PG No. 10 S_4 $\bar{4}$ [tetragonal] (axial, internal polar dipole)

* Harmonics for rank 0
* Harmonics for rank 1

$\vec{\mathbb{G}}_1^{(1,0)}[q](A)$

** symmetry

z

** expression

$$\frac{\sqrt{2}Q_x y}{2} - \frac{\sqrt{2}Q_y x}{2}$$

$\vec{\mathbb{G}}_{1,1}^{(1,0)}[q](E)$, $\vec{\mathbb{G}}_{1,2}^{(1,0)}[q](E)$

** symmetry

y

x

** expression

$$-\frac{\sqrt{2}Q_x z}{2} + \frac{\sqrt{2}Q_z x}{2}$$

$$\frac{\sqrt{2}Q_y z}{2} - \frac{\sqrt{2}Q_z y}{2}$$

* Harmonics for rank 2

$\vec{\mathbb{G}}_2^{(1,0)}[q](A,1)$

** symmetry

$$\frac{\sqrt{3}\,(x-y)\,(x+y)}{2}$$

** expression

$$\frac{\sqrt{2}Q_x yz}{2} + \frac{\sqrt{2}Q_y xz}{2} - \sqrt{2}Q_z xy$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](A,2)$

** symmetry

$$\sqrt{3}xy$$

** expression

$$-\frac{\sqrt{2}Q_x xz}{2} + \frac{\sqrt{2}Q_y yz}{2} + \frac{\sqrt{2}Q_z\,(x-y)\,(x+y)}{2}$$

$\vec{\mathbb{G}}_2^{(1,0)}[q](B)$

** symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

** expression

$$\frac{\sqrt{6}Q_x yz}{2} - \frac{\sqrt{6}Q_y xz}{2}$$

$\vec{\mathbb{G}}_{2,1}^{(1,0)}[q](E)$, $\vec{\mathbb{G}}_{2,2}^{(1,0)}[q](E)$

** symmetry

$$\sqrt{3}xz$$

$$\sqrt{3}yz$$

** expression

$$\frac{\sqrt{2}Q_x xy}{2} - \frac{\sqrt{2}Q_y\,(x-z)\,(x+z)}{2} - \frac{\sqrt{2}Q_z yz}{2}$$

$$\frac{\sqrt{2}Q_x\,(y-z)\,(y+z)}{2} - \frac{\sqrt{2}Q_y xy}{2} + \frac{\sqrt{2}Q_z xz}{2}$$

* Harmonics for rank 3

$\vec{\mathbb{G}}_3^{(1,0)}[q](A)$

** symmetry

$$-\frac{z\left(3x^2+3y^2-2z^2\right)}{2}$$

** expression

$$-\frac{\sqrt{3}Q_x y\left(x^2+y^2-4z^2\right)}{4}+\frac{\sqrt{3}Q_y x\left(x^2+y^2-4z^2\right)}{4}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,1)$
** symmetry

$$\sqrt{15}xyz$$

** expression

$$\frac{\sqrt{5}Q_x x\left(y-z\right)\left(y+z\right)}{2}-\frac{\sqrt{5}Q_y y\left(x-z\right)\left(x+z\right)}{2}+\frac{\sqrt{5}Q_z z\left(x-y\right)\left(x+y\right)}{2}$$

$\vec{\mathbb{G}}_3^{(1,0)}[q](B,2)$
** symmetry

$$\frac{\sqrt{15}z\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$\frac{\sqrt{5}Q_x y\left(x^2-y^2+2z^2\right)}{4}-\frac{\sqrt{5}Q_y x\left(x^2-y^2-2z^2\right)}{4}-\sqrt{5}Q_z xyz$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E,1)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E,1)$
** symmetry

$$-\frac{y\left(3x^2-2y^2+3z^2\right)}{2}$$

$$\frac{x\left(2x^2-3y^2-3z^2\right)}{2}$$

** expression

$$\frac{\sqrt{3}Q_x z\left(x^2-4y^2+z^2\right)}{4}-\frac{\sqrt{3}Q_z x\left(x^2-4y^2+z^2\right)}{4}$$

$$\frac{\sqrt{3}Q_y z\left(4x^2-y^2-z^2\right)}{4}-\frac{\sqrt{3}Q_z y\left(4x^2-y^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_{3,1}^{(1,0)}[q](E,2)$, $\vec{\mathbb{G}}_{3,2}^{(1,0)}[q](E,2)$
** symmetry

$$\frac{\sqrt{15}x\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{15}y\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$-\sqrt{5}Q_x xyz+\frac{\sqrt{5}Q_y z\left(2x^2+y^2-z^2\right)}{4}+\frac{\sqrt{5}Q_z y\left(2x^2-y^2+z^2\right)}{4}$$

$$\frac{\sqrt{5}Q_x z\left(x^2+2y^2-z^2\right)}{4}-\sqrt{5}Q_y xyz-\frac{\sqrt{5}Q_z x\left(x^2-2y^2-z^2\right)}{4}$$

* Harmonics for rank 4
$\vec{\mathbb{G}}_4^{(1,0)}[q](A,1)$
** symmetry

$$\frac{\sqrt{5}\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

** expression

$$-\frac{Q_x yz\left(3x^2-4y^2+3z^2\right)}{2}+\frac{Q_y xz\left(4x^2-3y^2-3z^2\right)}{2}-\frac{Q_z xy\left(x^2+y^2-6z^2\right)}{2}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](A,2)$

** symmetry

$$-\frac{\sqrt{5}xy\left(x^2+y^2-6z^2\right)}{2}$$

** expression

$$\frac{Q_x xz\left(x^2+15y^2-6z^2\right)}{4}-\frac{Q_y yz\left(15x^2+y^2-6z^2\right)}{4}-\frac{Q_z\left(x-y\right)\left(x+y\right)\left(x^2+y^2-6z^2\right)}{4}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,1)$
** symmetry

$$\frac{\sqrt{21}\left(x^4-3x^2y^2-3x^2z^2+y^4-3y^2z^2+z^4\right)}{6}$$

** expression

$$-\frac{\sqrt{105}Q_x yz\left(y-z\right)\left(y+z\right)}{6}+\frac{\sqrt{105}Q_y xz\left(x-z\right)\left(x+z\right)}{6}-\frac{\sqrt{105}Q_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,2)$
** symmetry

$$-\frac{\sqrt{15}\left(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4\right)}{12}$$

** expression

$$-\frac{\sqrt{3}Q_x yz\left(9x^2+2y^2-5z^2\right)}{6}+\frac{\sqrt{3}Q_y xz\left(2x^2+9y^2-5z^2\right)}{6}+\frac{7\sqrt{3}Q_z xy\left(x-y\right)\left(x+y\right)}{6}$$

$\vec{\mathbb{G}}_4^{(1,0)}[q](B,3)$
** symmetry

$$\frac{\sqrt{35}xy\left(x-y\right)\left(x+y\right)}{2}$$

** expression

$$-\frac{\sqrt{7}Q_x xz\left(x^2-3y^2\right)}{4}+\frac{\sqrt{7}Q_y yz\left(3x^2-y^2\right)}{4}+\frac{\sqrt{7}Q_z\left(x^2-2xy-y^2\right)\left(x^2+2xy-y^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E,1)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E,1)$
** symmetry

$$\frac{\sqrt{35}yz\left(y-z\right)\left(y+z\right)}{2}$$

$$-\frac{\sqrt{35}xz\left(x-z\right)\left(x+z\right)}{2}$$

** expression

$$\frac{\sqrt{7}Q_x\left(y^2-2yz-z^2\right)\left(y^2+2yz-z^2\right)}{4}-\frac{\sqrt{7}Q_y xy\left(y^2-3z^2\right)}{4}+\frac{\sqrt{7}Q_z xz\left(3y^2-z^2\right)}{4}$$

$$-\frac{\sqrt{7}Q_x xy\left(x^2-3z^2\right)}{4}+\frac{\sqrt{7}Q_y\left(x^2-2xz-z^2\right)\left(x^2+2xz-z^2\right)}{4}+\frac{\sqrt{7}Q_z yz\left(3x^2-z^2\right)}{4}$$

$\vec{\mathbb{G}}_{4,1}^{(1,0)}[q](E,2)$, $\vec{\mathbb{G}}_{4,2}^{(1,0)}[q](E,2)$
** symmetry

$$-\frac{\sqrt{5}xz\left(x^2-6y^2+z^2\right)}{2}$$

$$\frac{\sqrt{5}yz\left(6x^2-y^2-z^2\right)}{2}$$

** expression

$$-\frac{Q_x xy\left(x^2-6y^2+15z^2\right)}{4}+\frac{Q_y\left(x-z\right)\left(x+z\right)\left(x^2-6y^2+z^2\right)}{4}+\frac{Q_z yz\left(15x^2-6y^2+z^2\right)}{4}$$

$$\frac{Q_x\left(y-z\right)\left(y+z\right)\left(6x^2-y^2-z^2\right)}{4}-\frac{Q_y xy\left(6x^2-y^2-15z^2\right)}{4}+\frac{Q_z xz\left(6x^2-15y^2-z^2\right)}{4}$$