac{21\sqrt{22}G_x xyz\left(x^2+y^2-2z^2\right)}{44}+\frac{\sqrt{22}G_y z\left(3x^4-15x^2y^2-x^2z^2-18y^4+41y^2z^2-4z^4\right)}{44}+\frac{3\sqrt{22}G_z y\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$$\frac{\sqrt{22}G_x z\left(18x^4+15x^2y^2-41x^2z^2-3y^4+y^2z^2+4z^4\right)}{44}+\frac{21\sqrt{22}G_y xyz\left(x^2+y^2-2z^2\right)}{44}-\frac{3\sqrt{22}G_z x\left(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4\right)}{44}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u,2)$

** symmetry

$$\frac{\sqrt{35}\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{8}$$

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{77}G_x x\left(5x^4-46x^2y^2-4x^2z^2+21y^4+12y^2z^2\right)}{88}+\frac{\sqrt{77}G_y y\left(21x^4-46x^2y^2+12x^2z^2+5y^4-4y^2z^2\right)}{88}+\frac{9\sqrt{77}G_z z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{88}$$

$$\frac{\sqrt{77}G_x y\left(6x^4-11x^2y^2-3x^2z^2+y^4+y^2z^2\right)}{22}-\frac{\sqrt{77}G_y x\left(x^4-11x^2y^2+x^2z^2+6y^4-3y^2z^2\right)}{22}+\frac{9\sqrt{77}G_z xyz\left(x-y\right)\left(x+y\right)}{22}$$

$\vec{\mathbb{G}}_{4,1}^{(1,1)}[g](E_u,3)$, $\vec{\mathbb{G}}_{4,2}^{(1,1)}[g](E_u,3)$

** symmetry

$$-\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$$\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{11}G_x x\left(5x^4-4x^2y^2-46x^2z^2-9y^4+66y^2z^2+12z^4\right)}{44}$$
$$-\frac{\sqrt{11}G_y y\left(9x^4+4x^2y^2-66x^2z^2-5y^4+46y^2z^2-12z^4\right)}{44}-\frac{21\sqrt{11}G_z z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-2z^2\right)}{44}$$

$$\frac{\sqrt{11}G_x y\left(6x^4+5x^2y^2-51x^2z^2-y^4+5y^2z^2+6z^4\right)}{22}-\frac{\sqrt{11}G_y x\left(x^4-5x^2y^2-5x^2z^2-6y^4+51y^2z^2-6z^4\right)}{22}+\frac{21\sqrt{11}G_z xyz\left(x^2+y^2-2z^2\right)}{22}$$