

PG No. 37  $D_{3h}(1)$   $\bar{6}m2$  (-62m setting) [ hexagonal ] (axial, internal polar quadrupole)

\* Harmonics for rank 0

\* Harmonics for rank 1

$$\vec{\mathbb{G}}_1^{(2,1)}[q](A'_2)$$

\*\* symmetry

$z$

\*\* expression

$$\frac{\sqrt{30}Q_vxy}{5} - \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{10} + \frac{\sqrt{30}Q_{xzyz}}{10} - \frac{\sqrt{30}Q_{yzxz}}{10}$$

$$\vec{\mathbb{G}}_{1,1}^{(2,1)}[q](E''), \vec{\mathbb{G}}_{1,2}^{(2,1)}[q](E'')$$

\*\* symmetry

$x$

$y$

\*\* expression

$$-\frac{3\sqrt{10}Q_uyz}{10} - \frac{\sqrt{30}Q_vyz}{10} + \frac{\sqrt{30}Q_{xyxz}}{10} - \frac{\sqrt{30}Q_{xzyz}}{10} - \frac{\sqrt{30}Q_{yz}(y-z)(y+z)}{10}$$

$$\frac{3\sqrt{10}Q_{uxz}}{10} - \frac{\sqrt{30}Q_vxz}{10} - \frac{\sqrt{30}Q_{xyyz}}{10} + \frac{\sqrt{30}Q_{xz}(x-z)(x+z)}{10} + \frac{\sqrt{30}Q_{yzxy}}{10}$$

\* Harmonics for rank 2

$$\vec{\mathbb{G}}_2^{(2,-1)}[q](A'_1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{\sqrt{2}Q_{xzy}}{2} - \frac{\sqrt{2}Q_{yzx}}{2}$$

$$\vec{\mathbb{G}}_2^{(2,1)}[q](A'_1)$$

\*\* symmetry

$$-\frac{x^2}{2} - \frac{y^2}{2} + z^2$$

\*\* expression

$$\frac{5\sqrt{42}Q_vxyz}{14} - \frac{5\sqrt{42}Q_{xyz}(x-y)(x+y)}{28} - \frac{\sqrt{42}Q_{xzy}(x^2+y^2-4z^2)}{28} + \frac{\sqrt{42}Q_{yzx}(x^2+y^2-4z^2)}{28}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E'), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E')$$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$\frac{\sqrt{2}Q_{ux}}{2} + \frac{\sqrt{6}Q_{vx}}{6} + \frac{\sqrt{6}Q_{xyy}}{6} - \frac{\sqrt{6}Q_{xzz}}{6}$$

$$\frac{\sqrt{2}Q_{uy}}{2} - \frac{\sqrt{6}Q_{vy}}{6} + \frac{\sqrt{6}Q_{xyx}}{6} - \frac{\sqrt{6}Q_{yzz}}{6}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E'), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E')$$

\*\* symmetry

$$\sqrt{3}yz$$

$$-\sqrt{3}xz$$

\*\* expression

$$-\frac{\sqrt{42}Q_{ux}(x^2+y^2-4z^2)}{28} - \frac{\sqrt{14}Q_{vx}(x^2-9y^2+6z^2)}{28} - \frac{\sqrt{14}Q_{xyy}(3x^2-2y^2+3z^2)}{14} + \frac{\sqrt{14}Q_{xzz}(3x^2+3y^2-2z^2)}{14}$$

$$-\frac{\sqrt{42}Q_u y (x^2 + y^2 - 4z^2)}{28} - \frac{\sqrt{14}Q_v y (9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}Q_{xy} x (2x^2 - 3y^2 - 3z^2)}{14} + \frac{\sqrt{14}Q_{yz} z (3x^2 + 3y^2 - 2z^2)}{14}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,-1)}[q](E''), \vec{\mathbb{G}}_{2,2}^{(2,-1)}[q](E'')$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$\frac{\sqrt{6}Q_{xy}z}{3} - \frac{\sqrt{6}Q_{xz}y}{6} - \frac{\sqrt{6}Q_{yz}x}{6}$$

$$\frac{\sqrt{6}Q_v z}{3} - \frac{\sqrt{6}Q_{xz}x}{6} + \frac{\sqrt{6}Q_{yz}y}{6}$$

$$\vec{\mathbb{G}}_{2,1}^{(2,1)}[q](E''), \vec{\mathbb{G}}_{2,2}^{(2,1)}[q](E'')$$

\*\* symmetry

$$\frac{\sqrt{3}(x-y)(x+y)}{2}$$

$$-\sqrt{3}xy$$

\*\* expression

$$-\frac{5\sqrt{42}Q_u xyz}{14} + \frac{\sqrt{14}Q_{xy} z (3x^2 + 3y^2 - 2z^2)}{28} - \frac{\sqrt{14}Q_{xz} y (9x^2 - y^2 - 6z^2)}{28} + \frac{\sqrt{14}Q_{yz} x (x^2 - 9y^2 + 6z^2)}{28}$$

$$-\frac{5\sqrt{42}Q_u z (x-y)(x+y)}{28} + \frac{\sqrt{14}Q_v z (3x^2 + 3y^2 - 2z^2)}{28} - \frac{\sqrt{14}Q_{xz} x (2x^2 - 3y^2 - 3z^2)}{14} - \frac{\sqrt{14}Q_{yz} y (3x^2 - 2y^2 + 3z^2)}{14}$$

\* Harmonics for rank 3

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](A'_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{\sqrt{3}Q_v yz}{2} + \frac{\sqrt{3}Q_{xy}xz}{2} - \frac{\sqrt{3}Q_{xz}xy}{2} - \frac{\sqrt{3}Q_{yz}(x-y)(x+y)}{4}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A'_1)$$

\*\* symmetry

$$\frac{\sqrt{10}x(x^2 - 3y^2)}{4}$$

\*\* expression

$$-\frac{7\sqrt{2}Q_u yz (3x^2 - y^2)}{8} + \frac{\sqrt{6}Q_v yz (3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{6}Q_{xy}xz (3x^2 + 3y^2 - 4z^2)}{24}$$

$$-\frac{\sqrt{6}Q_{xz}xy (17x^2 - 11y^2 - 18z^2)}{24} + \frac{\sqrt{6}Q_{yz} (2x^4 - 21x^2y^2 + 9x^2z^2 + 5y^4 - 9y^2z^2)}{24}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](A'_2)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{30}Q_v xy}{10} + \frac{\sqrt{30}Q_{xy}(x-y)(x+y)}{20} + \frac{\sqrt{30}Q_{xz}yz}{5} - \frac{\sqrt{30}Q_{yz}xz}{5}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A'_2)$$

\*\* symmetry

$$-\frac{z(3x^2 + 3y^2 - 2z^2)}{2}$$

\*\* expression

$$-\frac{\sqrt{15}Q_vxy(x^2+y^2-6z^2)}{6} + \frac{\sqrt{15}Q_{xy}(x-y)(x+y)(x^2+y^2-6z^2)}{12} - \frac{\sqrt{15}Q_{xz}yz(3x^2+3y^2-4z^2)}{12} + \frac{\sqrt{15}Q_{yz}xz(3x^2+3y^2-4z^2)}{12}$$

$$\vec{\mathbb{G}}_3^{(2,-1)}[q](A_2'')$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2-y^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{3}Q_vxz}{2} + \frac{\sqrt{3}Q_{xy}yz}{2} + \frac{\sqrt{3}Q_{xz}(x-y)(x+y)}{4} - \frac{\sqrt{3}Q_{yz}xy}{2}$$

$$\vec{\mathbb{G}}_3^{(2,1)}[q](A_2'')$$

\*\* symmetry

$$\frac{\sqrt{10}y(3x^2-y^2)}{4}$$

\*\* expression

$$\frac{7\sqrt{2}Q_{u,xz}(x^2-3y^2)}{8} - \frac{\sqrt{6}Q_{v,xz}(3x^2+3y^2-4z^2)}{24} + \frac{\sqrt{6}Q_{xy}yz(3x^2+3y^2-4z^2)}{24} + \frac{\sqrt{6}Q_{xz}(5x^4-21x^2y^2-9x^2z^2+2y^4+9y^2z^2)}{24} + \frac{\sqrt{6}Q_{yz}xy(11x^2-17y^2+18z^2)}{24}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E'), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E')$$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$\frac{\sqrt{6}Q_u(x-y)(x+y)}{4} + \frac{\sqrt{2}Q_v(x^2+y^2-2z^2)}{4}$$

$$-\frac{\sqrt{6}Q_{u,xy}}{2} - \frac{\sqrt{2}Q_{xy}(x^2+y^2-2z^2)}{4}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E'), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E')$$

\*\* symmetry

$$\sqrt{15}xyz$$

$$\frac{\sqrt{15}z(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{\sqrt{3}Q_u(x-y)(x+y)(x^2+y^2-6z^2)}{6} - \frac{Q_v(x^4-12x^2y^2+6x^2z^2+y^4+6y^2z^2-2z^4)}{6} - \frac{7Q_{xy}xy(x-y)(x+y)}{6} + \frac{7Q_{xz}xz(x-z)(x+z)}{6} - \frac{7Q_{yz}yz(y-z)(y+z)}{6}$$

$$\frac{\sqrt{3}Q_{u,xy}(x^2+y^2-6z^2)}{3} + \frac{7Q_{v,xy}(x-y)(x+y)}{6} - \frac{Q_{xy}(5x^4-18x^2y^2-12x^2z^2+5y^4-12y^2z^2+4z^4)}{12} - \frac{7Q_{xz}yz(3x^2+y^2-2z^2)}{12} - \frac{7Q_{yz}xz(x^2+3y^2-2z^2)}{12}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,-1)}[q](E''), \vec{\mathbb{G}}_{3,2}^{(2,-1)}[q](E'')$$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2+y^2-4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2+y^2-4z^2)}{4}$$

\*\* expression

$$-\frac{\sqrt{15}Q_{u,yz}}{5} + \frac{3\sqrt{5}Q_{v,yz}}{10} - \frac{3\sqrt{5}Q_{xy,xz}}{10} + \frac{3\sqrt{5}Q_{xz,xy}}{10} - \frac{\sqrt{5}Q_{yz}(5x^2-y^2-4z^2)}{20}$$

$$\frac{\sqrt{15}Q_u xz}{5} + \frac{3\sqrt{5}Q_v xz}{10} + \frac{3\sqrt{5}Q_{xy}yz}{10} - \frac{\sqrt{5}Q_{xz}(x^2 - 5y^2 + 4z^2)}{20} - \frac{3\sqrt{5}Q_{yz}xy}{10}$$

$$\vec{\mathbb{G}}_{3,1}^{(2,1)}[q](E''), \vec{\mathbb{G}}_{3,2}^{(2,1)}[q](E'')$$

\*\* symmetry

$$-\frac{\sqrt{6}x(x^2 + y^2 - 4z^2)}{4}$$

$$-\frac{\sqrt{6}y(x^2 + y^2 - 4z^2)}{4}$$

\*\* expression

$$\frac{\sqrt{30}Q_{uyz}(3x^2 + 3y^2 - 4z^2)}{24} + \frac{\sqrt{10}Q_{vyz}(27x^2 - y^2 - 8z^2)}{24} - \frac{\sqrt{10}Q_{xyxz}(13x^2 - 15y^2 - 8z^2)}{24}$$

$$- \frac{\sqrt{10}Q_{xzy}(x^2 + y^2 - 6z^2)}{24} + \frac{\sqrt{10}Q_{yz}(2x^4 + 3x^2y^2 - 15x^2z^2 + y^4 - 9y^2z^2 + 4z^4)}{24}$$

$$- \frac{\sqrt{30}Q_{uxz}(3x^2 + 3y^2 - 4z^2)}{24} - \frac{\sqrt{10}Q_vxz(x^2 - 27y^2 + 8z^2)}{24} - \frac{\sqrt{10}Q_{xyyz}(15x^2 - 13y^2 + 8z^2)}{24}$$

$$- \frac{\sqrt{10}Q_{xz}(x^4 + 3x^2y^2 - 9x^2z^2 + 2y^4 - 15y^2z^2 + 4z^4)}{24} + \frac{\sqrt{10}Q_{yzxy}(x^2 + y^2 - 6z^2)}{24}$$

\* Harmonics for rank 4

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A'_1)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$\frac{3\sqrt{2}Q_{ux}(x^2 - 3y^2)}{8} + \frac{\sqrt{6}Q_vx(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}Q_{xyy}(x^2 + y^2 - 4z^2)}{8} + \frac{\sqrt{6}Q_{xzz}(x - y)(x + y)}{8} - \frac{\sqrt{6}Q_{yzxyz}}{4}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A'_1)$$

\*\* symmetry

$$\frac{\sqrt{70}yz(3x^2 - y^2)}{4}$$

\*\* expression

$$-\frac{3\sqrt{77}Q_{ux}(x^2 - 3y^2)(x^2 + y^2 - 8z^2)}{88} - \frac{\sqrt{231}Q_vx(x^4 - 16x^2y^2 + 6x^2z^2 + 7y^4 + 6y^2z^2 - 4z^4)}{88}$$

$$- \frac{\sqrt{231}Q_{xyy}(4x^4 - 7x^2y^2 - 3x^2z^2 + y^4 - 3y^2z^2 + 2z^4)}{44} + \frac{\sqrt{231}Q_{xzz}(4x^4 - 9x^2y^2 - 5x^2z^2 - y^4 + 5y^2z^2)}{44} + \frac{\sqrt{231}Q_{yzxyz}(x^2 - 11y^2 + 10z^2)}{44}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A'_1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$-\frac{\sqrt{105}Q_vxyz}{7} + \frac{\sqrt{105}Q_{xyz}(x - y)(x + y)}{14} - \frac{\sqrt{105}Q_{xzy}(x^2 + y^2 - 4z^2)}{28} + \frac{\sqrt{105}Q_{yzx}(x^2 + y^2 - 4z^2)}{28}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A'_1)$$

\*\* symmetry

$$\frac{3x^4}{8} + \frac{3x^2y^2}{4} - 3x^2z^2 + \frac{3y^4}{8} - 3y^2z^2 + z^4$$

\*\* expression

$$-\frac{7\sqrt{330}Q_vxyz(x^2 + y^2 - 2z^2)}{44} + \frac{7\sqrt{330}Q_{xyz}(x - y)(x + y)(x^2 + y^2 - 2z^2)}{88}$$

$$+ \frac{\sqrt{330}Q_{xzy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} - \frac{\sqrt{330}Q_{yzx}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88}$$

$$\vec{\mathbb{G}}_4^{(2,-1)}[q](A'_2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$-\frac{3\sqrt{2}Q_{uy}(3x^2 - y^2)}{8} - \frac{\sqrt{6}Q_{vy}(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}Q_{xyx}(x^2 + y^2 - 4z^2)}{8} - \frac{\sqrt{6}Q_{xzx}xyz}{4} - \frac{\sqrt{6}Q_{yzx}(x - y)(x + y)}{8}$$

$$\vec{\mathbb{G}}_4^{(2,1)}[q](A'_2)$$

\*\* symmetry

$$\frac{\sqrt{70}xz(x^2 - 3y^2)}{4}$$

\*\* expression

$$\frac{3\sqrt{77}Q_{uy}(3x^2 - y^2)(x^2 + y^2 - 8z^2)}{88} + \frac{\sqrt{231}Q_{vy}(7x^4 - 16x^2y^2 + 6x^2z^2 + y^4 + 6y^2z^2 - 4z^4)}{88} \\ - \frac{\sqrt{231}Q_{xyx}(x^4 - 7x^2y^2 - 3x^2z^2 + 4y^4 - 3y^2z^2 + 2z^4)}{44} - \frac{\sqrt{231}Q_{xzx}xyz(11x^2 - y^2 - 10z^2)}{44} - \frac{\sqrt{231}Q_{yzx}(x^4 + 9x^2y^2 - 5x^2z^2 - 4y^4 + 5y^2z^2)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E'), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E')$$

\*\* symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$-\frac{3\sqrt{14}Q_{ux}(x^2 + y^2 - 4z^2)}{56} - \frac{\sqrt{42}Q_{vx}(x^2 + 5y^2 - 8z^2)}{56} + \frac{\sqrt{42}Q_{xyy}(x^2 - 3y^2 + 8z^2)}{56} - \frac{\sqrt{42}Q_{xzx}(x^2 - 13y^2 + 4z^2)}{56} - \frac{\sqrt{42}Q_{yzx}xyz}{4} \\ - \frac{3\sqrt{14}Q_{uy}(x^2 + y^2 - 4z^2)}{56} + \frac{\sqrt{42}Q_{vy}(5x^2 + y^2 - 8z^2)}{56} - \frac{\sqrt{42}Q_{xyx}(3x^2 - y^2 - 8z^2)}{56} - \frac{\sqrt{42}Q_{xzx}xyz}{4} + \frac{\sqrt{42}Q_{yzx}(13x^2 - y^2 - 4z^2)}{56}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E'), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E')$$

\*\* symmetry

$$-\frac{\sqrt{10}yz(3x^2 + 3y^2 - 4z^2)}{4}$$

$$\frac{\sqrt{10}xz(3x^2 + 3y^2 - 4z^2)}{4}$$

\*\* expression

$$\frac{3\sqrt{11}Q_{ux}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} + \frac{\sqrt{33}Q_{vx}(x^4 - 12x^2y^2 + 2x^2z^2 - 13y^4 + 114y^2z^2 - 20z^4)}{88} \\ + \frac{\sqrt{33}Q_{xyy}(4x^4 + x^2y^2 - 27x^2z^2 - 3y^4 + 29y^2z^2 - 10z^4)}{44} \\ - \frac{\sqrt{33}Q_{xzx}(4x^4 + 15x^2y^2 - 13x^2z^2 + 11y^4 - 27y^2z^2 + 4z^4)}{44} + \frac{7\sqrt{33}Q_{yzx}xyz(x^2 + y^2 - 2z^2)}{44} \\ \frac{3\sqrt{11}Q_{uy}(x^4 + 2x^2y^2 - 12x^2z^2 + y^4 - 12y^2z^2 + 8z^4)}{88} + \frac{\sqrt{33}Q_{vy}(13x^4 + 12x^2y^2 - 114x^2z^2 - y^4 - 2y^2z^2 + 20z^4)}{88} \\ - \frac{\sqrt{33}Q_{xyx}(3x^4 - x^2y^2 - 29x^2z^2 - 4y^4 + 27y^2z^2 + 10z^4)}{44} + \frac{7\sqrt{33}Q_{xzx}xyz(x^2 + y^2 - 2z^2)}{44} \\ - \frac{\sqrt{33}Q_{yzx}(11x^4 + 15x^2y^2 - 27x^2z^2 + 4y^4 - 13y^2z^2 + 4z^4)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E'', 1)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2 - 2xy - y^2)(x^2 + 2xy - y^2)}{8}$$

$$\frac{\sqrt{35}xy(x - y)(x + y)}{2}$$

\*\* expression

$$\sqrt{3}Q_{vxyz} + \frac{\sqrt{3}Q_{xyz}(x - y)(x + y)}{2} - \frac{\sqrt{3}Q_{xzy}(3x^2 - y^2)}{4} - \frac{\sqrt{3}Q_{yzx}(x^2 - 3y^2)}{4} \\ - \frac{\sqrt{3}Q_{vz}(x - y)(x + y)}{2} + \sqrt{3}Q_{xyxyz} + \frac{\sqrt{3}Q_{xzx}(x^2 - 3y^2)}{4} - \frac{\sqrt{3}Q_{yzx}(3x^2 - y^2)}{4}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,-1)}[q](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,-1)}[q](E'', 2)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$-\frac{6\sqrt{7}Q_uxyz}{7} - \frac{\sqrt{21}Q_{xy}z(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{21}Q_{xz}y(2x^2-y^2+z^2)}{14} - \frac{\sqrt{21}Q_{yz}x(x^2-2y^2-z^2)}{14}$$

$$-\frac{3\sqrt{7}Q_uz(x-y)(x+y)}{7} - \frac{\sqrt{21}Q_vz(3x^2+3y^2-2z^2)}{14} + \frac{\sqrt{21}Q_{xz}x(x^2-5y^2+2z^2)}{28} + \frac{\sqrt{21}Q_{yz}y(5x^2-y^2-2z^2)}{28}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E'', 1), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E'', 1)$$

\*\* symmetry

$$\frac{\sqrt{35}(x^2-2xy-y^2)(x^2+2xy-y^2)}{8}$$

$$\frac{\sqrt{35}xy(x-y)(x+y)}{2}$$

\*\* expression

$$-\frac{9\sqrt{154}Q_uxyz(x-y)(x+y)}{22} + \frac{\sqrt{462}Q_vxyz(x^2+y^2-2z^2)}{44} + \frac{\sqrt{462}Q_{xy}z(x-y)(x+y)(x^2+y^2-2z^2)}{88}$$

$$-\frac{\sqrt{462}Q_{xz}y(9x^4-14x^2y^2-12x^2z^2+y^4+4y^2z^2)}{88} + \frac{\sqrt{462}Q_{yz}x(x^4-14x^2y^2+4x^2z^2+9y^4-12y^2z^2)}{88}$$

$$\frac{9\sqrt{154}Q_uz(x^2-2xy-y^2)(x^2+2xy-y^2)}{88} - \frac{\sqrt{462}Q_vz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{462}Q_{xy}xyz(x^2+y^2-2z^2)}{44}$$

$$+ \frac{\sqrt{462}Q_{xz}x(x^4-8x^2y^2-2x^2z^2+3y^4+6y^2z^2)}{44} + \frac{\sqrt{462}Q_{yz}y(3x^4-8x^2y^2+6x^2z^2+y^4-2y^2z^2)}{44}$$

$$\vec{\mathbb{G}}_{4,1}^{(2,1)}[q](E'', 2), \vec{\mathbb{G}}_{4,2}^{(2,1)}[q](E'', 2)$$

\*\* symmetry

$$-\frac{\sqrt{5}(x-y)(x+y)(x^2+y^2-6z^2)}{4}$$

$$\frac{\sqrt{5}xy(x^2+y^2-6z^2)}{2}$$

\*\* expression

$$\frac{21\sqrt{22}Q_uxyz(x^2+y^2-2z^2)}{44} + \frac{21\sqrt{66}Q_vxyz(x-y)(x+y)}{44} - \frac{\sqrt{66}Q_{xy}z(9x^4-24x^2y^2-10x^2z^2+9y^4-10y^2z^2+2z^4)}{44}$$

$$+ \frac{\sqrt{66}Q_{xz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} + \frac{\sqrt{66}Q_{yz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$

$$\frac{21\sqrt{22}Q_uz(x-y)(x+y)(x^2+y^2-2z^2)}{88} + \frac{\sqrt{66}Q_vz(3x^4-78x^2y^2+20x^2z^2+3y^4+20y^2z^2-4z^4)}{88} + \frac{21\sqrt{66}Q_{xy}xyz(x-y)(x+y)}{44}$$

$$+ \frac{\sqrt{66}Q_{xz}x(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44} - \frac{\sqrt{66}Q_{yz}y(x^4+2x^2y^2-12x^2z^2+y^4-12y^2z^2+8z^4)}{44}$$